OPHI

Oxford Poverty & Human Development Initiative
University of Oxford
www.ophi.org.uk

Summer School on Capability and Multidimensional Poverty

11-20 September, 2010

Amman, Jordan

We are grateful to IDRC for funding this summer school.

Unidimensional Inequality Measurement

Maria Emma Santos
(CONICET-UNS and OPHI)

Main Sources of this Lecture

- Foster (1985)
- Atkinson (1970)
- Foster and Sen (1997), Annexe to "On Economic Inequality"
- There are others: please see the readings list.

Introduction

- Focus of this lecture:
  - Unidimensional income
  - Measurement

Inequality Measurement

- ***Inequality Rankings:*** A rule for comparing distributions in terms of inequality.
  - ‘Complete rankings’ *vs*. ‘Partial Rankings’
  - Lorenz dominance: A partial ranking.

- ***Inequality Measures***

Inequality Measures
Four Basic Properties

Notation

- Let $x = \{x_1, \ldots\ldots, x_n\}$ be the income distribution, where x_i is the income of the *i*th person, and $n=n(x)$ is the population size.

- Let $D = \bigcup_{n \geq 1} R_{++}^{n}$ be the overall set of distributions under consideration.

- An inequality measure is a function $I : D \rightarrow R$ which, for each distribution x indicates the level $I(x)$ of inequality in the distribution.

Four Basic Properties for Inequality Measures

x is obtained from *y* by a ***permutation*** of incomes if $x=Py$, where P is a permutation matrix.

$$x = Py = \begin{bmatrix} 010 \\ 100 \\ 001 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 8 \end{bmatrix}$$

1. *SYMMETRY (Anonymity)*: *If x* is obtained from *y* by a ***permutation*** of incomes, then $I(x)=I(y)$.

Four Basic Properties…

x is obtained from y by a ***replication*** if the incomes in x are simply the incomes in y repeated a finite number of times.

$$x = \{y_1, y_1, y_2, y_2, ..., y_n, y_n\}$$

$$x = \{6,6,1,1,8,8\}$$

2. *Replication Invariance (Population Principle):*
   If x is obtained from y by a ***replication***, then $I(x)=I(y)$.

Four Basic Properties…

x is obtained from y by a ***proportional change (or scalar multiple)*** if the incomes in $x=\alpha y$, for some $\alpha>0$.

$$y = \{6,1,8\} \qquad x = \{12,2,16\}$$

3. *SCALE INVARIANCE (Zero-Degree Homogeneity): If x* is obtained from y by a ***proportional change***, then $I(x)=I(y)$.

Four Basic Properties…

x is obtained from *y* by a ***Pigou-Dalton regressive transfer*** if for some *i*, *j*:

i) $y_i \leq y_j$

ii) $y_i - t = x_i$

$y_j + t = x_j$ $\Rightarrow$ $y_i - x_i = x_j - y_j > 0$

y = {2,6,7} x = {1,6,8}

4. TRANSFER: *If x* is obtained from *y* by a ***regressive transfer***, then *I(x)>I(y)*.

Four Basic Properties for Inequality Measures

- Any measure satisfying the four basic properties (symmetry, replication invariance, scale invariance and transfer) is called a **relative inequality measure.**

Inequality Rankings:
The Lorenz Curve

The Lorenz Curve (Lorenz, 1905)

Given an income distribution of *n* people $x = \{x_1, \ldots\ldots, x_n\}$

Example: $x = \{8,6,1\}$

1. Order the population from lowest income to highest.

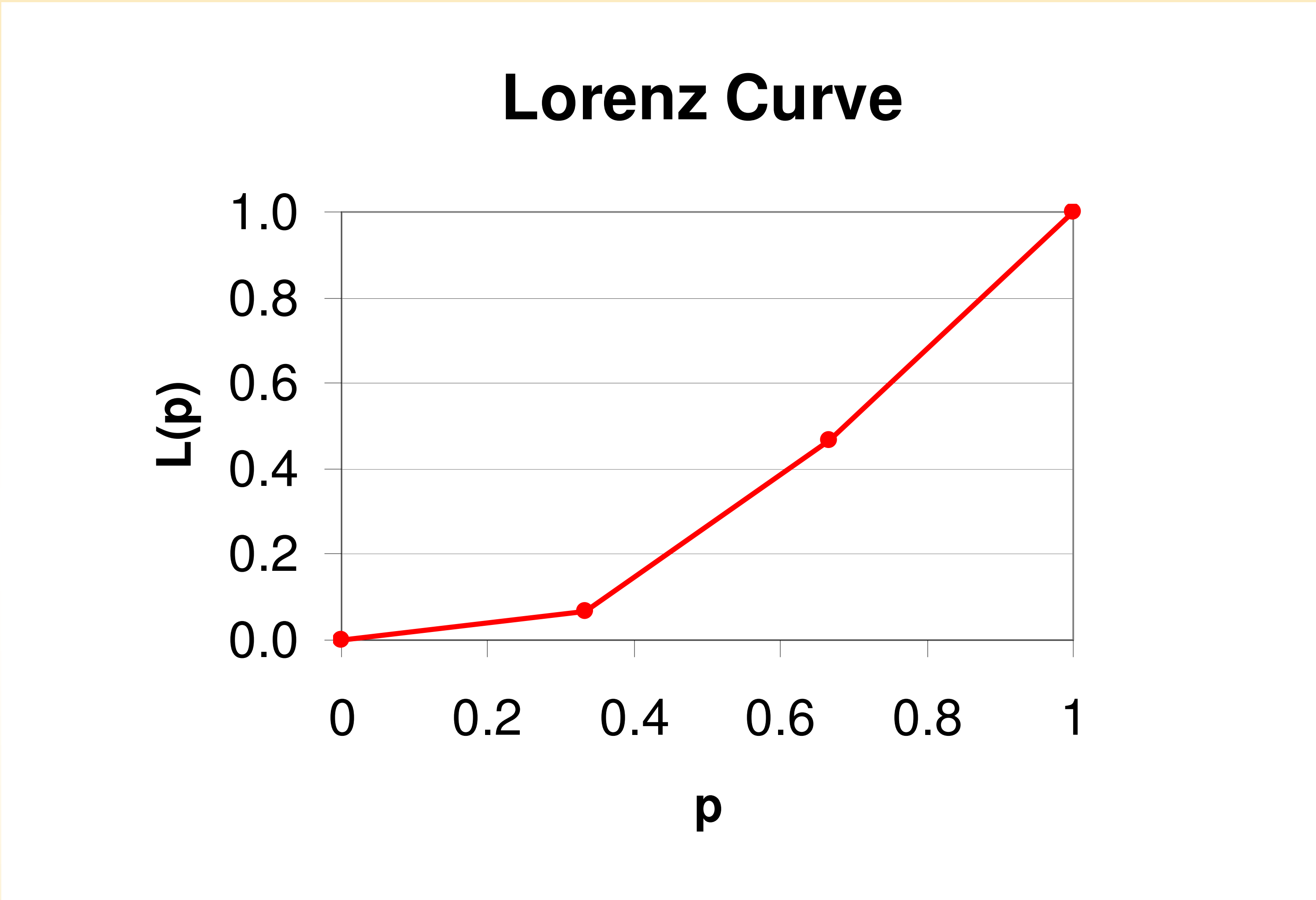
   Example: $\hat{x} = \{\hat{x}_1, \ldots\ldots, \hat{x}_n\}$

