Setting weights in multidimensional indices of well-being*

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Abstract

Multidimensional indices of well-being and deprivation have become increasingly popular, both in the theoretical and in the policy-oriented literature. By now, there is a wide range of methods to construct multidimensional well-being indices, differing in the way they transform, aggregate and weight the relevant dimensions. We use a unifying framework that allows us to compare the different approaches and to analyze the specific role of the dimension weights in each of them. In interplay with the choices on the transformation and aggregation, the weights play a crucial role in determining the trade-offs between the dimensions. Setting weights is hence inherently a delicate matter, reflecting important value judgements about the exact notion of well-being. From this perspective, we critically survey six methods that are proposed in the literature to set the weights.

Keywords: Weights, Multidimensional Well-being index, Multidimensional Poverty index

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1 Introduction

The notion that well-being is inherently multidimensioned has by now become well-established in the theoretical and policy-oriented literature. The following three examples will illustrate. First, rooted in a tradition going back to Aristotle, philosophers such as Rawls (1971), Dworkin (2000), Sen (1985) or Nussbaum (2000) have advocated a multidimensional perspective on the good life and well-being, exposing the deficiencies of a sole focus on income as indicator of well-being. Second, in the rapidly emerging literature on the determinants of happiness and life-satisfaction in psychology and economics there exists by now a broad consensus that people's happiness is affected by many aspects of life such as their health, employment, marital status, and material resources.[1] Third, in a large survey the World Bank collected the voices of more than 60,000 poor women and men from 60 countries, to understand poverty from the perspective of the poor themselves. One of the main conclusions of the survey is that for the poor, well-being and deprivation are multidimensional with both material and psychological dimensions (Narayan 2000).

When it comes to operationalizing the multidimensional approaches, one quickly runs into the crucial problem of how to describe individuals' multidimensional well-being by one single index. This is the so-called *indexing problem* (Rawls 1971, p. 80). In the literature this problem has been taken up in two different branches from a slightly different and complementary perspective. First, from an operational perspective rooted in measurement theory, the approach has been to provide clear and relatively simple guidelines on how to construct composite or social indicators in many fields, including well-being. The most popular example of such a well-being composite indicator is probably the Human Development Index (HDI), used to compare the performance of countries in terms of their combined achievements in income –as command over resources–, health and education.[2] Second, from a social choice perspective, the focus has been on defining measures of multidimensional welfare, inequality or poverty with an emphasis on the measurability and comparability of the different dimensions, and on desirable properties of obtained indices.[3] The results of this second strand are

[1] For an overview of this booming literature on happiness, see Kahneman and Krueger (2006). In his paper, Schokkaert (2007) deals with the links between the happiness and the capability literature, proposed by Sen (1985) and Nussbaum (2000).

[2] Many international institutions are active in using and promoting the use of composite indicators - see Nardo *et al.* (2005) for a survey of some alternative composite indicators. To give one example, the reader is referred to the detailed information server on composite indicators hosted by the European Commission on http://composite-indicators.jrc.ec.europa.eu/.

[3] See Weymark (2006) or Maasoumi (1999) for an overview of the literature on multidimensional inequality and Bourguignon and Chakravarty (2003) for a survey of the multidimensional poverty or deprivation literature.

mainly theoretical, although they are increasingly empirically applied.[4] In this paper, we survey the different approaches to deal with the indexing problem, particularly focussing on the question how to weight the different dimensions of well-being. Thereby we combine the insights from both the theoretical and operational branches of the literature.

To order and compare the plethora of existing well-being indices from the literature, the paper starts by proposing a unifying framework in section 2. This framework reduces the differences between the well-being indices to differences in the chosen transformations, the aggregation function and the weighting of the dimensions. The focus of this paper is on the weights. In section 3 we analyze the meaning of the weights within the proposed unifying framework. Together with the choices about transformation and aggregation, the weights will be shown to play a crucial role in the imposed trade-offs between the dimensions. Inescapably, the weights reflect important value judgements about the (vague) notion of well-being. Researchers should therefore be as clear as possible about how the weights are set. As Anand and Sen argued:

> "Since any choice of weights should be open to questioning and debating in public discussions, it is crucial that the judgments that are implicit in such weighting be made as clear and comprehensible as possible and thus be open to public scrutiny" (Anand & Sen 1997, p. 6)

Section 4 critically surveys six proposed methods to set the weights in multidimensional measures of well-being: equal weighting, frequency based weighting, most favorable weighting, multivariate statistical weighting, regression based weighting and normative weighting. We argue that whether the weights are set reasonably should be judged upon the acceptability of the implicitly imposed trade-offs by them. Section 5 concludes.

2 A unifying framework

Let us assume that agreement has been reached on which q dimensions or domains of well-being are relevant, and moreover that the individual achievement for all these dimensions can be measured in a interpersonal comparable way. Let x_j denote the achievement or outcome of an individual (or country) on dimension $j = 1, ..., q$, and let the well-being bundle $X = (x_1, ..., x_q) \in \mathbb{R}^q_{++}$ summarize these achievements across all dimensions.

The indexing problem can be summarized as the search for an appropriate *well-being index* I, that maps the well-being bundle on the real line, so that it can be naturally ordered and hence can be used to assess whether

[4] See Justino (2005) for an overview.

one individual is better, worse or similar to another one, and by how much. Inescapable, the choice of a specific well-being index entails important value judgements about the meaning of well-being. In the present paper we confine ourselves to the following wide class of well-being indices:[5]

$$I(X|\beta) = \frac{\left[w_1 I_1(x_1)^\beta + ... + w_q I_q(x_q)^\beta\right]^{1/\beta}}{w_1 + ... + w_q}. \quad (1)$$

The individual well-being index $I(X|\beta)$ is defined as a weighted mean of order β of the transformed achievements $I_j(x_j)$. The dimension-weights $w_1, \ldots w_q$ are all non-negative, and are often assumed to sum up to one so that the denominator of expression (1) drops out. The interpretation of these weights and how to set them, is the topic of this paper. Before turning to the weights, though, we discuss briefly the other two components of the well-being index, that is, the transformation functions $I_j(.)$ and the parameter β.

Appropriate transformation functions for well-being indices should satisfy at least two criteria. First, since the achievements x_j are often measured in different measurement units –such as income in pounds or euros, health in years or in an ordered scale–, they need to be transformed or standardized to a common basis before they can be sensibly aggregated. Transformation functions typically make the achievements scale independent. Second, the transformation functions should avoid that excessive relative importance is given to outliers or extreme values if the original distribution is skewed.[6]

Expression (1) can also be used to construct an index of multidimensional poverty or deprivation. In this setting, the transformation function $I_j(.)$ transforms the achievement in dimension j into the shortfall or poverty in that dimension. These transformation functions typically request also a dimension-specific poverty-line z_j to be defined.

