Summer School on Capability and Multidimensional Poverty

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HDCP-IRC

The Human Development, Capability and International Research Centre
Instituto Universitario di Studi Superiori
www.iusspavia.it

OPHI

Oxford Poverty & Human Development Initiative
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Measuring Multidimensional Poverty

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Outline

- Order of Aggregation and MD measures
- Axiomatic MD measure
- Discuss :
  - Substitutes and Complements
  - Weights
  - Axiomatic vs Information Theory vs Fuzzy
- Features vis a vis capability approach

MD Poverty & Capability Approach

- Focus on *Individuals* as unit of analysis when possible
- Each dimension might be of *intrinsic importance*, whether or not it is also instrumentally effective
- Normative Value Judgments:
  - Choice of dimensions
  - Choice of poverty lines
  - Choice of weights across dimensions

Order of Aggregation

- First across people, then across dimensions (e.g. HPI).
  - Aggregate data are widely available – so simple, less sophisticated.
  - Can combine different data sources
  - Can combine with distribution information
  - Cannot speak about breadth of poverty,
  - May not be able to decompose by state or smaller groups

Order of Aggregation

- First across dimensions, then across people (e.g. this class).
  - Coheres with a normative focus on individual deprivations.
  - Has information that can penalise breadth as well as depth of deprivation
  - Decomposable as far as data allows.
  - Can combine with distribution information
  - Requires all questions from same dataset
  - [if desired, the measure can represent interaction – substitutability/complementarity – between dimensions]

Bourguignon & Chakravarty 2003 express an emerging preference for aggregation first across dimensions:

- "The fundamental point in all what follows is that a multidimensional approach to poverty defines poverty as a shortfall from a threshold on each dimension of an individual's well being. In other words, *the issue of the multidimensionality of poverty arises because individuals, social observers or policy makers want to define a poverty limit on* each *individual attribute: income, health, education, etc…*"

Multidimensional Poverty- our challenge:

- A government would like to create an official multidimensional poverty indicator
- Desiderata
  - It must understandable and easy to describe
  - It must conform to “common sense” notions of poverty
  - It must be able to target the poor, track changes, and guide policy.
  - It must be technically solid
  - It must be operationally viable
  - It must be easily replicable
- What would you advise?

Multidimensional Poverty [or well-being] Comparisons

- **How do we create an Index?**
  - Choice of Unit of Analysis (indy, hh, cty)
  - Choice of Dimensions
  - Choice of Variables/Indicator(s) for dimensions
  - Choice of Poverty Lines for each indicator/dimension
  - Choice of Weights for indicators within dimensions
  - If more than one indicator per dimension, aggregation
  - Choice of Weights across dimensions
  - Identification method
  - Aggregation method – within and across dimensions.

  Particular Challenges:
  - Needs to be technically robust for policy analysis
  - Needs to be valid for Ordinal data

How to Measure?

- Variables
- Identification
- Aggregation

Our Proposal

- Variables – Assume given
- Identification – Dual cutoffs
- Aggregation – Adjusted FGT

Review: Unidimensional Poverty

Variable – income

Identification – poverty line

Aggregation – Foster-Greer-Thorbecke '84

Example Incomes = (7,3,4,8) poverty line z = 5

Deprivation vector $g^0 = (0,1,1,0)$

Headcount ratio $P_0 = \mu(g^0) = 2/4$

Normalized gap vector $g^1 = (0, 2/5, 1/5, 0)$

Poverty gap = $P_1 = \mu(g^1) = 3/20$

Squared gap vector $g^2 = (0, 4/25, 1/25, 0)$

FGT Measure = $P_2 = \mu(g^2) = 5/100$

Multidimensional Data

Matrix of well-being scores for n persons in d domains

$$\begin{array}{cc} & \text{Domains} \\ \boldsymbol{y} = & \begin{bmatrix} \mathbf{13.1} & \mathbf{14} & \mathbf{4} & \mathbf{1} \\ \mathbf{15.2} & \mathbf{7} & \mathbf{5} & \mathbf{0} \\ \mathbf{12.5} & \mathbf{10} & \mathbf{1} & \mathbf{0} \\ \mathbf{20} & \mathbf{11} & \mathbf{3} & \mathbf{1} \end{bmatrix} & \text{Persons} \end{array}$$

Multidimensional Data

Matrix of well-being scores for *n* persons in *d* domains

$$
\begin{array}{cc}
\text{Domains} & \\
y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & 7 & 5 & 0 \\ 12.5 & 10 & 1 & 0 \\ 20 & 11 & 3 & 1 \end{bmatrix} & \text{Persons}
\end{array}
$$

$$
z \quad (13 \quad 12 \quad 3 \quad 1) \quad \text{Cutoffs}
$$

Multidimensional Data

Matrix of well-being scores for n persons in d domains

$$
\begin{array}{ccc}
 & \text{Domains} & \\
y = & \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix} & \text{Persons}
\end{array}
$$

$$
z \quad (13 \quad 12 \quad 3 \quad 1) \quad \text{Cutoffs}
$$

<u>These entries fall below cutoffs</u>

Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

$$y = \begin{array}{c} \text{Domains} \\ \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix} \end{array} \quad \text{Persons}$$

Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

$$\boldsymbol{g}^{0} = \begin{array}{c} \text{Domains} \\ \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} \end{array} \quad \text{Persons}$$