   $\hat{x} = \{1,6,8\}$

2. On the horizontal axis plot the cumulative share of population.

3. On the vertical axis plot the cumulative share of the income received by each cumulative population share.

Cum. Population Share p	Cum. Income Share $L(x, p)$
1/3	1/15
2/3	7/15
1	15/15

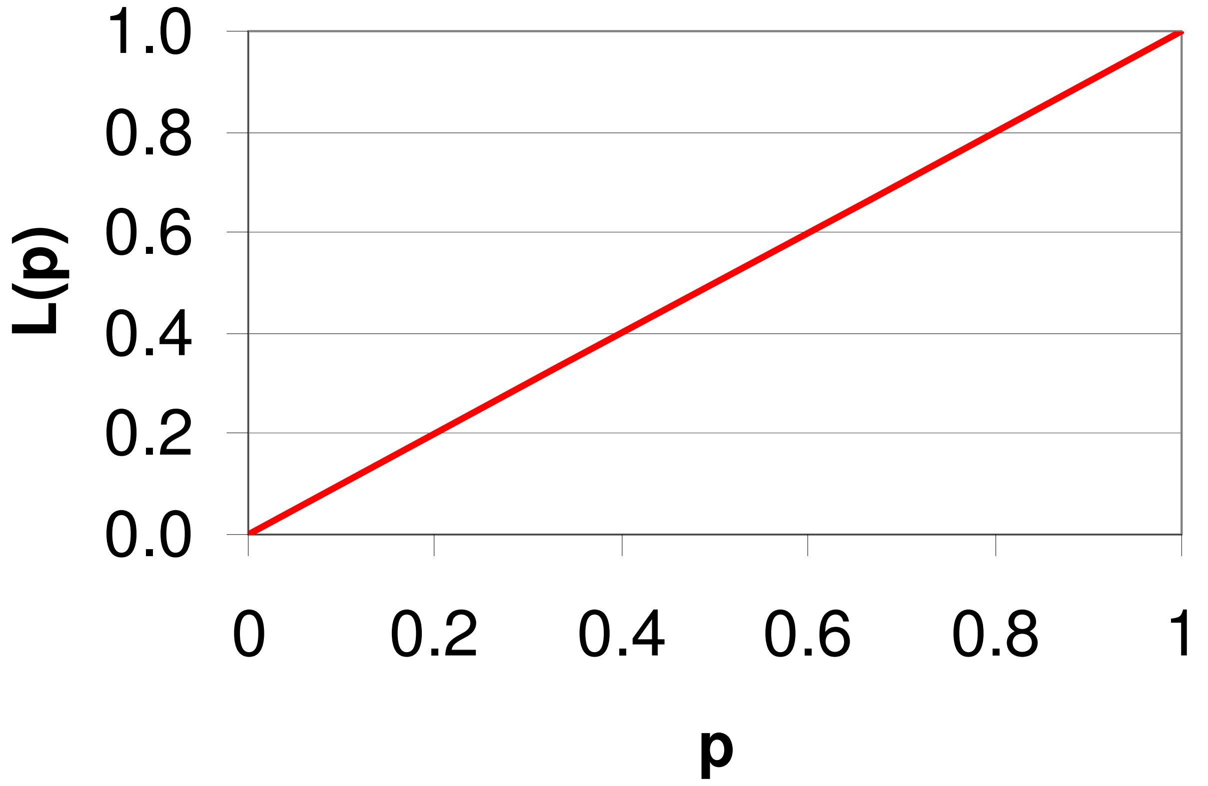


- At any point, the Lorenz curve gives the cumulative share of total income received by each poorest cumulative share of the population.

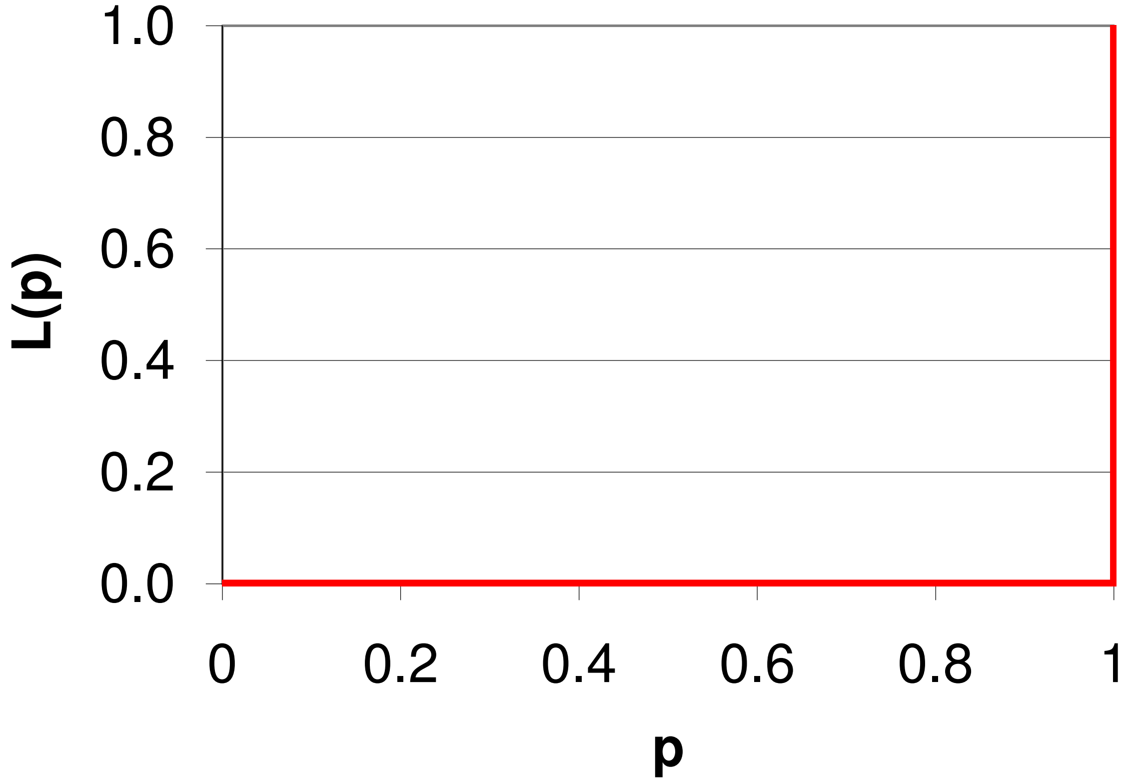
The Lorenz Curve: Characteristics

1. Starts in (0,0); ends in (1,1).
2. Always increasing and convex (because population is ordered from poorest to richest).
3. Lorenz curve of a *perfectly equal* distribution?
4. Lorenz curve of a *perfectly unequal* distribution?

The Lorenz Curve of a Perfectly Equal Distribution



The Lorenz Curve of a Perfectly Unequal Distribution



Lorenz Dominance

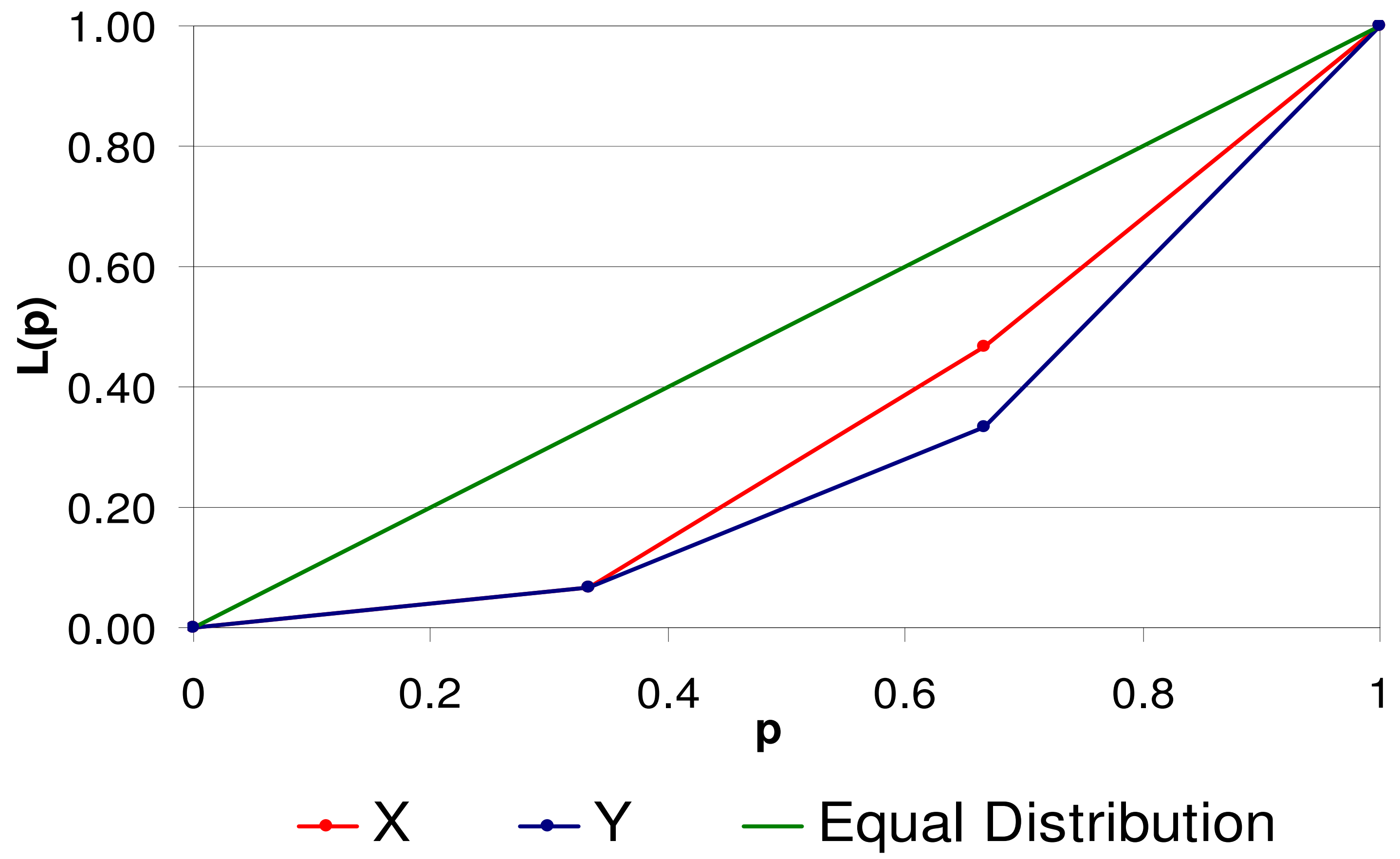
- Given two distributions *x* and *y*, *x* ***Lorenz-dominates*** *y* (*x is less unequal than y*) if and only if:

