Multidimensional Well-Being and Inequality Indices

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    - The human development index (UNDP)

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- Multidimensional indices

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- HDI $= \frac{1}{3} \times 0.58 + \frac{1}{3} \times 0.64 + \frac{1}{3} \times 0.61 = 0.61$

2004 HDI Table: Top Ten Countries

Rank	**Country**	**HDI**	**LE Index**	**Edu Index**	**GDP Index**
1	Norway	0.965	0.909	0.993	0.993
2	Iceland	0.960	0.931	0.981	0.968
3	Australia	0.957	0.925	0.993	0.954
4	Ireland	0.956	0.882	0.990	0.995
5	Sweden	0.951	0.922	0.982	0.949
6	Canada	0.950	0.919	0.970	0.959
7	Japan	0.949	0.953	0.945	0.948
8	United States	0.948	0.875	0.971	0.999
9	Switzerland	0.947	0.928	0.946	0.968
10	Netherlands	0.947	0.892	0.987	0.962

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    - Foster, López-Calva, Székely (2005)

Hicks (1997): Inequality Adjusted HDI (IAHDI)

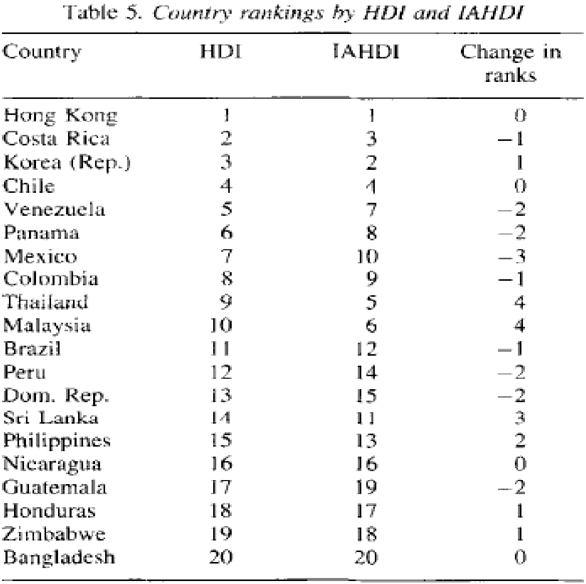

Table 5. *Country rankings by HDI and IAHDI*

Country	HDI	IAHDI	Change in ranks
Hong Kong	1	1	0
Costa Rica	2	3	−1
Korea (Rep.)	3	2	1
Chile	4	4	0
Venezuela	5	7	−2
Panama	6	8	−2
Mexico	7	10	−3
Colombia	8	9	−1
Thailand	9	5	4
Malaysia	10	6	4
Brazil	11	12	−1
Peru	12	14	−2
Dom. Rep.	13	15	−2
Sri Lanka	14	11	3
Philippines	15	13	2
Nicaragua	16	16	0
Guatemala	17	19	−2
Honduras	18	17	1
Zimbabwe	19	18	1
Bangladesh	20	20	0

Foster et. al. (2005): Generalized Mean HDI (HDI-GM)

