

Qualitative Response

Examples:

Poor or not

Catastrophic health expenditures or not

Employed or not

Model:

$$y = \begin{cases} 1 & \text{if poor} \\ 0 & \text{otherwise} \end{cases}$$

Postulate existence of an underlying latent variable y^* , say vulnerability, such that

$$y^* = x'\beta + \varepsilon$$

Then, $y = 1 \Leftrightarrow y^* > \text{threshold, say } c$

$$\begin{aligned} \Rightarrow P(y=1) &= P(y^* > c) = P(x'\beta + \varepsilon > c) \\ &= P(\varepsilon > c - x'\beta) = F(c + x'\beta) \end{aligned}$$

and

$$P(y=0) = 1 - F(c + x'\beta)$$

Estimation by Maximum Likelihood:

$$\max_{\beta} \log \text{Likelihood} =$$

$$\sum_{i \in \text{poor}} \log F(c + x_i'\beta) + \sum_{i \notin \text{poor}} \log [1 - F(c + x_i'\beta)]$$

Qualitative Response (contd.)

Marginal Effects:

$$\begin{aligned} \frac{\partial P(y=1)}{\partial x_k} &= \frac{\partial F(c+x'\beta)}{\partial x_k} = \frac{\partial F}{\partial (c+x'\beta)} \frac{\partial (c+x'\beta)}{\partial x_k} \\ &= f(-c+x'\beta) \cdot \beta_k \end{aligned}$$

Choice of F:

F = Normal Distribution \rightarrow Probit Model

F = Logistic Distribution \rightarrow Logit Model

Remark

For identification purposes we set $c=0$, $V(u_i)=1 \forall i$

Extensions:

- Multinomial:
 - More than 2 outcomes, say m outcomes
 - No hierarchy in response/outcome
- Ordered Categorical
 - More than 2 outcomes, say m outcomes
 - There is a hierarchy in the outcomes/responses.