  $L(x,p) \geq L(y,p)$ for all *p*, with > for some *p*

- Example:

  x = {1,6,8} y = {1,5,9}

Lorenz Curves for Two Distributions



Lorenz Criterion & Relative Inequality Measures: the *link* between the two

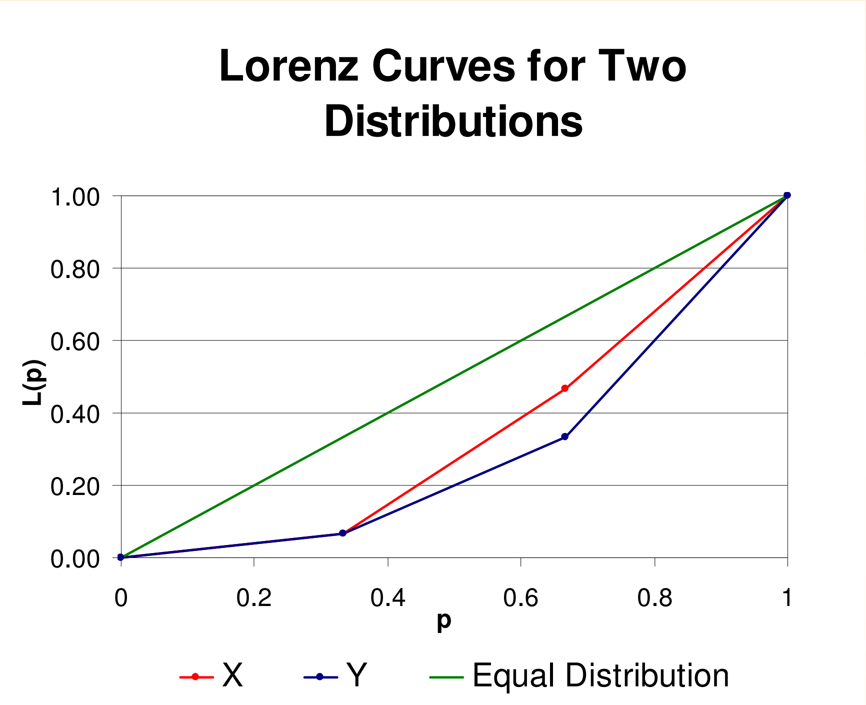
- How does the Lorenz Criterion relates to the Relative Inequality Measures (those satisfying the 4 basic axioms?)

The Lorenz Curve and the Four Axioms

- **Symmetry** and **Replication** are satisfied since permutations and replications leave the curve unchanged.

- Proportional changes in incomes do not affect the LC, since it is normalized by the mean income. Only *shares* matter. So it is **scale invariant**.

- A regressive transfer will make the Lorenz curve to be further away from the diagonal. So it satisfies **transfer**.



$$x = \{1,6,8\}$$

$$y = \{1,5,9\}$$

Lorenz Consistency (Foster, 1985)

- An inequality measure $I: D \to R$ is Lorenz Consistent if for all x and y in D:

$$xLy \Rightarrow I(x) < I(y)$$

- An inequality measure $I: D \to R$ is Lorenz Consistent **if and only if** it satisfies Symmetry, Replication Invariance, Scale Invariance and Transfer, ie: if and only if it is a relative inequality measure.

Incomplete & Complete Rankings

- Lorenz Ranking is incomplete. When Lorenz curves cross, the Lorenz criterion can not decide between the two distributions.
  - **If Lorenz dominance holds, then all relative measures agree.**
  - **If Lorenz dominance fails, then a measure of inequality might be used to get a complete ranking. But different measures may rank the distributions differently.**
- Shorrocks and Foster (1987) provide additional conditions by which two distributions can be ranked when their Lorenz curves cross once. Still, these do not eliminate *all* the incompleteness.

Atkinson's Theorem (Atkinson, 1970)

- Motivating question: Given two income distributions x and y, which one produces a higher **social welfare**?

Stochastic Dominance

- Atkinson's theorem draws on the stochastic dominance literature, developed for risk analysis (how to choose betw 2 lotteries?).
- One distribution is said to *stochastically dominate* another if it yields **higher expected utility** for all utility functions in a given class.
- Three common stochastic dominance relationships: First Order (FSD), Second Order (SSD), Third Order (TSD).

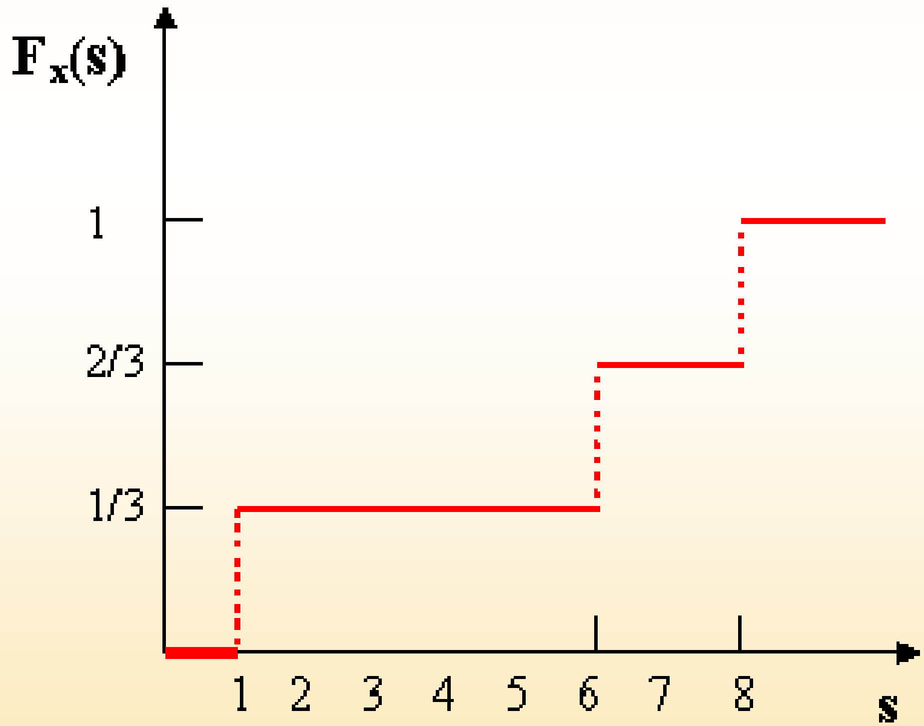
Cumulative Distribution Function (cdf)

- Given the income distribution x, the **cumulative distribution function** associated with x, $F_x(s)$ is the proportion of persons i such that $x_i \leq s$.
- The c.d.f. of a random variable is clearly a **monotonously increasing** (or more precisely, non decreasing) function from 0 to 1

Cumulative Distribution Function (cdf)

- Example (discrete case): x={1,6,8}: What proportion of people have an income lower than 1? Lower than 6? Lower than 8?

$$F_x(s) = \begin{cases} 0 & s < 1 \\ 1/3 & s < 6 \\ 2/3 & s < 8 \\ 1 & 8 \leq s \end{cases}$$



Stochastic Dominance : Definition

Let F(s) and G(s) be the cdf of x and y, respectively.

F(s) dominates G(s) iff

$$\int_0^y u(s)dF(s) \geq \int_0^y u(s)dG(s)$$

for all s with > for some s

Stochastic Dominance: Order and conditions on the utility functions

- FIRST ORDER (FSD)

Positive Marginal Utility (MU): ***u'(s)>0***

- SECOND ORDER (SSD)

Positive and decreasing MU: ***u'(s)>0 and u''(s)<0***

- THIRD ORDER (TSD)

Positive, decreasing and convex MU:

u'(s)>0 ; u''(s)<0 ; u'''(s)<0

Stochastic Dominance: Equivalent Conditions using the cdf

- FIRST ORDER

  F(s) FSD G(s) iff

  $$F(s) \le G(s)$$

  for all s, with < for some s

- SECOND ORDER

  F(s) FSD G(s) iff

  $$\int_0^y F(s)ds \le \int_0^y G(s)ds$$

  for all s, with < for some s

- THIRD ORDER

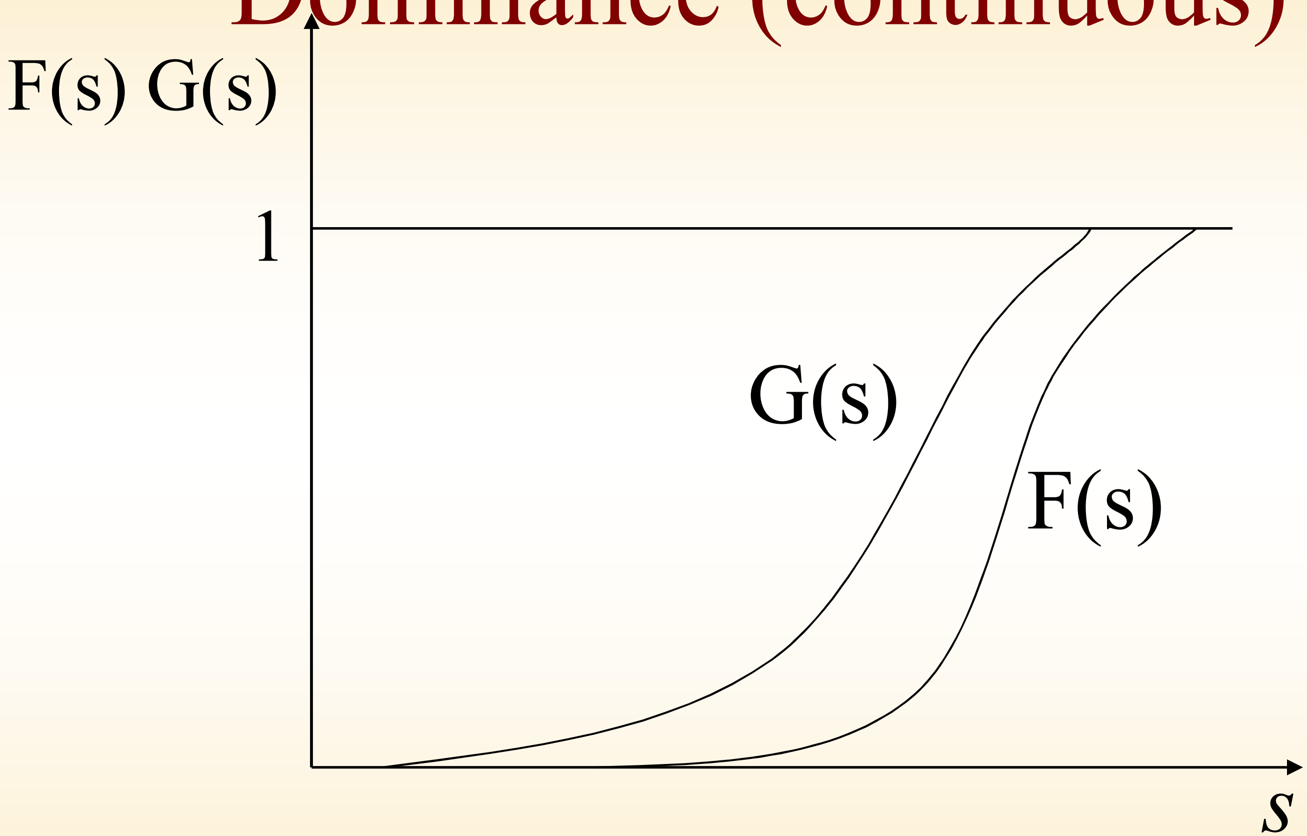
  F(s) FSD G(s) iff $$\int_0^x \int_0^y F(s)ds \le \int_0^x \int_0^y G(s)ds$$

  for all s, with < for some s

Stochastic Dominance:

- FSD→SSD →TSD (lower orders of dominance imply higher orders of dominance).
- The converse does not hold.

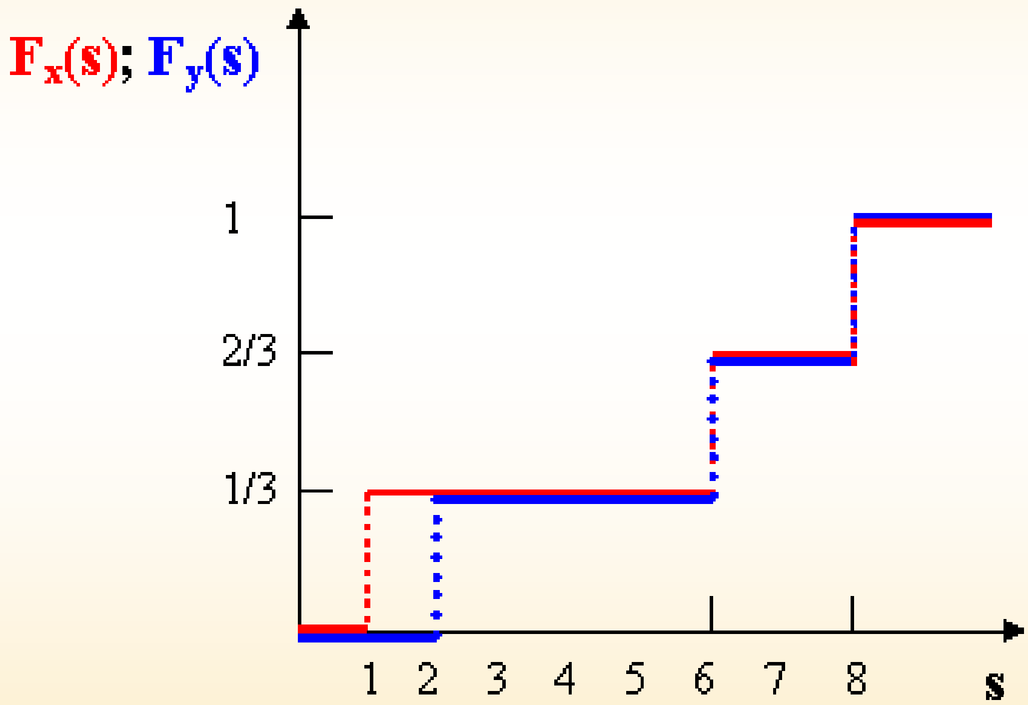
Example of First Order Stochastic Dominance (continuous)



F(s) FSD G(s) because F(s)≤G(s) for all s, with < for some s

Example of First Order Stochastic Dominance (discrete):

x={1,6,8} y={2,6,8}



$F_y(s)$ **FSD** $F_x(s)$: Note that the cdf of y is at every point lower or equal than that of x

First Order Stochastic Dominance (discrete)

- Also note that the condition of FSD of the cdf implies 'vector dominance', ie: each element of the (ordered) dominating vector is not lower than the element of the dominated one and strictly greater for some:

$$x=\{1,6,8\} \quad y=\{2,6,8\}:$$

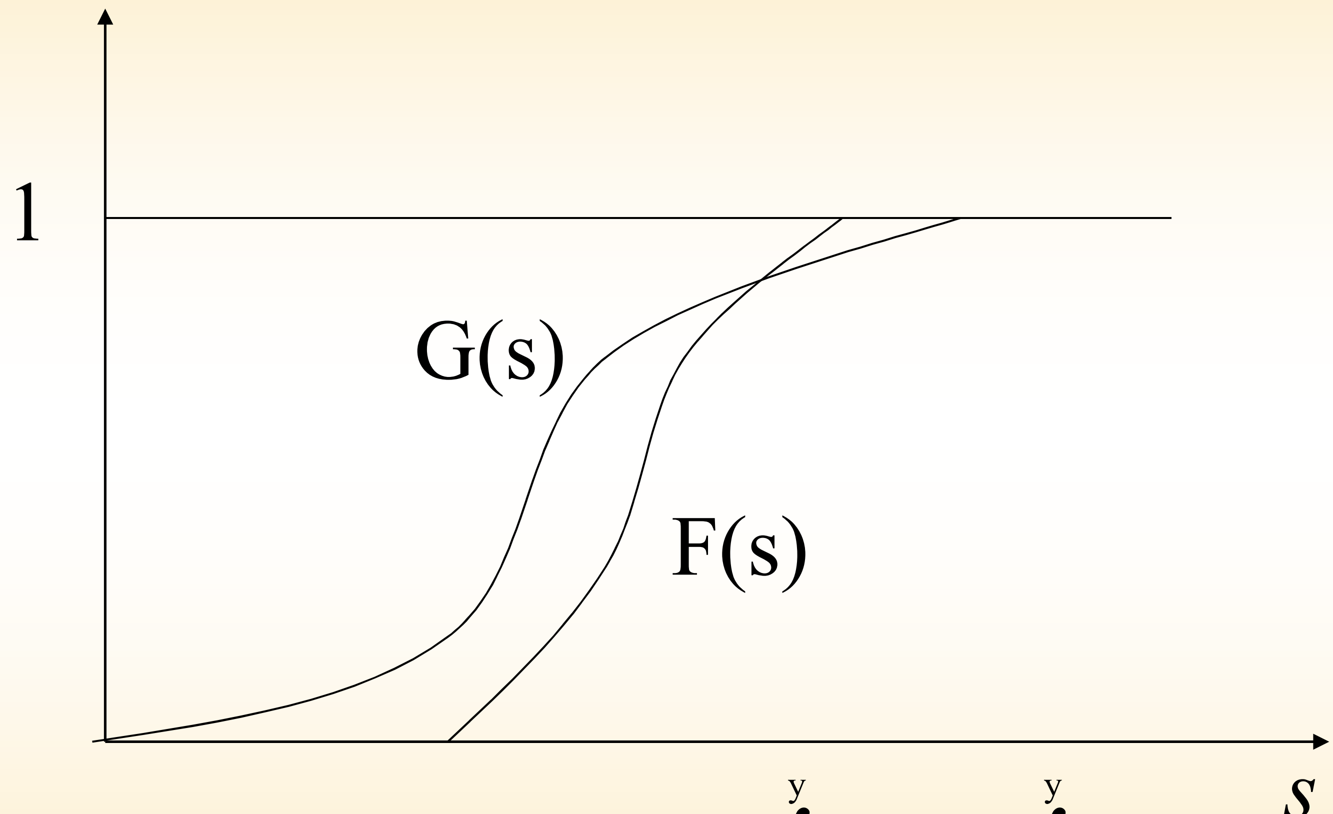
$$1<2$$

$$6=6$$

$$8=8$$

- Note that for this check the two vectors need to have the same number of elements. In some cases replications may be required to make this comparison.

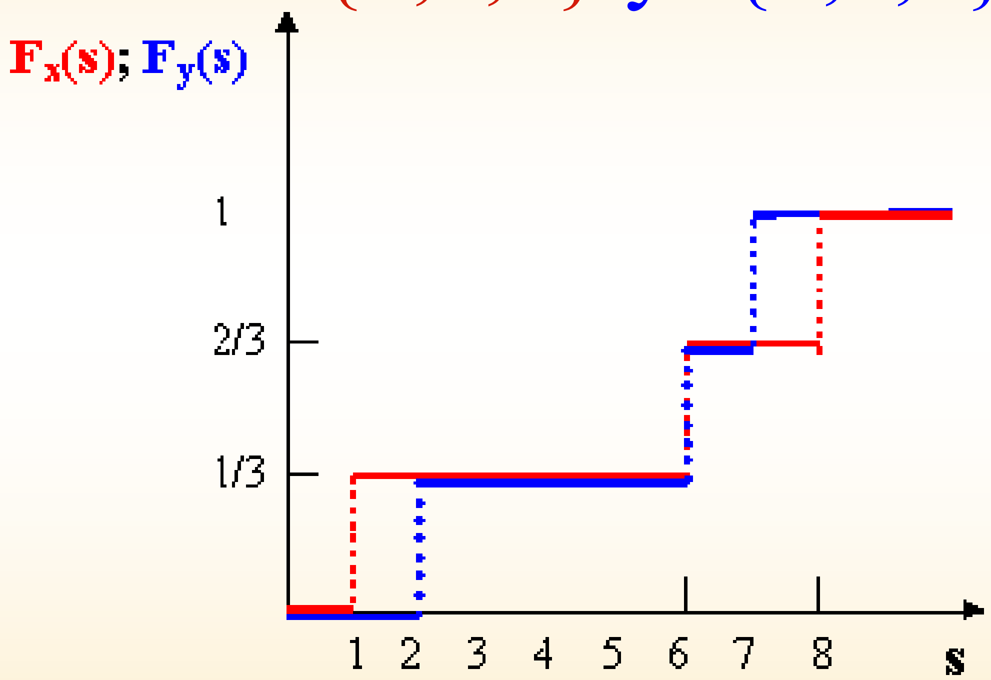
Second Order Stochastic Dominance



F(s) SSD G(s) because $\int_0^y F(s)ds \le \int_0^y G(s)ds$ with < for some s.

