Generalised Regression

Model: $$y = X\beta + \varepsilon$$

Assumptions:

A1. X non-stochastic (or indep. of ε) of rank k

A2. $\varepsilon \sim (0, \sigma^2 V)$ V positive def.

A3. For ML, $\varepsilon \sim N(0, \sigma^2 V)$

GLS: (V known)

$$\hat{\beta} = (X' V^{-1} X)^{-1} X' V^{-1} y$$

$$E(\hat{\beta}) = \beta$$

$$V(\hat{\beta}) = \sigma^2 (X' V^{-1} X)^{-1}$$

$$\hat{\sigma}^2 = \frac{1}{T-K} \hat{\varepsilon}' V^{-1} \hat{\varepsilon} \quad ; \quad E(\hat{\sigma}^2) = \sigma^2$$

ML: (V known)

$$\hat{\beta} = (X' V^{-1} X)^{-1} X' V^{-1} y$$

$$\hat{\sigma}^2 = \frac{1}{T} \hat{\varepsilon}' V^{-1} \hat{\varepsilon}$$

Feasible GLS (V unknown, $V = V(\underset{p\times 1}{\theta})$)

Two stages:

(1) Estimate θ consistently $\rightarrow \hat{\theta}$
(usually from an OLS estimation)

(2) $\hat{V} = V(\hat{\theta})$ and apply GLS with $\hat{V}$ instead of V

Feasible GLS and GLS have the same limiting distribution.

ML: (for unknown V) Simultaneous maximisation of log-likelihood with respect to β, σ^2 and θ.

$$R^2 = 1 - \frac{\hat{\varepsilon}'\hat{\varepsilon}}{(y - \iota\bar{y})'(y - \iota\bar{y})}$$ is

not necessarily bounded between 0 and 1
(can become negative if fit poor)