Table 1 in the Appendix surveys some widely used transformation functions in the literature. In this paper, we do not prioritize one transformation method over another, but limit ourselves to presenting them while high-

[5]Blackorby and Donaldson (1982) provide an axiomatic characterization of the weighted mean of order β. In the literature on multidimensional inequality, Maasoumi (1986) provides an information-theoretic justification of this class of well-being indices. Further, it belongs to the wider class of well-being indices proposed by Bourguignon (1999). Foster *et al* (2005) propose a similar formulation for a distribution-sensitive measure of human development. Decancq and Lugo (2008) axiomatize it as part of a multidimensional Gini measure. Recently, Decancq et al. (2007) have used it to analyze the trend in multidimensional global inequality. Furthermore, in the related literature on the measurement of multidimensional poverty and deprivation, this class of indices has been suggested by Anand and Sen (1997) and is a special case of the class proposed by Bourguignon and Chakravarty (2003).

[6]Noble *et al* (2006, 2008) include also two other criteria for standardization: first it should imply an appropriate degree of substitutability or cancelation, second it should facilitate the identification of the most deprived.

lighting the crucial role they play on the interpretation of relative weights -as shown in the next section. We refer the interested reader to Jacobs *et al.* (2004) and Nardo *et al.* (2005) for an extensive survey of the alternative transformation methods and their properties. In general, we see that transformation functions used to construct a well-being index are increasing, whereas the transformation functions used to construct an index of deprivation are decreasing in the achievements.

The parameter β equals $1 - 1/\sigma$, where σ is the elasticity of substitution. In other words, β captures the degree of substitutability between the transformed achievements. The smaller the β, the smaller the allowed substitutability between dimensions. For $\beta = 1$, the weighted mean of order β is reduced to the standard weighted arithmetic mean where the dimensions are perfect substitutes,

$$I(X|1) = \frac{w_1 I_1(x_1) + \cdots + w_q I_q(x_q)}{w_1 + \cdots + w_q}. \tag{2}$$

Due to its simplicity and clarity of procedure, expression (2) is used frequently to construct composite indices.[7] However, the consequence of setting $\beta = 1$ might not always be desirable. Especially not in the light of measuring well-being or human development. For instance, in the Human Development Report (2005) we read:

> "Losses in human welfare linked to life expectancy, for example, cannot be compensated for by gains in other areas such as income or education." UNDP Report (2005)

At the same time, the leading index of the Human Development Report – the Human Development Index– makes use of a linear aggregation assuming *perfect* substitutability between the transformed achievements.

It should be noted, though, that other equally simple choices are also available which are worth exploring here. For instance, when $\beta = 0$, the well-being index becomes the geometric mean,

$$I(X|0) = I_1(x_1)^{w_1/(w_1+\cdots+w_q)} * \cdots * I_q(x_q)^{w_q/(w_1+\cdots+w_q)}. \tag{3}$$

In this case, the well-being index has unit elasticity of substitution between all pairs of dimensions, which means that a one percent decrease in one of the dimensions can be compensated by a one percent increase in another dimension.[8] In general, for $\beta \leq 1$ the well-being index is weakly convex, which reflects a preference for well-being bundles that are more equally

[7] See Jacobs *et al.* (2004) and Nardo *et al.* (2005).

[8] A potential problem of setting $\beta = 0$ is that when a person has no achievement in one of the (transformed) dimensions the overall well-being index will be insensitive to the achievements in the other dimensions.

distributed. When expression (1) is used to summarize multidimensional poverty or deprivation, a $\beta \geq 1$ seems more appropriate.

If β goes to $-\infty(+\infty)$ the elasticity of substitution becomes 0, and the well-being index becomes the minimum (maximum) of the transformed achievements across the dimensions,

$$I(X|-\infty) = min[I_1(x_1), ..., I_q(x_q)]. \tag{4}$$

In this extreme case, there is no substitution between dimensions possible, which seems to reflect better the philosophy of the above quote.[9]

The choice of the substitutability parameter β is intimately linked to the choice of the transformation function $I_j(.)$. In an interesting paper, Ebert and Welsch (2004) investigate to what extent the ordering of the well-being bundles is invariant to the choice of the specific transformation function. Building on results from social choice theory, they conclude that the multiplicative aggregation ($\beta = 0$) is the only aggregation form that makes the ordering of the well-being bundles robust to the choice of a *dimension-specific* transformation of the rescaling type (row 4 in table 1). Since well-being indices typically aggregate very different dimensions, a dimension-specific transformation is most often needed. On the other hand, the aggregation used in expression (1) is robust for transformation of the rescaling type that involve the *same rescaling across all dimensions.* (See Ebert and Welsch (2004) and the reference therein for more details.) In other words, apart from some very restricted choices for β and the transformation functions $I_j(.)$, the decision which transformation function to use, typically affects the ordering of the bundles and therefore that decision should be handled with care and preferably within a theoretic framework about the true meaning of "well-being".

In sum, the framework proposed reduces the decisions to be made to three: the value for parameter β, the transformation functions $I_1(.), \dots, I_q(.)$, the weights $w_1, \dots, w_q$. Table 2 gives an overview of the common choices made in the literature with respect to these decisions.[10] These choices reflect alternative viewpoints on the meaning of the notion "well-being" and will potentially have a non-trivial impact on the resulting ordering of bundles. A striking example can be found in the work by Becker, *et al.* (1987). The

[9]For simplicity, we assume in this paper that the degree substitutability between dimensions is constant. Nonetheless, this might not be always a sensible assumption to make. One alternative is to use a nested approach where, first, several subsets of dimensions are aggregated using expression (1) where each subset has a different β and, second, these subsets are combined using again the same expression. Another alternative is to allow the substitutability parameter β to be a function of the achievements, as in Bourguignon and Chakravarty (2003).

[10]In the table we include indices that are widely used in practice, such as the HDI, and studies that provide empirical applications -that is, the table does not include papers that are solely theoretical.

authors studied the quality of life in 329 metropolitan areas of the U.S. by ordering them according to standard variables such as quality of climate, health, security, economical performance. The authors find that, depending on the weighting scheme chosen, there were 134 cities that could be ranked first, and 150 cities that could be rank last. Moreover, there were 59 cities that could be rated either first or last, using the same data, but by selecting alternative weighting schemes. Based on this example, Diener and Suh (1997) conclude that a procedure for resolving how to weight the dimensions is lacking. In the next section we go deeper into the meaning of the weights. Before we will do so, we introduce another example of how changing one of the parameters in expression (1), can lead to different orderings. We will use this stylized example throughout the rest of the paper.