Example of Second Order Stochastic Dominance (discrete):

$x=\{1,6,8\}$ $y=\{2,6,7\}$



$F_y(s)$ **SSD** $F_x(s)$: Note that the <u>cumulative</u> area below the cdf of y is at every point lower or equal than that of x.

Example of Second Order Stochastic Dominance (discrete):

- In terms of comparisons of the ordered vectors, for SSD one can check the ‘cumulatives’. Those of the dominating vector are not lower than those of the dominated one, and strictly higher at least for some:

$x=\{1,6,8\}$ $y=\{2,6,7\}$:

$1<2$

$1+6<2+6$

$1+6+8=2+6+7$

Link between Lorenz Curve and the cdf (Foster, 1985):

- Formal Definition of the Lorenz Curve for the discrete case. Given a distribution x, one defines an ordered version of it (from lowest to highest) $\hat{x}$

$$L_x(x,p) = \sum_{i=1}^{pn} \hat{x}_i \Big/ X \qquad p \in [0,1]$$

$$X = \sum_{i=1}^{n} \hat{x}_i$$

Link between Lorenz Curve and the cdf

- Given the cdf of a distribution $F_x(s):R \rightarrow [0,1]$, we can define its inverse $F^{-1}_x(t):[0,1] \rightarrow R$.

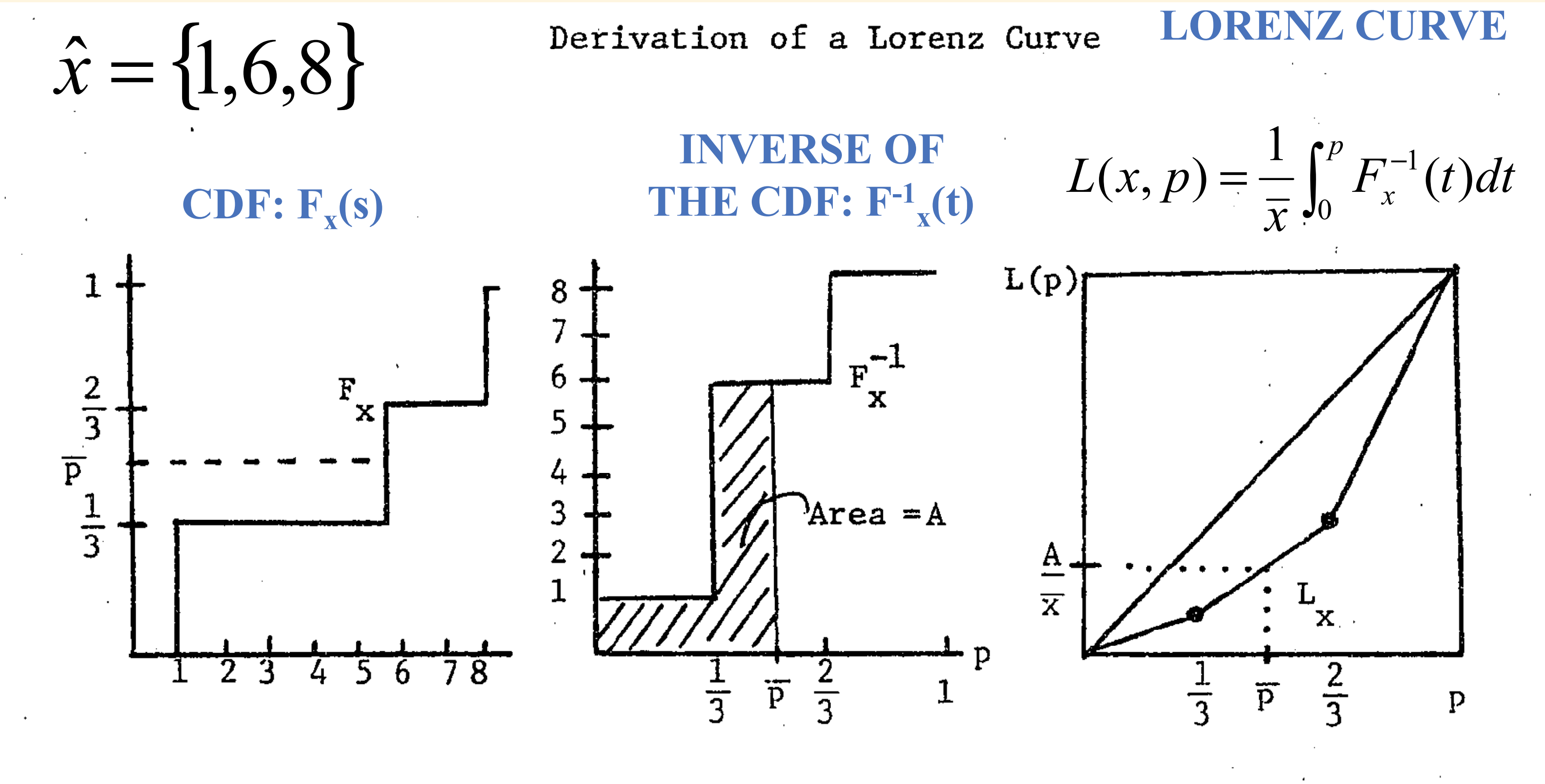
- Then, the Lorenz Curve associated with *x* is given by:

$$L(x,p) = \frac{1}{\bar{x}} \int_0^p F_x^{-1}(t)dt$$

with $\bar{x}$ being the mean of *x*.

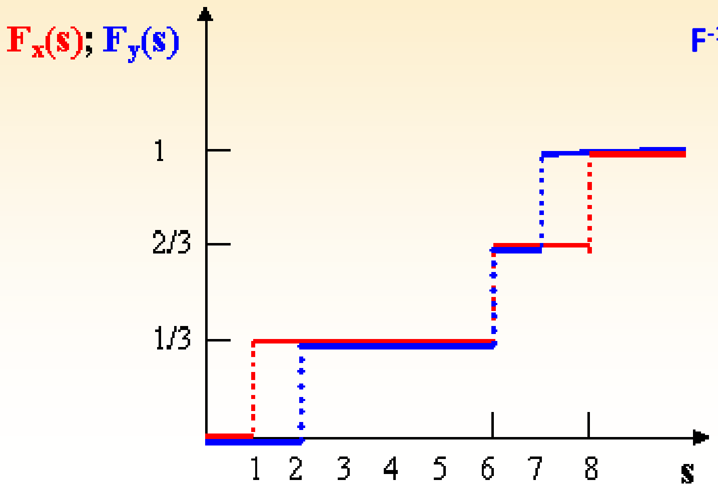
Link between Lorenz Curve and the cdf graphically:

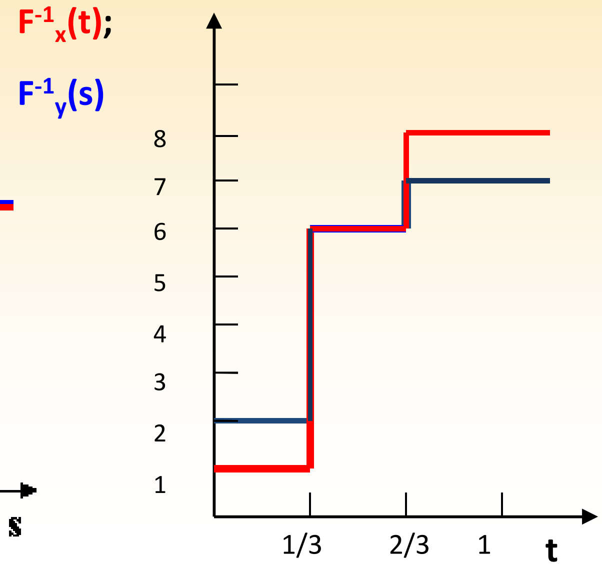
$\hat{x} = \{1,6,8\}$



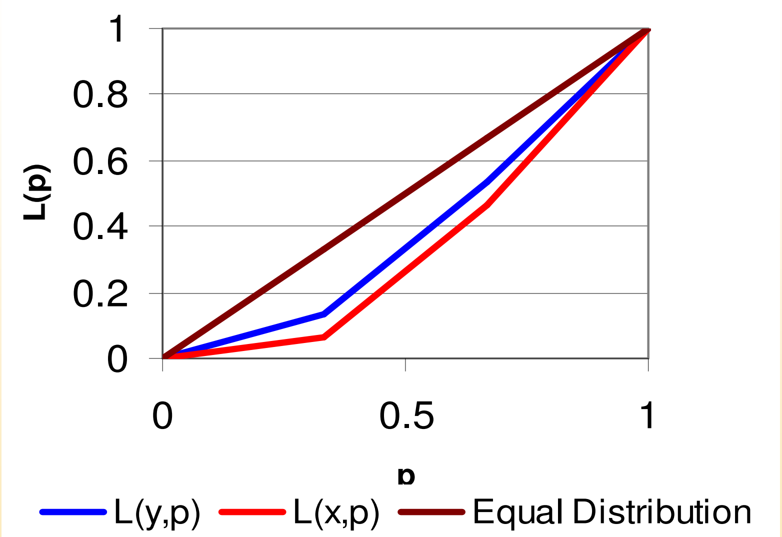
Source: Foster (1985), p. 17.

