

# Instrumental Variable Estimation (IV)

Model :  $y = X\beta + \varepsilon$

Assumptions :  $X$  stochastic,  $T \times k$   
 $X$  correlated with  $\varepsilon$

Instrument Matrix  $Z$  :  $T \times m$ ,  $m \geq k$

- $Z$  correlated with  $X$
- $Z$  uncorrelated with  $\varepsilon$

Then, transform model by  $z'$  and apply GLS on the transformed model:

If  $V(\varepsilon) = \sigma^2 I$ , then

$$\hat{\beta}_{IV} = [X'Z (Z'Z)^{-1} Z'X]^{-1} X'Z (Z'Z)^{-1} Z'y$$

If  $V(\varepsilon) = \sigma^2 V$ , then

$$\hat{\beta}_{IV} = [X'Z (Z'VZ)^{-1} Z'X]^{-1} X'Z (Z'VZ)^{-1} Z'y$$