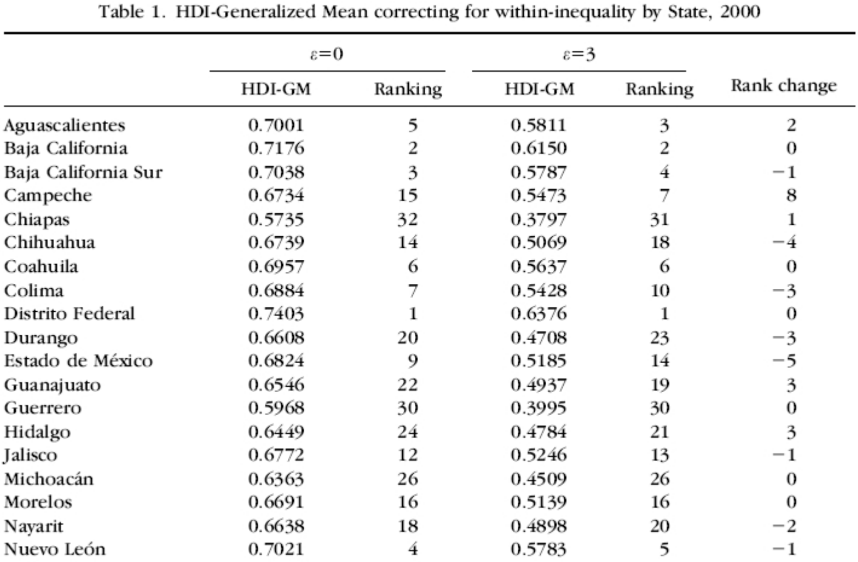

Table 1. HDI-Generalized Mean correcting for within-inequality by State, 2000

	$\varepsilon=0$		$\varepsilon=3$		
	HDI-GM	Ranking	HDI-GM	Ranking	Rank change
Aguascalientes	0.7001	5	0.5811	3	2
Baja California	0.7176	2	0.6150	2	0
Baja California Sur	0.7038	3	0.5787	4	−1
Campeche	0.6734	15	0.5473	7	8
Chiapas	0.5735	32	0.3797	31	1
Chihuahua	0.6739	14	0.5069	18	−4
Coahuila	0.6957	6	0.5637	6	0
Colima	0.6884	7	0.5428	10	−3
Distrito Federal	0.7403	1	0.6376	1	0
Durango	0.6608	20	0.4708	23	−3
Estado de México	0.6824	9	0.5185	14	−5
Guanajuato	0.6546	22	0.4937	19	3
Guerrero	0.5968	30	0.3995	30	0
Hidalgo	0.6449	24	0.4784	21	3
Jalisco	0.6772	12	0.5246	13	−1
Michoacán	0.6363	26	0.4509	26	0
Morelos	0.6691	16	0.5139	16	0
Nayarit	0.6638	18	0.4898	20	−2
Nuevo León	0.7021	4	0.5783	5	−1

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- Infant survival rate (health variable) varies across municipalities

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- Symmetry in Persons (SP). Personal identity does not matter
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  - Both sequences of aggregation yield the same result. **Path Independence** (PI) - *sequence of aggregation is not important* (Foster, López-Calva, Székely (2005)

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- First Stage: Generalized mean across persons yields $(0.41, 0.41, 0.42)$.

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- Therefore, both W_H and W_F are sensitive to inequality across persons

Policy Exercise

- Reconsider the achievement matrix

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- X =

	Dim 1	Dim 2	Dim 3
Person 1	0.8	0.8	0.3
Person 2	0.4	0.3	0.8
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Policy Exercise

H =

	Dim 1	Dim 2	Dim 3
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- Where should the dollar be spent from an ethical point of view?

Policy Exercise

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  - Suppose, capability of the n^{th} individual $= (x_{n1} + x_{n2} + x_{n3})/3$

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  - Suppose, capability of the n^{th} individual $= \left(x_{n1} + x_{n2} + x_{n3}\right)/3$
    - Achievement vector across individuals: $(0.63, 0.5, 0.37)$

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Association Sensitivity

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  - $H = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.4 & 0.3 & 0.8 \\ 0.3 & 0.4 & 0.4 \end{bmatrix}, H' = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.4 & \mathbf{0.4} & 0.8 \\ 0.3 & \mathbf{0.3} & 0.4 \end{bmatrix}$

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- Association Sensitivity
  - Strictly decreasing in increasing association (SDIA) - $W(H') < W(H)$
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- **Proposition:** A well-being index that aggregates across persons first and then across dimensions is not sensitive to association among dimensions

Association Sensitivity

- **Corollary:** No path independent well-being index is sensitive to association among dimensions

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    - First stage: aggregates across dimensions by $\mu_\beta(\cdot)$. Second stage: aggregates across persons by $\mu_\alpha(\cdot)$

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- $\mathcal{W}$ satisfies NM, LH, SP, M, PRI, CN, SC, UM, and
    - SDIA if and only if $\alpha < \beta \leq 1$

Policy Exercise

H =

	Dim 1	Dim 2	Dim 3
Person 1	0.8	0.8	0.3
Person 2	0.4	0.3	0.8
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- Where should the dollar be spent according to $\mathcal{W}$?

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	Dim 1	Dim 2	Dim 3
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- Where should the dollar be spent according to $\mathcal{W}$?
  - Suppose, $\alpha = -2$ and $\beta = 0.1$

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	Dim 1	Dim 2	Dim 3
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  - Spend the dollar on dim 1 of person 3

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- Where should the dollar be spent according to $\mathcal{W}$?
  - Suppose, $\alpha = -2$ and $\beta = 0.1$
  - Spend the dollar on dim 1 of person 3
    - Total well-being is $= 0.465$

Policy Exercise

H =

	Dim 1	Dim 2	Dim 3
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    - Suppose, $\alpha = -2$ and $\beta = 0.1$
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Policy Exercise