Example of link btw Lorenz & cdf





$x=\{1,6,8\}$ $y=\{2,6,7\}$



To think…

Lorenz dominance & SSD…?

Atkinson's Theorem (1970)

- Conditions:
  1. Social Welfare is the <u>sum</u> of individual utility functions
  $$W(s)=\frac{1}{n}\sum_{i=1}^{n}u(s_i)$$
  2. Utility functions are increasing and concave
  $$u'(s)>0 \qquad u''(s)<0$$
  3. The two distributions have the same mean income $\bar{x}=\bar{y}$

Then: xLy *iff* $W(x)>W(y)$

Atkinson's Theorem

- Atkinson's Theorem proved that for distributions with equal means, Lorenz Dominance is equivalent to SSD and so we can judge distributions using the Lorenz curve.

- The theorem has a strong policy implication: For distributions with equal mean income, to increase welfare we just need to make the distribution more equal!

- If the means differ, SSD implies that the mean of the distribution that dominates can be no lower than the mean of the dominated one.

Summary so far…

(Theorem 1 in Shorrocks & Foster, 1987)

- For *x*, *y* having equal means, the following statements are equivalent:
  a) *x* Lorenz dominates *y*
  b) I(y)>I(x) for <u>all</u> relative inequality measures I(.)
  c) *x* SSD *y*
  d) $\hat{x}$ can be obtained from $\hat{y}$ by a non-empty sequence of rank preserving progressive transfers.

Extensions of Atkinson’s Theorem

- Sen extended Atkinson’s theorem to the more general case in which welfare is non-aditive ($W=\varphi(s_1, \ldots s_n)$) and strictly S-concave.

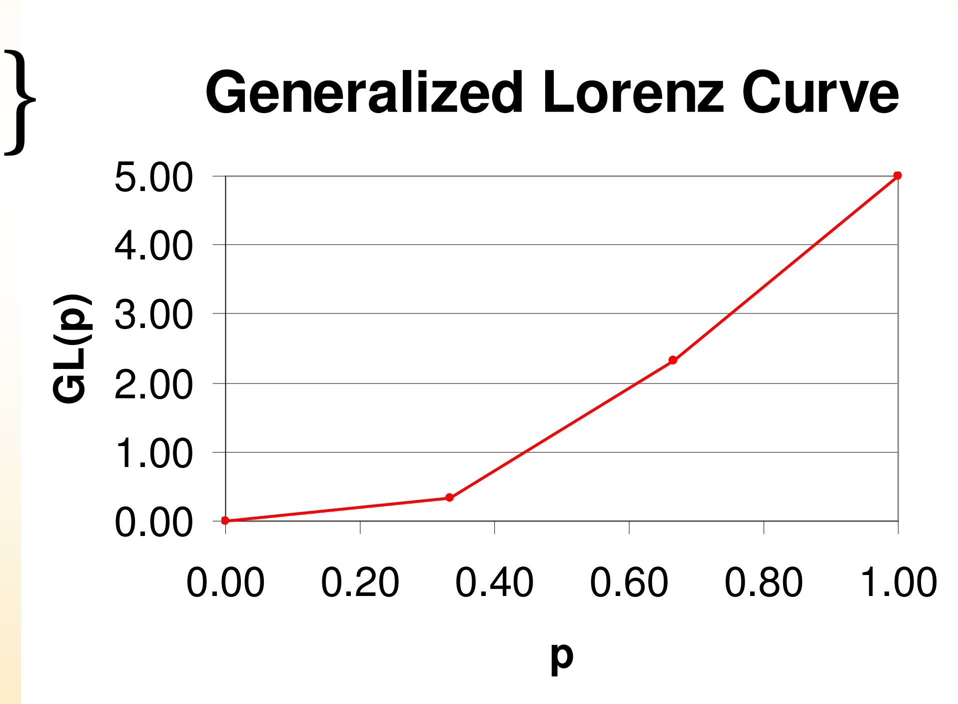
- Shorrocks (1983) extended Atkinson’s theorem to the case of <u>different means</u>, using the Generalized Lorenz Curve.

Generalized Lorenz Curve

- It is the Lorenz Curve scaled by the mean for each population share $GL(p) = \bar{x}L(p)$ for each p

- Example: $\hat{x} = \{1,6,8\}$

p	$L(p)$	$GL(p)$
1/3	1/15	1/3
2/3	7/15	7/3
1	15/15	5



Extension of Atkinson's Theorem

- Conditions:
  1. Social Welfare is the sum of individual utility functions
  $$W(s) = \frac{1}{n}\sum_{i=1}^{n} u(s_i)$$
  2. Utility functions are increasing and concave
  $$u'(s) > 0 \qquad u''(s) < 0$$

Then:

$$xGLy \qquad iff \qquad W(x) > W(y)$$

(When means differ, GL Dominance is equivalent to SSD).

Summary

INEQUALITY MEASUREMENT

Measures

Four basic principles: Symmetry, Replication Invariance, Scale Invariance & Transfer: **Relative Inequality Measures**

Rankings

Lorenz Criterion/Dominance (from Lorenz Curve)

For equal means...

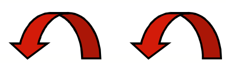
Generalized Lorenz Criterion/Dominance (from Generalized Lorenz Curve=LC*Mean)

Different means...

Second Order Stochastic Dominance

Transfer Sensitivity

- Motivating Question:
  - How should these transfers be reflected in an inequality measure?



x={2,4,6,8} x'={3,3,6,8} x''={2,4,7,7}

- Intuitively, a transfer-sensitive inequality measure places more emphasis on transfers at the lower end of the distribution.
- Shorrocks and Foster (1987) formalize the idea.

Subgroup Consistency

- Subgroup Consistency: If $I(x')>I(x)$ and $I(y')=I(y)$, and $n(x')=n(x)$, $n(y')=n(y)$ and $\bar{x}'=\bar{x}$, $\bar{y}'=\bar{y}$, then $I(x',y')>I(x,y)$.
- Crucially important for policy design & evaluation, regional vs. national
- Yet…
  - There are arguments in favour of considering the ***interdependence*** of incomes, their relative positions, issues that are ignored when subgroup consistency is satisfied. See example in Foster & Sen (1997), p.160.

Additive Decomposability

- An inequality measure I(.) is additively decomposable if:

$$I(x,y) = I[W] + I[B] = w_x I(x) + w_y I(y) + I(\bar{x},\bar{y})$$

where the weights vary according to the Inequality Measure, being the population shares in many cases, and $(\bar{x},\bar{y})$ being the 'smoothed' group distributions, with each member of the respective group having the mean income of that group.

- Additive Decomposability implies Subgroup Consistency. The converse is not true.

Example of Additive Decomp.

- Distribution x=(2,8,6,10)

- Say we have two groups of people (by some characteristic, say region): first two and second two: x_A=(2,8) x_B=(6,10).

- Inequality ***within*** groups is the *weighted* sum of I(x_A)=I(2,8) and I(x_B)=I(6,10). (Usually weighted by population shares)

Example of Additive Decomp.

- Inequality ***between*** groups: replace the income of each person by the mean income of his group; ie. create *smoothed* distributions $x^S_A=(5,5)$ y $x^S_B=(8,8)$. Then inequality between is I(5,5,8,8).
- Additive Decomposable measures are such that:

$$I(2,8,6,10)=w_A I(2,8)+w_B I(6,10)+I(5,5,8,8)$$

Normalisation

If every individual has the same income, then there is complete equality and the degree of inequality is normalised to zero, ie. $I(x)=0$

Continuity

I(x) is continuous if a small change in any income does not result in an abrupt change in the inequality index I(x)

- *CONTINUITY:* For any sequence x^k, if x^k converges to x, then $I(x^k)$ converges to I(x)

Inequality Measures

Non-Lorenz Consistent Inequality Measures

From now on: $\mu = \bar{x}$

- Range: $$R(x) = \frac{1}{\mu}(x_{\max} - x_{\min})$$

- It is the gap between the highest and the lowest income as a ratio to the mean income.

- Ignores the distribution between the extremes: **Violates transfer.**

Non-Lorenz Consistent Inequality Measures

- Kuznets Ratio

$$K(x)=\frac{\sum_{i=n(1-R)+1}^{n}\hat{x}_i}{X}\Bigg/\frac{\sum_{i=1}^{nP}\hat{x}_i}{X}$$

- It is the share of income earned by the richest R% relative to the share of income earned by the poorest p%. The 90-10 ratio is typical.
- The ratios are formed of ‘pieces’ of the Lorenz Curve.
- Like the range, it ignores the distribution between the R% and the P%, and therefore violates transfer.

