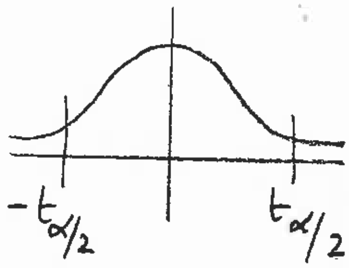
Hypothesis Testing in Classical Regression

Tests of Significance

$H_0: \ \beta_j = 0 \qquad (H_a: \ \beta_j \neq 0)$

$$t_j = \frac{\hat{\beta}_j}{\sqrt{\hat{V}(\hat{\beta}_j)}} \sim t_{T-k}$$



$|t_j| > t_{\alpha/2} \Rightarrow H_0$ rejected

Tests of Linear Restrictions

$H_0: \ R\beta = \alpha$; R $(m \times k)$ of rank m

$(H_a: \ R\beta \neq \alpha)$

3 approaches: Wald, LM, LR

Wald

$$W = (\hat{\alpha} - \alpha)' \left[V(\hat{\alpha})\right]^{-1} (\hat{\alpha} - \alpha) \sim \chi^2_m$$

where $\hat{\alpha} = R\hat{\beta}$, $V(\hat{\alpha})$ = true variance of $\hat{\alpha}$ (with true σ^2)

$W > W^*_\alpha \Rightarrow H_0$ rejected

If σ^2 unknown, estimate by $\hat{\sigma}^2$.

Then

$$F = (\hat{\alpha} - \alpha)' \left[\hat{V}(\hat{\alpha})\right]^{-1} (\hat{\alpha} - \alpha)/m \sim F_{m,T-K}$$

$F > F^*_\alpha \Rightarrow$ reject H_0

(Here $\hat{V}(\hat{\alpha})$ is with $\hat{\sigma}^2$ instead of σ^2)

Also

$$F = \frac{(SS_r - SS)/m}{SS/T-K} \sim F_{m,T-K}$$

Lagrange Multiplier (LM)

$$LM_1 = \hat{\lambda}' \left[V(\hat{\lambda})\right]^{-1} \hat{\lambda} \sim \chi^2_m$$

where λ = multiplier associated with $(R\beta = \alpha)$ in the constrained estimation procedure

With σ^2 replaced by $\hat{\sigma}^2$,

$$LM_2 = \hat{\lambda} \left[\hat{V}(\hat{\lambda})\right]^{-1} \hat{\lambda} \sim F_{m,T-K}$$

<u>Note</u>: If σ^2 known, $W = LM_1$

Likelihood Ratio (LR)

$$LR = -2 \ln \left(\frac{L_r^*}{L^*} \right) \sim \chi^2_m \quad \left(\text{with } \sigma^2 \text{ known}\right)$$

If σ^2 unknown, distribution (exact) of LR not known.

Asymptotically, $LR \sim \chi^2_m$

Note: If σ^2 known, $W = LM_1 = LR$

Zero Slopes test

$H_0: \ \beta_2 = \cdots = \beta_K = 0$

Restricted Model: $y_t = \beta_1 + \varepsilon_t$

Unrestricted Model: $y_t = \beta' x_t + \varepsilon_t$

Then the F-test becomes

$$F = \frac{R^2/(K-1)}{(1-R^2)/(T-K)} \sim F_{K-1,\, T-K}$$

(R^2 is that of the Unrestricted Model)