We compare the well-being of two persons –Ann and Bob– in two dimensions –income and health–, denoted y and h with the former being measured in dollars and the latter in years of life expectancy. Ann is healthier than Bob, her life expectancy is 90 years whereas his is only 50 years. But Bob is richer; he has an income of 2,000 dollar, whereas she earns only 1,000 dollar. We use expression (1) to evaluate who who is better off of the two. Figures 1 to 3 depict the position of Ann and Bob in the income-health space, and the iso-well-being curves connect all points leading to the same level of the well-being index, for different definitions of the index.

Let us first look at a benchmark case with equal dimension weights $w_y = w_h = \frac{1}{2}$, where $\beta = 1$, and where the transformations are a rescaling by the median achievement (x_j/Me_j for $j = y, h$), which is 2,500 dollars of income and a life expectancy of 80 years. The dotted lines represents the iso-well-being curves for the benchmark case. In figure 1 we see that Ann's bundle is to the right of Bob's iso-well-being curve, so using the benchmark well-being index Ann is clearly better off than Bob. Let us now look at three alternative parameter choices. First, we increase the relative weight assigned to income so that $w_y = \frac{3}{4}$ and $w_h = \frac{1}{4}$. The corresponding iso-well-being curves are represented in figure 1 by the solid lines. The iso-well-being curves are steeper than the benchmark case and Bob is now considered to be better off than Ann, hence reversing the ordering.

In the second case –figure 2– we change the transformation function used. Let us assume that the achievements of other individuals in the society have deteriorated, leading to a drop of the median achievement to an income of 1,000 dollars and a life expectancy of 60 years. In the new situation (solid line in figure 2, once more Bob turns out to be better off than Ann.

Finally, in figure 3, we decrease the degree of substitutability between dimensions, from $\beta = 1$ to $\beta = 0.1$. The previously linear iso-well-being curves become now convex. Once more, Bob turns out to be better off than Ann.

These stylized examples illustrate that the ordering of the well-being

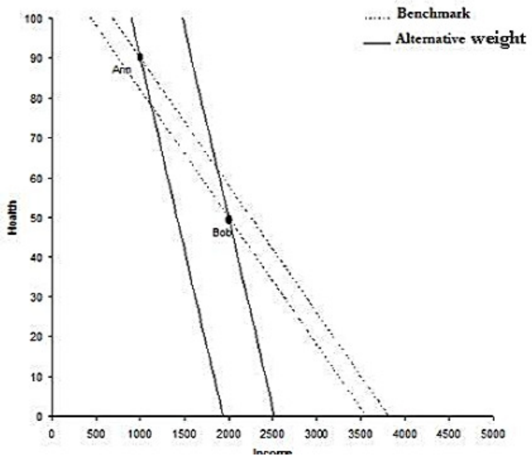

Figure 1: Iso- well-being curves. Alternative dimension-weights.

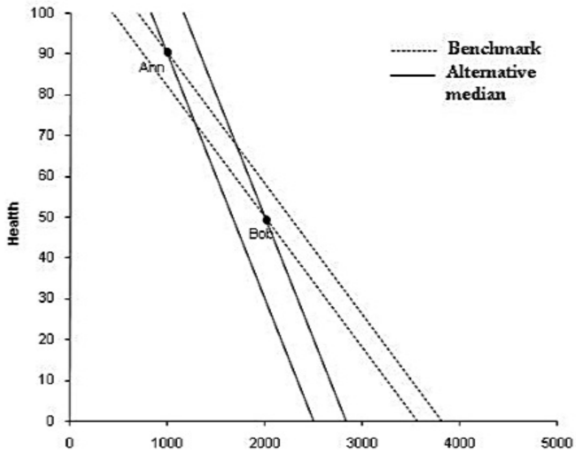

Figure 2: Iso- well-being curves. Alternative transformation (median).

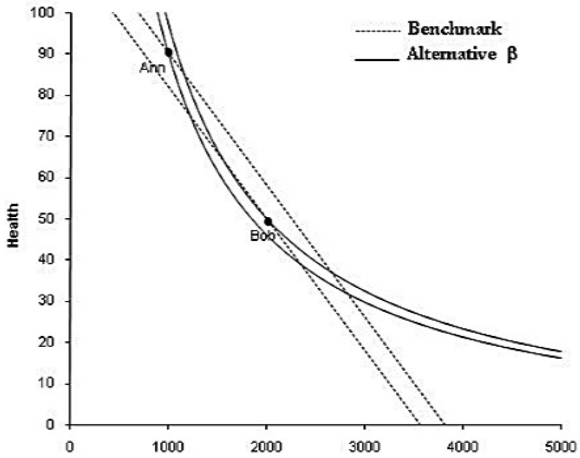

Figure 3: Iso- well-being curves. Alternative β.

bundles can be very sensitive to the choice of the parameters. The lesson is that care should be taken when deciding about the parameters. In the following section we go deeper into the meaning of the parameters, in general, and of the weights, in particular.

3 What do weights mean?

A straightforward way to look at the meaning of the weights within the framework of the previous section, is to study some of the properties of the well-being index in terms how it reacts to changes in the parameters – weights, in particular– and the achievements in the different dimensions. We do this by analyzing the partial derivatives and the corresponding marginal rate of substitution, and by looking specifically at the role played therein by the weights. (See also Anand and Sen (1997) for a similar approach).

Let us start by looking at the derivative of the well-being index $I(.)$ with respect the weight of dimension j. This will tell us how the well-being index reacts to small changes in w_j, while keeping all other parameters and achievements constant.

Proposition 1. *For all well-being indices defined by expression (1), it holds that:*

$$\frac{\partial I(X|\beta)}{\partial w_j} = \frac{\left[I_j(x_j)^\beta - I(X)^\beta\right]}{\beta\left[w_1 + ... + w_q\right] I(X)^{\beta-1}}. \tag{5}$$

Proof. The proof uses some straightforward algebraic manipulations and is extended to the appendix, together with all other proofs of this section. □

Irrespective of the selected β, an increase in the weight of dimension j leads to an increase the well-being index if the transformed achievement in dimension j is larger than the total well-being, in other words if the individual is performing relatively well in dimension j. Indeed, it is intuitive that increasing the weight of a dimension on which the individual performs well, leads to an increase in well-being, whereas increasing the weight of a dimension on which the individual performs relatively weak, leads to a decrease in her well-being.

Expression (5) also provides interesting insights, to the cases when the obtained well-being index $I(.)$ is *in*sensitive to the exact choice of the weight w_j. For instance, the well-being index is insensitive to changing the weighting scheme, when $I_j(x_j)^\beta = I(X)^\beta$ for all$j$, in other words when the transformed achievements are very alike across dimensions, or when there is a lot of correlation between the dimensions.