Non-Lorenz Consistent Inequality Measures

- Relative Mean Deviation

$$M(x) = \frac{\sum_{i=1}^{n} | \mu - x_i |}{n\mu}$$

- Ratio of the sum of the absolute value of the distance between each income in the distribution and the mean income, to total income.

- It is not sensitive to transfers between people on the same side of the mean income. Violates transfer.

Non-Lorenz Consistent Inequality Measures

- Variance

$$V(x) = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$$

- By squaring the gaps of each income to the mean, the bigger gaps receive a higher weight. It satisfies transfer.
- However, the variance depends on the mean. It violates scale invariance.

Non-Lorenz Consistent Inequality Measures

- Variance of Logarithms

$$\tilde{x} = \{\ln(x_1), \ldots\ldots, \ln(x_n)\} \quad V_L(\tilde{x}) = \frac{\sum_{i=1}^{n} (\mu_{\tilde{x}} - \tilde{x}_i)^2}{n}$$

- Applies the variance to the distribution of log-incomes.
- It is mean-independent (scale invariant).
- But it violates transfer when relatively high incomes are involved. Not Lorenz consistent.

Lorenz Consistent Inequality Measures

- Squared Coefficient of Variation

$$CV^2(x) = \left(\frac{1}{\mu} \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}} \right)^2 = \left(\frac{\sigma}{\mu} \right)^2$$

- By taking the variance over the normalised distribution, it is scale invariant. It satisfies the four basic axioms.
- It is not transfer sensitive. A regressive transfer t has the same impact on C regardless of the part of the distribution in which it took place.
- It is additively decomposable, the weights are

$$w_x = (n_x / n)(\mu_x / \mu)^2$$

Lorenz Consistent Inequality Measures

- Gini Coefficient – Equivalent expressions:

$$G(x) = \frac{1}{2n^2\mu}\sum_{i=1}^{n}\sum_{i=j}^{n}| x_i - x_j | =$$

$$= 1 - \frac{1}{n^2\mu}\sum_{i=1}^{n}\sum_{j=1}^{n} Min(x_i, x_j) =$$

$$= 1 + \frac{1}{n} - \left(\frac{2}{n^2\mu}\right)\left[x_1 + 2x_2 + ...+ nx_n\right]$$

$$x_1 \geq x_2 \geq ..... \geq x_n$$

Inequality is the sum of all pairwise comparisons of ‘two person inequalities’ that can possibly be made.

An easy way to compute the Gini manually (from Prof. Foster):

- X=(1,2,3,4)

$$G(x) = \frac{1}{2n^2\mu}\sum_{i=1}^{n}\sum_{i=j}^{n}|x_i - x_j|$$

- Create a double-entry table to compute the numerator of the above expression

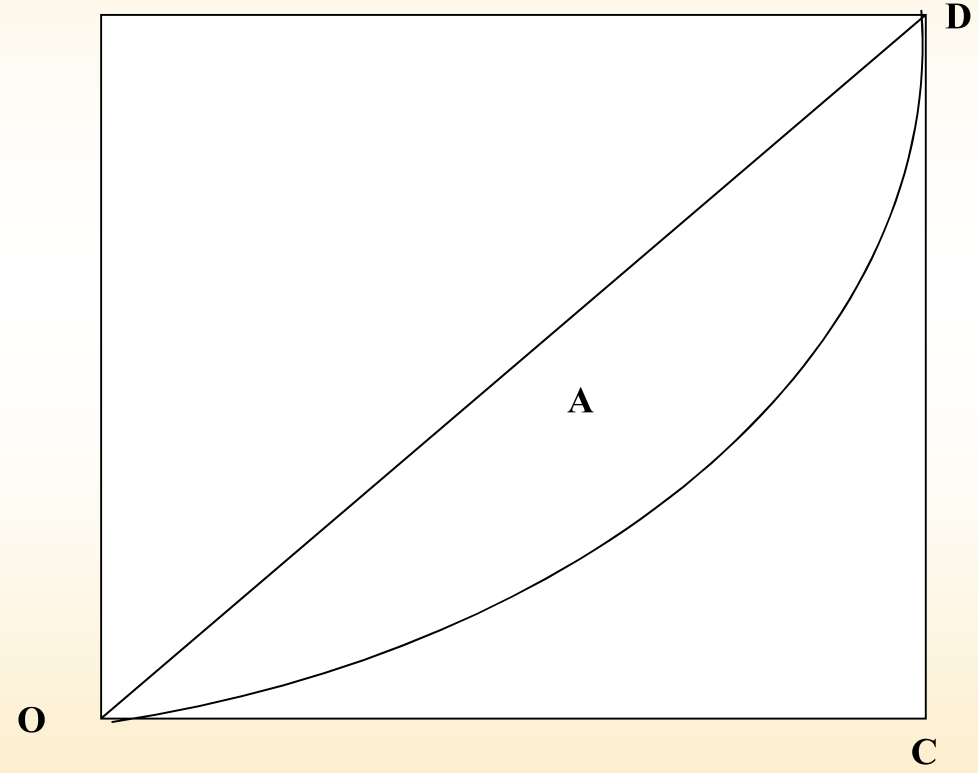
	1	**2**	**3**	**4**
1	0	1	2	3
2	1	0	1	2
3	2	1	0	1
4	3	2	1	0

$$G(x) = \frac{2(1+2+3+1+2+1)}{(2)(4)^2(10/4)} = \frac{20}{80} = 0.25$$

Lorenz Consistent Inequality Measures

- Gini Coefficient: Relationship with the Lorenz Curve

$$G = \frac{A}{OCD} = 2A$$



Lorenz Consistent Inequality Measures

Gini Coefficient:

- It satisfies the four basic axioms.

- It is not transfer-sensitive in the 'traditional way', where the impact of a transfer on inequality depends on the income levels. Because it is rank-based, the sensitivity of the Gini depends on the number of people in between. The higher the number of people in between, the bigger the impact of the transfer.

- It is not subgroup consistent.

- It can not be decomposed into the between-within group terms but in this way:

$$G(x,y) = G[W] + G[B] + R = \left[\left(\frac{\mu_x n_x^2}{\mu n^2}\right)G(x) + \left(\frac{\mu_y n_y^2}{\mu n^2}\right)G(y)\right] + G(\bar{x},\bar{y}) + R$$

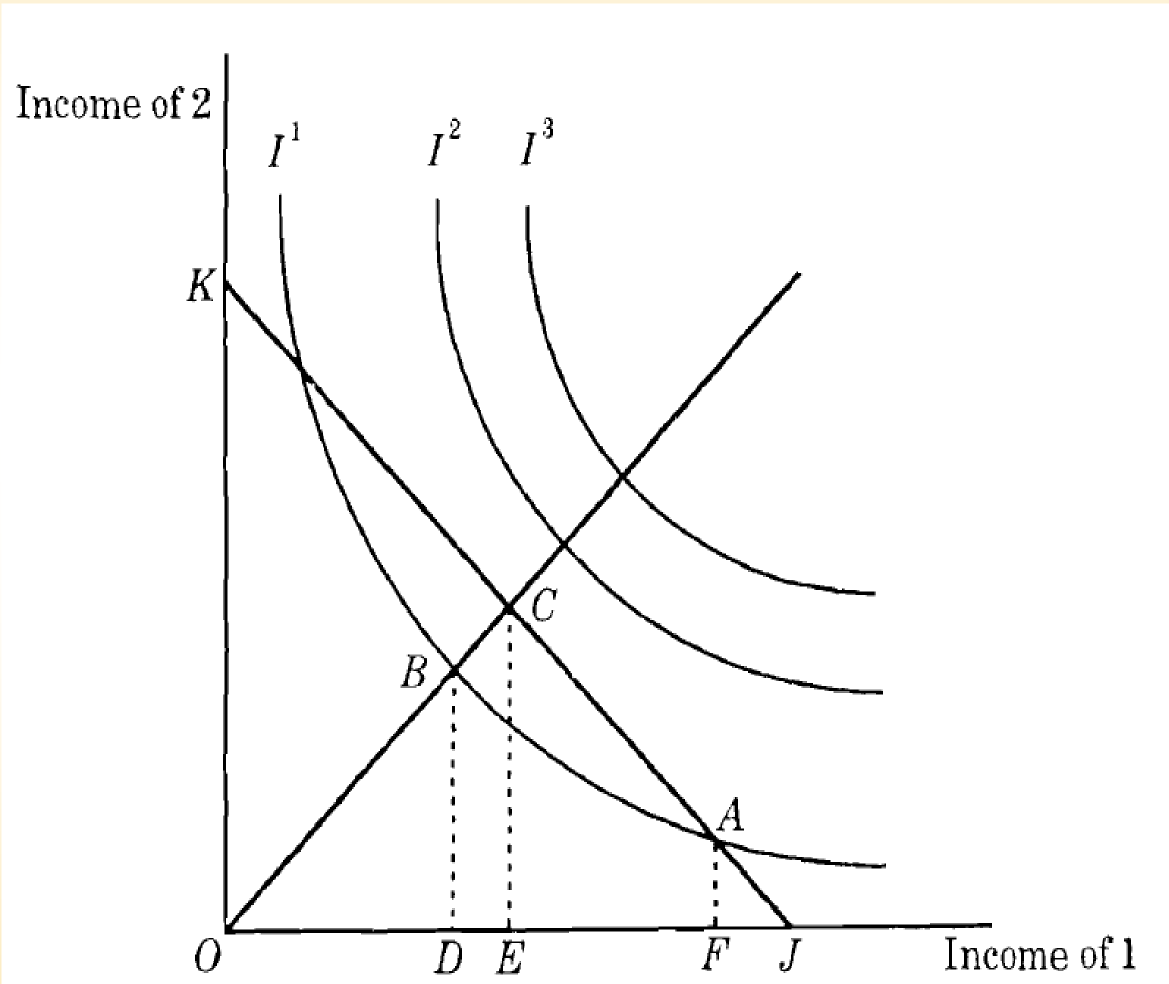
R is a non-negative residual term that balances the equation. It indicates the extent to which the subgroups' distributions overlap.

