Classical Regression: Reminder

Model: $y = X\beta + \varepsilon$ (T observations, K columns in X of rank K)

Assumptions:

A1. X non-stochastic (or independent of ε)

A2. $\varepsilon \sim (0, \sigma^2 I_T)$

A3. For ML, $\varepsilon \sim N(0, \sigma^2 I)$

OLS:

$$\hat{\beta} = (X'X)^{-1}X'y \; ; \quad E(\hat{\beta}) = \beta$$
$$V(\hat{\beta}) = \sigma^2 (X'X)^{-1}$$

$$\hat{\sigma}^2 = \frac{1}{T-K}\hat{\varepsilon}'\hat{\varepsilon}, \quad \hat{\varepsilon} = y - X\hat{\beta}$$
$$E(\hat{\sigma}^2) = \sigma^2$$
$$V(\hat{\sigma}^2) = \frac{2}{T-K}\sigma^4 \text{ under A3}$$

OLS is BLUE

$$R^2 = 1 - \frac{\hat{\varepsilon}'\hat{\varepsilon}}{(y-\iota\bar{y})'(y-\iota\bar{y})}, \quad \bar{y} = \frac{1}{T}\iota_T'y$$

ML:

$$\hat{\beta} = (X'X)^{-1}X'y$$
$$\hat{\sigma}^2 = \frac{1}{T}\hat{\varepsilon}'\hat{\varepsilon}, \quad E(\hat{\sigma}^2) \neq \sigma^2$$

ML is BUE (OLS = ML = BUE)