Proposition 2. *For all well-being indices defined by expression (2), it holds that:*

$$\frac{\partial I(X|1)}{\partial w_j} = \frac{I_j(x_j) - I(X)}{w_1 + ... + w_q}. \tag{6}$$

If the well-being index is defined as a weighted mean, $\beta = 1$, the impact of increasing the weight of dimension j equals the ratio between the difference between the transformed achievement in dimension j and total well-being, on the one hand, and the sum of the weights, on the other. Again, an increase in the weight of dimension j leads to an increase the well-being index if the transformed achievement in dimension j is larger than the total well-being $I(X)$.

Intuitively, one expects changes in dimensions with a higher weight to have more impact on total well-being, than dimensions with a lower weight. In a recent paper, Chowdhury and Squire (2006) write:

> “The ideal approach would presumably involve using as weights the impact of each component on the ultimate objective ...”. Chowdhury and Squire (2006, p. 762)

Therefore, we investigate the first derivative of the well-being index with respect the achievement in dimension j itself. This derivative captures how the well-being index of an individual reacts to small changes of her achievement in a given dimension keeping all parameters constant.

Proposition 3. *For all well-being indices defined by expression (1), it holds that:*

$$\frac{\partial I(X|\beta)}{\partial x_j} = \frac{w_j}{w_1 + ... + w_q} I'_j(x_j) \left[\frac{I_j(x_j)}{I(X)}\right]^{\beta-1}, \tag{7}$$

where $I'_j = \frac{\partial I_j(x_j)}{\partial x_j}$.

The impact of a small change of the achievements of dimension j on total well-being depends on three terms. The first term is the relative dimension-specific weight. As one might expect, the larger the relative weight of a certain dimension, the larger the impact of a small change in the achievement of that dimension. Secondly, the impact depends on the derivative of the transformation curve. The larger this derivative, in other words, the steeper the transformation curve, the larger the effect of a small increase in the achievement on the transformed achievement and hence on total well-being. For multidimensional well-being indices, the derivative tends to be positive, whereas it is negative for multidimensional poverty indices. Finally, the effect depends on the ratio $\frac{I_j(x_j)}{I(X)}$ to the power $\beta - 1$. For values of $\beta \leq 1$, if the person performs worse in that dimension than in the overall well-being, an increase in such dimension will have a large effect on overall well-being. Note the the parameter β offers an instrument to increase the relative impact of a dimension on total well-being. The lower the β the more sensitive the index is to weak performing dimensions. A policy maker seeking to maximize the well-being $I(X)$ will spend more effort on the relatively weak performing dimensions if $\beta \leq 1$, leading to a more equalized development across dimensions.

For the simple additive well-being indices ($\beta = 1$), the term between square brackets in expression (7) drops out, and the effect of a small change of one of the achievements only depends on its relative weight and the steepness of the transformation function.

Proposition 4. *For all well-being indices defined by expression (2), it holds that:*

$$\frac{\partial I(X|1)}{\partial x_j} = \frac{w_j I'_j(x_j)}{w_1 + ... + w_q}. \tag{8}$$

If, moreover, the transformation function is the identity function (so its derivative equals 1), the impact of a small change in achievement j is only determined by the relative weight of dimension j. Hence, for this specific choice of parameters, the relative weight of a dimension captures the impact of small change in the achievement of that dimension. The total well-being is indeed more sensitive to changes in a dimension with larger weight.

An alternative but related meaning of the weights is as substitution rates between two dimensions –i and j– denoted MRS_{ij} .[11] Let us reconsider the previous example of Ann and Bob where dimension i represents health and dimension j is income. The marginal rate of substitution between these

[11] See, for instance, Munda and Nardo, (2005).

dimensions is the amount of health an individual would like to gain if she were to sacrifice one unit of income, while maintaining the same level of well-being. In graphical terms, the MRS_{ij} reflects the slope of the iso-well-being curves and is formally defined as:

$$MRS_{ij} = -\frac{dx_i}{dx_j} = \frac{\partial I(X|\beta)}{\partial x_j} \Big/ \frac{\partial I(X|\beta)}{\partial x_i}. \tag{9}$$

By substituting expression (7) into (9) we obtain the following expression.

Proposition 5. *For all well-being indices defined by expression (1), it holds that:*

$$MRS_{ij} = \frac{w_j}{w_i} \frac{I_j'(x_j)}{I_i'(x_i)} \left[\frac{I_j(x_j)}{I_i(x_i)} \right]^{\beta-1}. \tag{10}$$

The marginal rate of substitution between dimension i and j also consists of three parts. We will relate each of these components to the cases illustrated in the previous section in figures 1 to 3. The first component is the ratio of the dimension-specific weights w_j/w_i. The larger the weight w_j the more the amount of x_i that the person needs to gain to compensate for the loss of one unit x_j. Going back to figure 1, the new (alternative) income weight is increased, leading to a larger ratio w_j/w_i, a larger MRS_{ij} and a steeper iso-well-being curve. The second part of expression 10 is the ratio of the derivatives of the transformation functions of dimension j and i. The steeper the transformation function of x_j –or equally, the flatter the transformation function of x_i– the larger the amount of dimension i necessary to compensate for the loss in x_j. In figure 2, the deteriorated medians of the society lead to a larger ratio $I_j'(x_j)/I_i'(x_i)$,[12] and hence to a steeper iso-well-being curve. Finally, the marginal rate of substitution depends on the ratio of the transformed achievements to the power $\beta - 1$. For $\beta < 1$, the amount of dimension i needed to compensate for the loss in dimension j is greater, the smaller the original achievement in dimension j. This makes sense; achievements are more valuable as they become more scarce. In figure 3 using the alternative iso-well-being curves, the poorer the person the steeper the iso-well-being curve becomes. Ann should be given more health than Bob to compensate for a unit decrease in income.

In the linear case ($\beta = 1$), the trade-off is assumed constant at all levels of achievements.

Proposition 6. *For all well-being indices defined by expression (2), it holds that:*

$$MRS_{ij} = \frac{w_j}{w_i} \frac{I_j'(x_j)}{I_i'(x_i)}. \tag{11}$$

[12] To be precise, the ratio raise from $80/2500 = 0.032$ to $60/1000 = 0.06$.

If, in addition, the ratio of the derivatives of the transformation functions is unity, the marginal rate of substitution between two dimensions is uniquely defined by their weights.

In short, the analysis of some properties of the general class of the well-being index proposed in expression (1), shows that in close interplay with the other parameters, the dimension-weights affect the impact of small changes in one dimension on total well-being. Moreover, they form part of the trade-off between dimensions but can be interpreted directly as *the* trade-offs only under certain assumptions –perfect substitutability between dimensions and no transformation of the original variables. In the next section we survey some procedures to set the weights from this perspective.