Atkinson's Measures of Inequality

- Core Concept:

  Equally Distributed Equivalent Income (EDE): The income level which, if assigned to all individuals produces the same social welfare than the observed distribution.

Atkinson's Inequality



- OJ: Total given income
- JK: Set of all possible distributions of OJ.
- I1, I2, I3: Social Welfare Levels
- A: Actual Distribution (1:OF, 2: AF)
- CE: Mean Income
- BD: Equally Distributed Equivalent Income

Atkinson's Measures of Inequality

- Assuming this utility function (Constant Relative Risk Aversion) and additive welfare

$$u(x_i) = \begin{cases} A + B\dfrac{x_i^\beta}{\beta} & \beta < 1, \beta \neq 0 \\ \ln x_i & \beta = 0 \end{cases} \qquad W(x) = \frac{1}{n}\sum_{i=1}^{n} u(x_i)$$

The income level which, if assigned to all individuals produces the same social welfare than the observed distribution

$$A + B\frac{x_{EDE}^\beta}{\beta} = \frac{1}{n}\sum_{i=1}^{n}\left(A + B\frac{x_i^\beta}{\beta}\right)$$

$$x_{EDE} = \begin{cases} \left[\dfrac{1}{n}\displaystyle\sum_{i=1}^{n} x_i^\beta\right]^{1/\beta} & \beta \leq 1, \beta \neq 0 \\ \prod_{i=1}^{n} x_i^{1/n} & \beta = 0 \end{cases}$$

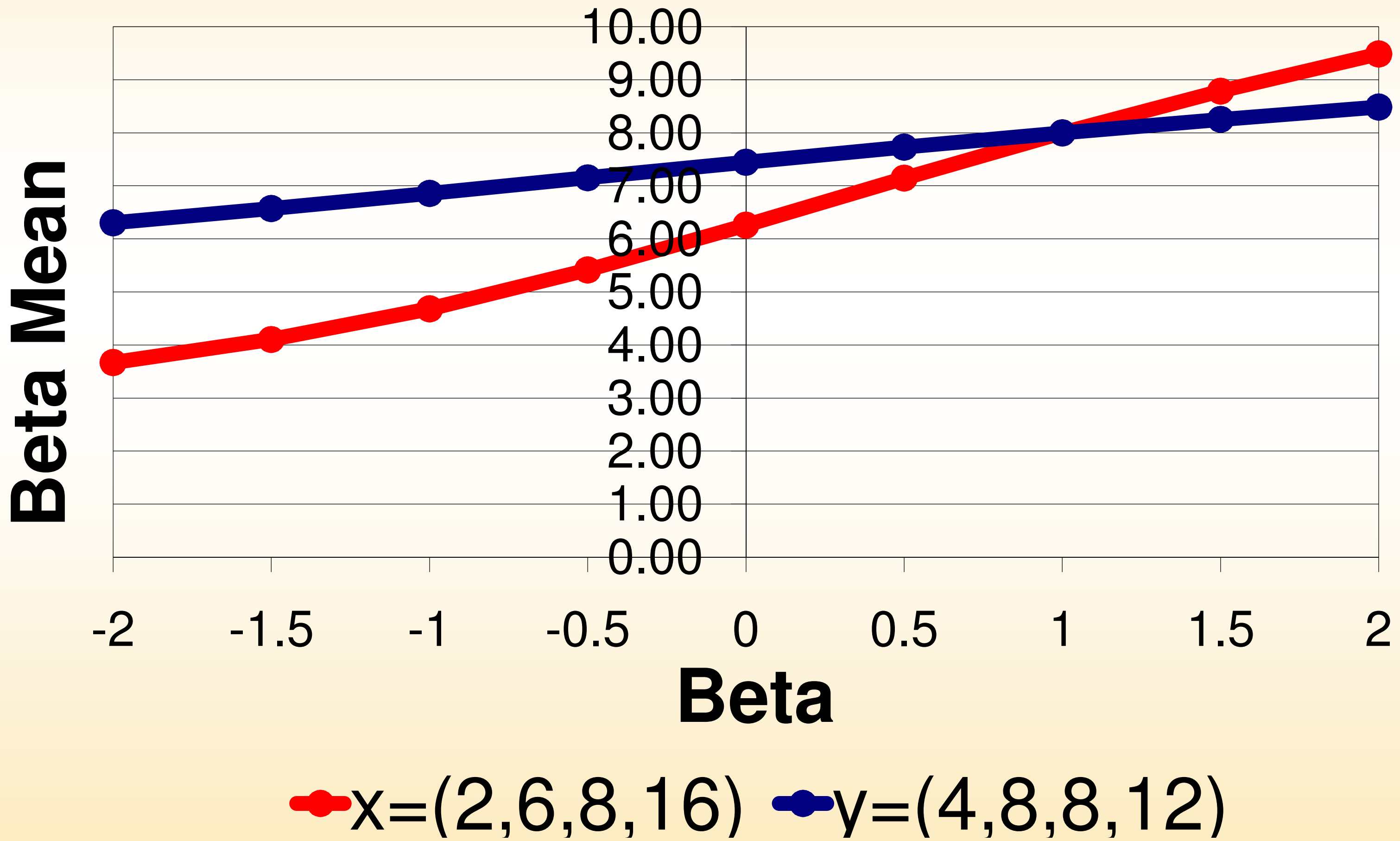
These are also known as the General Means of Order Beta (in this case for α≤1)

Atkinson's EDE income and Gral Means

$$\mu_\beta(x_i) = \begin{cases} [(1/n)\sum_{i=1}^{n}(x_i)^\beta]^{1/\beta} & \beta \neq 1 \\ \prod_{i=1}^{n}(x_i)^{1/n} & \beta = 0 \end{cases}$$

- Increasing in β
- When β =1, arithmetic mean.
- When β >1, more weight on higher incomes.
- When β <1, more weight on lower incomes. This is Atkinson's EDE income. The higher the inequality, the lower is the β-mean with respect to the mean.

Atkinson's EDE income and Gral Means



Lorenz Consistent Inequality Measures

- Atkinson's Measures

$$A = 1 - \frac{x_{EDE}}{\mu} \qquad A_\beta(x) = \begin{cases} 1 - \left[\frac{1}{n} \sum_{i=1}^{n} \left(\frac{x_i}{\mu} \right)^{\beta} \right]^{1/\beta} & \beta < 1, \beta \neq 0 \\ 1 - \prod_{i=1}^{n} \left(\frac{x_i}{\mu} \right)^{1/n} & \beta = 0 \end{cases}$$

- All the members in the family are Lorenz consistent.
- Parameter β is a measure of 'inequality aversion' or relative sensitivity of transfers at different income levels. The lower is β, the higher is the aversion to inequality and more weight is attached to transfers at the lower end of the distribution.
- Each member is subgroup consistent but it is not additively decomposable.

Lorenz Consistent Inequality Measures

- Generalized Entropy Measures

$$I_\beta(x) = \begin{cases} \dfrac{1}{\beta(1-\beta)} \dfrac{1}{n} \sum_{i=1}^{n} \left[1 - \left(\dfrac{x_i}{\mu} \right)^{\beta} \right] & \beta \neq 0,1 \\ I_1(x) = T(x) = \dfrac{1}{n} \sum_{i=1}^{n} \dfrac{x_i}{\mu} \ln\left(\dfrac{x_i}{\mu} \right) & \beta = 1 \\ I_0(x) = D(x) = \dfrac{1}{n} \sum_{i=1}^{n} \ln\left(\dfrac{\mu}{x_i} \right) & \beta = 0 \end{cases}$$

- When β=1, it is the **Theil first measure**.
- When β=0, it is the **Theil second measure**, also known as Mean Logarithmic Deviation
- When β=2, it is a multiple of the Squared Coefficient of Variation.

Lorenz Consistent Inequality Measures

- Generalized Entropy Measures

- Each I_β is a monotonic transformation of Atkinson's measure.
- The family is Lorenz Consistent.
- Parameter β is an indicator of 'inequality aversion' (more averse as β falls). It also indicates the measure's sensitivity to transfers at different parts of the distribution:
  - With $\beta = 2$, it is 'transfer neutral'. Multiple of the CV^2.
  - With $\beta < 2$, it favours transfers at the lower end of the distribution (includes both Theil's measures).
  - With $\beta > 2$ it shows a kind of 'reverse sensitivity' stressing transfers at higher incomes. (These are not used)

Lorenz Consistent Inequality Measures

- Generalized Entropy Measures GE
- They are additively decomposable with weights being:

$$w_x = (n_x / n)(\mu_x / \mu)^{\beta}$$

- Only the Theil's Measures (β=1 and β=0) are the ones with weights that sum up exactly to 1.
- *I* is a Lorenz consistent, normalized, continuous and **additively decomposable** inequality measure iff it is a positive multiple of a GE measure. (Shorrocks,1980, 1984) **That means there is only one class of ineq. measures that are additively decomposable.**