

# Generalised Method of Moments (GMM)

Model:  $y = X\beta + \varepsilon$

Assumption:  $X$  stochastic say  $T \times K$   
 $X$  correlated with  $\varepsilon$

Instrument matrix  $Z$ :  $T \times m$ ,  $m \geq k$

- $Z$  uncorrelated with  $\varepsilon$
- $Z$  correlated with  $X$

Then orthogonality / moment conditions:

$$E(z'e) = 0 \quad \text{or} \quad E(z_t \varepsilon_t) = 0$$

Optimisation problem of GMM:

$$\min_{\beta} \|z'e\|_W \quad (\text{norm of } z'e \text{ in metric } W \text{ to be determined})$$

$$\min_{\beta} \varepsilon' z W z' \varepsilon \quad \text{s.t. Asy. Var } (\hat{\beta}) \text{ minimal}$$

$$\Rightarrow \text{Solution: } \hat{\beta}_{\text{gmm}} = [X'Z(Z'Z)^{-1}Z'X]^{-1} X'Z(Z'Z)^{-1} Z'y$$

if  $V(\varepsilon) = \sigma^2 I$

or

$$\hat{\beta}_{\text{gmm}} = [X'Z(Z'VZ)^{-1}Z'X]^{-1} X'Z(Z'VZ)^{-1} Z'y$$

if  $V(\varepsilon) = \sigma^2 V$