4 How can we set weights in a reasonable way?

In the previous section we concluded that whether the weights are set reasonably or not, can and should be evaluated based on the trade-offs they imply between the dimensions of well-being. In this section we survey from this perspective some of the most commonly used methods to set the weights in practice.[13]

4.1 Equal weights

The most commonly used approach to weighting in multidimensional indices of well-being has been equal weighting. Despite its popularity,[14] equal weighting is far from uncontroversial. Chowdhury and Squire refer to equal weighting as "obviously convenient but also universally considered to be wrong." (Chowdhury & Squire 2006, p. 762).

Equal weighting has often been defended from an *agnostic viewpoint*, by its simplicity or indeed from the recognition that all indicators are equally important. As an example of the agnostic viewpoint, Mayer and Jencks defend equal weighting by remarking that: "ideally we would have liked to weight ten hardships according to their relative importance in the eyes of legislators and the general public, but we have no reliable basis for doing this" (Mayer & Jencks 1989, p. 96).

[13]An alternative method, not reviewed here, would be to use market or personalized prices as weights, so that the well-being index (with identity transformations and $\beta = 1$) coincides with the individual's expenditures. Srinivasan (1994) advocates such an approach. However, as stated by Foster and Sen (1997), prices do not exist for many dimensions of well-being and are in general inappropriate for well-being comparisons, a task for which they are not constructed.

[14]Examples are the Human Development Index, the Human Poverty Indices, the Commitment to Development Index (Roodman 2007), the English Index of Local Conditions (Department of Environment, 1994), and the Townsend Material Deprivation Score (Townsend, Phillimore & Beattie 1988), among others.

However, there is a fallacy in setting the weights equally motivated from an agnostic viewpoint. As has been shown in the previous section, there is no escape from the fact that the weights reflect an important aspect of the trade-offs between the dimensions. As any other weighting scheme, the equal weighting scheme implies in interplay with choices about the transformation and substitutability specific trade-offs between the dimensions, that can and should be made explicit, and might be considered reasonable or not. In a paper on the HDI, Ravallion (1997) looks at the implied marginal rates of substitution in the HDI and finds that: "The HDIs implicit monetary valuation of an extra year of life rises from a remarkably low level in poor countries to a very high level in rich ones. In terms of both absolute dollar values and the rate of GDP growth needed to make up for lower longevity, the construction of the HDI assumes that life is far less valuable in poor countries than in rich ones; indeed, it would be nearly impossible for a rich country to make up for even one year less of life on average through economic growth, but relatively easy for a poor country" He concludes: "The value judgements underlying these trade-offs built into the HDI are not made explicit, and they are questionable." (Ravallion 1997, p. 633).[15]

In sum, researchers that would like to avoid the hazardous question of how to set the weights, and therefore chose for equal weighting, should be aware that the equal weighting scheme is actually a weighting scheme as any other without specific normative attractiveness, and just as any other weighting scheme it implies trade-offs that might be reasonable or not.

4.2 Data-driven weighting schemes

Many methods to obtain a reasonable weights, rely in some way or another on the data at hand to come to a weighting scheme. We compare four approaches and will criticize them on similar grounds.

4.2.1 Frequency-based weights

A second method to determine the weights, is to set them relative to the proportion of the population suffering deprivation in that dimension. Two different approaches can be found in the literature, taking quite opposite perspectives. First, in the context of multidimensional deprivation measurement, Desai and Shah (1988) and Cerioli and Zani (1990) argue that the smaller the proportion of individuals with a certain deprivation, the higher should be the weight, on the grounds that a hardship shared by few has more impact than one shared by many.[16] On the other hand, in their work on

[15]Decanq *et al.* (2007) make a similar point, based on the most recent calculation method of the Human Development Index.

[16]In a recent paper, Brandolini (2007) points out that when applying Desai and Shah's weighting formula to Italian data, he comes to a rather questionable and unbalanced weighting scheme.

well-being indices, Osberg and Sharpe (2002) make use of frequency-based weights when weighting the subcomponents of the risk dimension of well-being. However, they use a procedure that attributes a smaller weight to dimensions with a smaller proportion at risk.

A related way of setting the weights, is by setting the weights relative to the quality of the data. Jacobs et al. (2004) suggest to give less weight to those variables where data problems exist or with large amounts of missing values. The advantage is that the reliability of the well-being index can be improved by giving more weight to good quality data.

Apart from the apparent disagreement how the weights should depend on the relative proportions, the fundamental question seems to be *why* the weights and the implied trade-offs should depend on the relative proportion achieved by the population or on the data quality. For instance, should the substitution rate between literacy and unemployment of an individual depend on how many unemployed there are in the society he lives in, or how accurately unemployment is measured?

4.2.2 Most favorable weights

When applying the same weighting scheme to all individuals, some of them might feel that the evaluation of their well-being is submitted so someone else perspective on what well-being exactly is.[17] Therefore, a researcher might want to give all individuals the "benefit of the doubt" and select for each individual the most favorable weighting scheme. This method has originally be proposed for evaluating macro-economic performance (Melyn & Moesen 1991) and has recently been used in the construction of composite indicators. The weights are individual-specific and endogenously determined such that they maximize the obtained well-being of the individual.[18] The highest relative weights are given to those dimensions on which the individual performs best. To avoid that all weight is given to one dimension (the best dimension of the individual), extra constraints can be imposed upon the weights assuring that minimal weight is given to all dimensions of well-being.

[17] In recent social choice theory, the question of *whose* preferences or ideas about trade-offs should matter, has been taken up under the label of the "indexing dilemma", see Fleurbaey (2007). In his elegant paper, Fleurbaey investigates, loosely speaking, the apparent impossibility of finding a weighting scheme that is individual specific and at the same time not susceptible to the critiques formulated against welfarism, as they are formulated by Sen (1985). He argues in favor of a way out the impossibility based on an approach that takes information on the individual iso-well-being curves into account.

[18] In case $\beta = 1$, this problem reduces to linear programming problem, see Cherchye *et al.* (2006) for technical details. The authors provide an overview composite indicators that set the weights based on this most favorable weighting scheme, which is an application of so-called Data Envelopment Analysis. Despotis (2005) proposes a specific application to the Human Development Index, and Ramos and Silber (2005) compare the approach to alternative ones. See also Mahlberg and Obersteiner (2001).

Drawbacks of this approach are the following: First, since every individual has her own weighting scheme, the comparison of well-being levels across individuals is not straightforward. Second, the obtained results depend highly on the exact formulation of the technical constraints chosen by the analyst, making it a less transparent procedure. Finally, and most importantly, there is no guarantee that the most favorable weights lead to reasonable trade-offs between the dimensions. There seems to be no *a priori* reason, why a certain dimension on which the individual performs relatively well should have a larger impact on total well-being, *because* the individual performs well on that dimension.