H =

	Dim 1	Dim 2	Dim 3
Person 1	0.8	0.8	0.3
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- Where should the dollar be spent according to $\mathcal{W}$?
  - Suppose, $\alpha = -2$ and $\beta = 0.1$
  - Spend the dollar on dim 1 of person 3
    - Total well-being is $= 0.465$
  - Spend the dollar on dim 2 of person 2
    - Total well-being is $= 0.456$

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  - Suppose, $\alpha = -2$ and $\beta = 0.1$
  - Spend the dollar on dim 1 of person 3
    - Total well-being is $= 0.465$
  - Spend the dollar on dim 2 of person 2
    - Total well-being is $= 0.456$
  - Spend the dollar on dim 3 of person 1

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	Dim 1	Dim 2	Dim 3
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- Where should the dollar be spent according to $\mathcal{W}$?
  - Suppose, $\alpha = -2$ and $\beta = 0.1$
  - Spend the dollar on dim 1 of person 3
    - Total well-being is $= 0.465$
  - Spend the dollar on dim 2 of person 2
    - Total well-being is $= 0.456$
  - Spend the dollar on dim 3 of person 1
    - Total well-being is $= 0.452$

Application to Mexico (Income, Education, and Health)

State	HDI (W_A)	W_F $\alpha = -2$	$\mathcal{W}$ $\beta = -1$ $\alpha = -3$
San Luis Potosí	0.716 (24)	0.258 (21)	0.223 (22)
Sinaloa	0.751 (17)	0.268 (20)	0.232 (18)
Sonora	0.790 (07)	0.386 (06)	0.309 (06)
Tabasco	0.719 (22)	0.296 (15)	0.254 (14)
Tamaulipas	0.771 (12)	0.349 (08)	0.287 (08)
Tlaxcala	0.736 (19)	0.309 (13)	0.258 (12)
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- A sequence of association increasing transfers for Tabasco

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- A sequence of association increasing transfers for Tabasco
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State	HDI (W_A)	W_F $\alpha = -2$	$\mathcal{W}$ $(\beta = -1, \alpha = -3)$
Tabasco	0.719 (22)	0.296 (15)	0.244 (15)

Summary

- Additive Indices are not sensitive to inequality across persons

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- The first form fails to provide proper policy implication
- Dimensional interactions are important
- Aggregation must take place across dimensions first, and then across persons
- We treated dimensions symmetrically; we could also apply weighted generalized mean
- A well-being index yet to be derived that takes different elasticity of substitution into account

Inequality Measures

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Inequality Measures

- Study of inequality is important on its own
- Multiple dimensions vs single-dimension
- Construction of inequality indices
- Association Sensitivity: aggregate first across dimensions
- Weighted generalized mean of order γ
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  - $\gamma = 1$: Weighted AM $\left(\mu_{1,a}\right)$; $\gamma = 0$: Weighted GM $\left(\mu_{0,a}\right)$; $\gamma = -1$: Weighted HM $\left(\mu_{-1,a}\right)$

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- Continuity (CN). I(H) does not change abruptly due to a change in any of the elements in H

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- Problems: role of inequality aversion parameter is not clear

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$$\mathrm{I}_M = \begin{cases} \frac{1}{\alpha(1-\alpha)} \frac{1}{N} \sum_{i=1}^{n} \left(1 - \left(\frac{U_n}{\bar{S}}\right)^{\alpha}\right) & \text{for } \alpha \neq 0, 1. \\ \frac{1}{N} \sum_{i=1}^{n} \log\left(\frac{\bar{S}}{U_n}\right) & \text{for } \alpha = 0 \\ \frac{1}{N} \sum_{i=1}^{n} \frac{U_n}{\bar{S}} \log\left(\frac{U_n}{\bar{S}}\right) & \text{for } \alpha = 1 \end{cases}$$

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- Problems: Not sure what restrictions on parameter satisfies different transfer properties

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- Problem: Tsui parameters are not interpretable.

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$$U_d = \sum_{n=1}^{N} \left(\frac{2n-1}{N^2} \right) \tilde{x}_n$$

where $\tilde{x}$ is obtained by arranging $\{x\}_{n=1}^{N}$ in a descending order.

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$$\mathrm{I}_{DL} = \sum_{n=1}^{N} \left(\frac{2n-1}{N^2} \right) \tilde{U}_n$$

where $\tilde{U}_n$ is obtained by arranging $\left\{ U_n \right\}_{n=1}^{N}$ in a descending order.

Summary

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- Association sensitivity requires aggregation across dimenisons first and then across persons