

Tests of Overidentifying Restrictions:

(in an instrumental variable or GMM context)

Model: $y = X\beta + \varepsilon$, $V(\varepsilon) = \sigma^2 I$
($N \times k$)

Instruments: z : $E(z_i \varepsilon_i) = 0$
($N \times p$)

Denote: $\hat{m}_N = \frac{1}{N} \sum_i z_i \varepsilon_i = \frac{1}{N} z' \varepsilon$

$$\hat{S}_N = \frac{1}{N} \sum_i \varepsilon_i^2 z_i z_i' = \sigma^2 \frac{1}{N} z' z$$

Then $N \cdot \hat{m}_N(\hat{\beta})' \hat{S}_N^{-1} \hat{m}_N(\hat{\beta}) \sim \chi_{p-k}^2$

Note: One can take $\hat{S}_N = \frac{1}{N} \sum_i \varepsilon_i^2 z_i z_i'$

High values indicate problems with the instruments.

This test is known as Sargan's test of overidentifying restrictions as Sargan derived it first in the linear case and Hansen extended it in the non-linear GMM context.