4.2.3 Multivariate Statistical weights

There are two sets of techniques that are employed to choose weights for multidimensional indices: descriptive and explanatory models.[19]

The first approach, a descriptive one, relies on multivariate statistical techniques to set the weights that summarize the data. The most commonly used techniques are based on principal components (Klasen 2000, Noorbaksh 1998) and cluster analysis (Hirschberg, Maasoumi & Slottje 1991). The use of these statistical techniques is motivated by a concern for the so-called problem of *double counting*. In many empirical applications the dimensions of well-being are found to be strongly correlated.[20] Loosely speaking, most multivariate statistical techniques adjust for the correlation between indicators by either choosing the dimensions that are not correlated or by adjusting the weights, so that correlated dimensions get less weight (Nardo, Saisana, Saltelli & Taranto 2005). For instance, in principal component analysis, a given set of dimensions is transformed into an equal number of mutually uncorrelated linear combinations of dimensions. One can compute the proportion of the variance explained by each linear combination. In a small group of those linear combinations can explain a large proportion of the variance, then the information contained by the initial dimensions is largely contained in the small group of combinations that are by definition uncorrelated, which solves the double counting problem. The two most commonly used methods to obtain weights from the linear combinations, is to use either the principal component that explains the largest proportion

[19] For a detailed overview of the statistical properties of some methods to set the weights based on multivariate statistics, we refer the reader to (Krishnakumar & Nadar 2008).

[20] For instance, Srinivasan (1994) reports a correlation coefficient of about 0.8 between the dimensions of the Human Development Index. Whether double counting is really a problem, is open for discussion. One could argue that the correlation between the dimensions in a society reflects an important aspect of the real situation and as such it should be included, not eliminated from the analysis. The pluralistic egalitarian notion of Walzer (1981), for instance, considers that the correlation between the dimensions is one of the most essential characteristics of the society. From that perspective, correcting for correlation between the dimension might be completely inappropriate.

of the variance, or to use a weighted average of all the linear combinations, obtaining the weights by the proportion of the total variance explained by that linear combination.

The second approach, sometimes known as *latent variable models* is an explanatory approach that assumes that some observed variables (dimensions) are dependent on a certain number of unobserved latent variables (Krishnakumar & Nadar 2008). Factor analysis is possibly the simplest case of latent variable model, imposing that the observed dimensions are in fact different manifestations of the latent component, called factor. In the context of well-being and deprivation indices, factor analysis have been widely employed (Maasoumi & Nickelsburg 1988, Schokkaert & Van Ootegem 1990, Nolan & Whelan 1996, Noble, McLennan, Wilkinson, Whitworth, Barnes & Dibben 2008). More advanced latent models include other exogenous variables that also might influence the latent variable but are not part of the selected set of dimensions used to construct the index. In this line, Multiple Indicator and Multiple causes model (MIMIC) and structural equation model (SEM) have been proposed to construct multidimensional indices, particularly among those supporting the capability approach (Di Tommaso 2006, Kuklys 2005, Krishnakumar 2007, Krishnakumar & Ballon 2007).

There are, however, some drawbacks to these multivariate statistical approaches. First, the obtained linear combinations of dimensions might be hard to interpret as a facet of human well-being (Srinivasan 1994). Additionally, statistical approaches can lead to normatively inappropriate results. For instance, in the construction of the environmental sustainability index, the principal component method was found to assign negative weights to some sub-indicators (World Economic Forum, 2002). We obtain the same result when we apply this method to the example of Ann and Bob. [21] Brandolini (2007) warns the reader that "we should be cautious in entrusting a mathematical algorithm with a fundamentally normative task". Multivariate statistical techniques, especially principal component analysis, are developed to summarize the data in a *statistically* reasonable and parsimonious way. As such, they can be useful to aggregate indicators within dimensions. But this is quite a different task than looking for weights that are *normatively* reasonable.

4.2.4 Regression based weights

Another way to set the weights, also based on data, is to estimate the coefficients α_j of the following equation:

$$Y_i = \alpha_1 I_1(x_{1i}) + ... + \alpha_q I_q(x_{qi}), \tag{12}$$

[21] The weights w_h, w_y obtained by principal component of the transformed data (z-scores) are respectively -0.7071 and 0.7071.

where Y_i is some output variable capturing the well-being of individual i. This expression shows great similarity with the linear well-being index as defined in expression (2), with the role played by the coefficients α_j corresponding to the weights w_j. The only problem to operationalize this approach to find for every individual a reasonable Y_i, approximating her well-being.

In a recent paper, Schokkaert (2007) proposes to rely on the emerging measures of life satisfaction to obtain a proxy for individual well-being. He writes "On the one hand, the robust statistical relationship between functionings and life satisfaction may provide useful information on the relative weights to be given to the various dimensions in the calculation of individual living standards. On the other hand, from a non-welfarist point of view we do not want idiosyncratic individual factors to wipe out the effects of conditions of material deprivation, linked, for example, to unemployment or job satisfaction" (Schokkaert 2007, p.423). Schokkaert proposes an approach in which individual life satisfaction S_i is used as lefthand-side variable in expression (12), and where all idiosyncratic individual factors are set at their mean value for the population, so that they do not effect the obtained weights. His approach can be made more realistic, by allowing for non-linearities in expression (12) or by allowing the coefficients α_j to vary across different groups in the population.

In general the regression based weights have the drawback that if the well-being could be measured in an appropriate way by the single variable Y_i, there would be not need to construct a well-being index in the first place. Moreover, the coefficients of α_j might suffer from the problem of multicollinearity in case the dimensions of well-being as strongly correlated.

Data-driven approaches offer an interesting way to obtain weights for multidimensional well-being indices. There is an expanding literature proposing their use and perfecting the methods so that they are more than just data summarizing techniques. Two points of caution are in order. First, most of these methods generally assumed a linear form, hence $\beta = 1$. Some of the techniques could overcome this problem. Second, an inconvenient property of the weights obtained by statistical techniques is that are sensitive to adding new observations to the data-set (Nardo et al. 2005). In other words, given that weights are data-specific they can change from one point in time to the next, and from country to country, which makes any meaningful comparison of situations problematic.

4.3 Normative weights

From the previous section on the meaning of the weights we recall that the weights are crucial in determining the trade-offs between the dimensions of well-being. A third approach is to obtain more normatively inspired weights.

Unfortunately, there are very few guidelines in the ethical or philosophical literature on how the obtain reasonable trade-offs between dimensions of well-being. Fleurbaey (2008) states: "One can of course invoke the ethical preferences of the observer and ask her, for instance, how she trades the suicide rate off against the literacy rate, but there is little philosophical or economic theory that gives us clues about how to form such preferences." (Fleurbaey 2008, p. 21) .

One approach would be to ask all individuals in the society how they personally would trade-off the different dimensions, and then aggregate these opinions somehow. In practice, however, asking all individuals in a society might not be feasible, therefore one often relies on the preferences of a limited group of people that are thought to represent, to some extent, the rest of the society.[22] Generally, two sets of groups are considered: policy makers -usually deciding where and how to spend resources- and 'experts' from the academic and international organization communities.

In the literature, there exist some methods to elicit the preferred trade-offs between the dimensions of the (representative group of) individuals. A first method is to survey how the individuals or their representatives would trade off different dimensions of well-being. Similar approaches have been used in health economics to obtain an estimation how much health gain one is prepared to sacrifice for a reduction in health inequality (See for instance Shah et al. 2001 and Jacobs et al. 2004). A similar question could be asked about trade-offs between dimensions of well-being or deprivation. In an interesting paper, Chowdury and Squire (2006) use electronic surveys to elicit weighting schemes to assess whether the equal weighting scheme of the Human Development Index had support from the 'expert community', understood as development researchers throughout the world placed in academic institutions. Each person was asked to weight each component of the HDI from 0 to 10 in order of importance, and the average of these weighting schemes was considered. Interestingly enough, they find that the average weighting scheme does not statistically differ from the present equal weighting scheme.

A second and related method is to use budget allocation. The members of the representative group are asked to distribute a budget of points to a number of dimensions, paying more for those dimensions whose importance they want to stress. Moldan and Billharz (1991) report a case study in which 400 German experts were asked to allocate a budget to a set of environmental indicators related to air pollution, leading to very consistent results, where experts came form very different social backgrounds.

[22]However, public opinion polls have been used in problems of eliciting the public concerns about environmental issues. In that way the concern the public opinion attaches to the different environmental subindicators is determined. Parker (1991 p.95-98) advocates such an approach: "public opinion polls have been extensively employed for many years for many purposes, including the setting of weights".

A third method is the analytic hierarchy process. This has been proposed by Saaty (1987) originating from multi-attribute decision making. In this procedure, all members of the representative group are asked to compare pairwise the dimensions by asking the question: "Which of the two is the more important? - and by how much?". The strength of the preference is expressed on a semantic scale of 1-9. These comparisons result in a comparison matrix from which the relative weights can be calculated using an eigenvector technique (see Nardo et al (2005) and the references therein for a detailed treatment).

Although these methods have there own disadvantages, they are in nature closer related to the meaning of the weights as trade-offs, and as such they can be expected to lead to more normative reasonable results.

After having surveyed these six methods, a final remark. Researchers might find it difficult to pinpoint a unique weighting scheme, whereas they might find it easier to obtain "ranges" in which reasonable values of weights can be found. Foster and Sen (1997, p. 206) state that while "the possibility of arriving at a unique set of weights is rather unlikely, that uniqueness is not really necessary to make agreed judgements in many situations". Such an approach of working with ranges of weights, rather than exact values has the advantage of allowing for some degree of agnosticism. However, that agnosticism comes at a price: an approach based on ranges of weights, is likely to lead to a partial ordering of the well-being bundles. How incomplete the ordering becomes, or how many bundles will become incomparable, depends on the allowed width of the ranges and the correlation between the achievements of the individuals across the dimensions. The stronger the correlation between the dimensions, the less important the exact specification of the weights. A sensitivity analysis[23] for alternative weighting schemes can be very helpful in determining how sensitive the well-being index and the implied ordering of the bundles is for alternative weighting schemes. Although it is clear that a sensitivity analysis can never answer the question how to set weights in a reasonable way, it might give an idea how important the answer is for the obtained results and how much room there is for agnosticism, concerning the weights.

[23]For example, in the context of the measurement of multidimensional global welfare, Decancq and Ooghe (2008) propose a normative framework in which they carry out a sensitivity analysis for all possible weighting schemes. They find that the obtained trend in increasing welfare is robust for almost all weighting schemes, except for the one giving almost all weight to life-expectancy. Foster *et al* (2008) propose a way to easily test the robustness of weights. They apply a rank-robustness technique to assess Human Development Index weights.

5 Conclusion

In this paper, we surveyed different approaches for setting weights in multidimensional indices. We provided a general framework where most methods fit in. This framework allowed us to understand the meaning of weights as crucial factors determining the trade-off between dimensions. Dimension-weights are, however, not the only component determining this trade-off. The form of the transformation of the original variables into commensurable units and the parameter of substitution between dimensions also play an important role. However, these components are, more often than not, ignored in the literature.

We reviewed six approaches used to set dimension weights, highlighting their advantages and drawbacks. Ultimately, the definite test for any weighting scheme should be in terms of its reasonability in terms of implied trade-offs between the dimensions. As long as there is no widely accepted theoretical framework how to set these trade-offs, the researcher has no choice than to rely on her common sense and to be very cautious in interpreting the obtained orderings of the well-being bundles. In all cases, robustness tests to determine whether results are driven solely by the specific value of weights selected, should be called upon.

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Appendix

Starting from expression (1):

$$I(X|\beta) = \left[\frac{w_1\left[I_1(x_1)\right]^\beta + ... + w_q\left[I_q(x_q)\right]^\beta}{w_1 + ... + w_q}\right]^{\frac{1}{\beta}} \tag{13}$$

For propostion 1:

$$\begin{aligned}\frac{\partial I(X|\beta)}{\partial w_j} &= \frac{1}{\beta}\left[w_1\left[I_1(x_1)\right]^\beta + ... + w_q\left[I_q(x_q)\right]^\beta\right]^{\frac{1}{\beta}-1}\left[I_j(x_j)\right]^\beta\left[w_1 + ... + w_q\right]^{-\frac{1}{\beta}} \\ &-\frac{1}{\beta}\left[w_1 + ... + w_q\right]^{-\frac{1}{\beta}-1}\left[w_1\left[I_1(x_1)\right]^\beta + ... + w_q\left[I_q(x_q)\right]^\beta\right]^{\frac{1}{\beta}}\end{aligned}$$

$$\begin{aligned}\frac{\partial I(X|\beta)}{\partial w_j} &= \frac{1}{\beta}\left[w_1\left[I_1(x_1)\right]^\beta + ... + w_q\left[I_q(x_q)\right]^\beta\right]^{\frac{1-\beta}{\beta}}\left[w_1 + ... + w_q\right]^{-\frac{1}{\beta}} * \\ &\left[\left[I_j(x_j)\right]^\beta - I(X)^\beta\right]\end{aligned}$$

$$\frac{\partial I(X|\beta)}{\partial w_j} = \frac{\left[I_j(x_j)^\beta - I(X)^\beta\right]}{\beta\left[w_1 + ... + w_q\right]I(X)^{\beta-1}} \tag{14}$$

For proposition 3:

$$\frac{\partial I(X|\beta)}{\partial x_j} = \left[\frac{w_1\left[I_1(x_1)\right]^\beta + ... + w_q\left[I_q(x_q)\right]^\beta}{w_1 + ... + w_q}\right]^{\frac{1-\beta}{\beta}}\frac{w_j}{w_1 + ... + w_q}\left[I_j(x_j)\right]^{\beta-1}I'_j(x_j)$$

$$\frac{\partial I(X|\beta)}{\partial x_j} = \frac{w_j}{w_1 + ... + w_q}\left[\frac{I_j(x_j)}{I(X|\beta)}\right]^{\beta-1}I'_j(x_j) \tag{15}$$

For proposition 5:

$$MRS_{ij} = \frac{\frac{\partial I(X|\beta)}{\partial x_j}}{\frac{\partial I(X|\beta)}{\partial x_i}} = \frac{\left[\frac{w_1\left[I_1(x_1)\right]^\beta + ... + w_q\left[I_q(x_q)\right]^\beta}{w_1 + ... + w_q}\right]^{\frac{1-\beta}{\beta}}\frac{w_j}{w_1 + ... + w_q}\left[I_j(x_j)\right]^{\beta-1}I'_j(x_j)}{\left[\frac{w_1\left[I_1(x_1)\right]^\beta + ... + w_q\left[I_q(x_q)\right]^\beta}{w_1 + ... + w_q}\right]^{\frac{1-\beta}{\beta}}\frac{w_i}{w_1 + ... + w_q}\left[I_i(x_i)\right]^{\beta-1}I'_i(x_i)}$$

$$MRS_{ij} = \frac{\frac{\partial I(X|\beta)}{\partial x_j}}{\frac{\partial I(X|\beta)}{\partial x_i}} = \frac{w_j}{w_i}\frac{I'_j(x_j)}{I'_i(x_i)}\left[\frac{I_j(x_j)}{I_i(x_i)}\right]^{\beta-1} \tag{16}$$

Tables

Table 1: Transformation functions

Transformation method	Formulae	Characteristics	Examples	$I'_j(x_j)$
Ranking	$I_i j = rank(x_{ij})$	Uses ordinal information only, hence discarding all level information	Medicare health care performance index	—
z-scores	$I_{ij} = \frac{x_{ij}-\bar{x}_j}{sd(\bar{x}_j)}$	Imposes a standard normal distribution. Some individuals will have negative values. Extreme values could be be given a large weight, which may or may not be desirable.	Environment Sustainability Index	$\frac{1}{sd}$
Re-scaled values	$I_{ij} = \frac{x_{ij}-\min(x_j)}{\max(x_j)-\min(x_j)}$	Standardization is based on the range so it is robust to outliers.	HDI	$\frac{1}{max-min}$
Distance		Less robust to outliers.		
- from mean	$I_{ij} = \frac{x_{ij}}{\bar{x}_j}$			$\frac{1}{\bar{x}_j}$
- from best performer	$I_{ij} = \frac{x_{ij}}{\max(x_j)}$		Environmental Policy Performance Indicator	$\frac{1}{max_j}$
- over time	$I_{ij} = \frac{x^t_{ij}-x^{t-1}_{ij}}{x^t_{ij}}$	Only feasible with longitudinal data	Economic Sentiment Indicator	
Number of indicators above/below mean	$I_{ij} = \frac{x_{ij}}{\bar{x}_j} - (1+p)$	p arbitrary threshold above/below the mean		$\frac{1}{\bar{x}_j}$
Logarithmic	$I_{ij} = \ln(x_{ij})$	Coefficients are interpreted as elasticities. Higher weight to changes at the bottom		$\frac{1}{x_j}$
Exponential	$R = 23 * \ln\{1R * [1e^{100/23}]\}$	In 2 steps: (1) rank each x_j to range [0,1] (2) transform scores R_j to a truncated exponential distribution using	Multiple Deprivation Index	

Table 2: Some composite well-being indices

Composite Index	Formula	standardisation	dimensions weights w_j	β
Human Development Index	$HDI = \sum_{j=1}^{3} w_j I(x_j)$ 3 dimension: income, educ, health	$I(x_j) = \frac{x_j - \min}{\max - \min}$	equal weight	1
Human Poverty Index - 1	$HPI-1 = \left[\sum_{j=1}^{3} w_j I(x_j)^{\beta}\right]^{1/\beta}$ x_j expressed in rates 3 dimensions: survival, educ, economic	$I(.) = x_j$	equal weight	3
Human Poverty Index - 2	$HPI-2 = \left[\sum_{j=1}^{4} w_j I(x_j)^{\beta}\right]^{1/\beta}$ x_j expressed in rates 4D: health, knowledge, economic, social exclusion	$I(.) = x_j$	equal weight	3
Gender-related development index	$GDI = \left[\sum_{j=1}^{4} w_j E(x_j)\right]$ $E(x_j) = \left[fI(x_{jf})^{\beta} + fI(x_{jm})^{\beta}\right]^{1/\beta}$ x_{jf}: achievement of female (m: male) f: female population share (m: male)	$I(x_j) = \frac{x_j - \min}{\max - \min}$	equal weight gender weight: freq weight	-1
Index of Multiple Deprivations	$IMD = \sum_{j=1}^{7} w_j I(x_j)$ 7 domains 38 indicators	Exponential (D)	Two stages: D: participatory I:ML Factor analsyis and equal weight	1
Some underlying well-being indices in multidimensional inequality and poverty measures with empirical applications				
Brandolini (2007)			range [0,1]	{1,2,5,10,100,500}
Decancq et al (2007)		$I(x_j) = \frac{x_j - \min}{\max - \min}$	equal weight	[-5,1]
		$I(x_j) = \frac{x_j}{\bar{x}_j}$	principal component	
		$I(x_j) = \frac{x_j - \bar{x}_j}{sd(x_j)}$		
		$I(x_j) = ln(x_j)$		
Deutsch and Silber (2005)			equal weight/freq weights	1
Justino (2005)		$I(x_j) = \frac{x_j - \min}{\max - \min}$	equal weight/range	[-1/3, 1]
Lugo (2007)		$I(x_j) = \frac{x_j - \min}{\max - \min}$	equal weight	{-20,-4,0,0.33,0.5,1}
Nilsson (2007)		$I(x_j) = \frac{x_j - \min}{\max - \min}$	equal weight	{-20,-1,0.5,0,0.5,1}
Maasoumi-Lugo (2008)		$I(.) = x_j$	equal weight/arbitrary	[-3, 1]