Normalized Gap Matrix

Matrix of well-being scores for *n* persons in *d* domains

$$\begin{array}{ccl} & \text{Domains} & \\ y = & \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix} & \text{Persons} \\ z & (\,13 \quad 12 \quad 3 \quad 1\,) & \text{Cutoffs} \end{array}$$

<u>These entries fall below cutoffs</u>

Gaps

Normalized gap = $(z_j - y_{ji})/z_j$ if deprived, 0 if not deprived

$$
\begin{array}{cc}
 & \text{Domains} \\
\mathbf{y} = & \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix} \quad \text{Persons}
\end{array}
$$

$$\mathbf{z} \quad (\,13 \quad 12 \quad 3 \quad 1) \quad \text{Cutoffs}$$

<u>These entries fall below cutoffs</u>

Normalized Gap Matrix

Normalized gap = $(z_j - y_{ji})/z_j$ if deprived, 0 if not deprived

$$
\begin{array}{cc}
 & \text{Domains} \\
\mathbf{g}^1 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0.08 & 0 & 0 \end{bmatrix} \text{Persons}
\end{array}
$$

Squared Gap Matrix

Squared gap = $[(z_j - y_{ji})/z_j]^2$ if deprived, 0 if not deprived

$$
\begin{array}{cc}
 & \text{Domains} \\
\mathbf{g}^1 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0.08 & 0 & 0 \end{bmatrix} \text{ Persons}
\end{array}
$$

Squared Gap Matrix

Squared gap = $[(z_j - y_{ji})/z_j]^2$ if deprived, 0 if not deprived

$$
\begin{array}{cc}
 & \text{Domains} \\
g^2 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.176 & 0 & 1 \\ 0.002 & 0.029 & 0.449 & 1 \\ 0 & 0.006 & 0 & 0 \end{bmatrix} \text{Persons}
\end{array}
$$

Identification

$$
\begin{array}{cc}
 & \text{Domains} \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \text{Persons}
\end{array}
$$

Matrix of deprivations

Identification – Counting Deprivations

$$\begin{array}{ccccc} & \text{Domains} & & c & \\ g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & & \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix} & \text{Persons} \end{array}$$

Identification – Counting Deprivations

Q/ Who is poor?

$$\begin{array}{cc} & \text{Domains} \\ g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \quad \begin{array}{c} c \\ 0 \\ 2 \\ 4 \\ 1 \end{array} \quad \text{Persons}$$

Identification – Union Approach

Q/ Who is poor?

A1/ Poor if deprived in any dimension $c_i \geq 1$

$$
\begin{array}{cc}
 & \text{Domains} \qquad \qquad c \\
g^0 = & \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad \begin{matrix} \mathbf{0} \\ \mathbf{2} \\ \mathbf{4} \\ \mathbf{1} \end{matrix} \quad \text{Persons}
\end{array}
$$

Identification – Union Approach

Q/ Who is poor?

A1/ Poor if deprived in any dimension $c_i \geq 1$

$$
\begin{array}{cccc}
 & \text{Domains} & & c & \\
\mathbf{g^0} = & \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} & & \begin{array}{c} \mathbf{0} \\ \underline{\mathbf{2}} \\ \underline{\mathbf{4}} \\ \underline{\mathbf{1}} \end{array} & \text{Persons}
\end{array}
$$

Difficulties

Single deprivation may be due to something other than poverty (UNICEF)

Union approach often predicts *very* high numbers - political constraints.

Identification – Intersection Approach

Q/ Who is poor?

A2/ Poor if deprived in all dimensions $c_i = d$

$$
\begin{array}{cc}
 & \text{Domains} \qquad\qquad\qquad c \\
\mathbf{g}^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{array}{c} \mathbf{0} \\ \mathbf{2} \\ \mathbf{4} \\ \mathbf{1} \end{array} \quad \text{Persons}
\end{array}
$$

Identification – Intersection Approach

Q/ Who is poor?

A2/ Poor if deprived in all dimensions $c_i = d$

$$
\begin{array}{cccc}
 & \text{Domains} & & c \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & & \begin{matrix} 0 \\ 2 \\ \underline{4} \\ 1 \end{matrix} & \text{Persons}
\end{array}
$$

Difficulties

Demanding requirement (especially if d large)

Often identifies a very narrow slice of population

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $\mathbf{c_i \geq k}$

$$
\begin{array}{ccc}
 & \text{Domains} & c \\
\boldsymbol{g}^{0} = & \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} & \begin{array}{c} \mathbf{0} \\ \mathbf{2} \\ \mathbf{4} \\ \mathbf{1} \end{array} & \text{Persons}
\end{array}
$$

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $\mathbf{c_i \geq k}$ **(Ex: k = 2)**

$$
\begin{array}{ccc}
 & \text{Domains} & c \\
\mathbf{g^0} = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 1 \end{matrix}
\end{array}
\quad \text{Persons}
$$

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $\mathbf{c_i \geq k}$ **(Ex: k = 2)**

$$
\begin{array}{cc}
 & \text{Domains} \\
g^0 = & \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}
\end{array}
\quad
\begin{array}{c}
c \\ \mathbf{0} \\ \underline{\mathbf{2}} \\ \underline{\mathbf{4}} \\ \mathbf{1}
\end{array}
\quad \text{Persons}
$$

Note

Includes both union and intersection

Identification – Dual Cutoff Approach

Q/ Who is poor?

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$$
\begin{array}{cc}
 & \text{Domains} \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}
\end{array}
\qquad
\begin{array}{c} c \\ 0 \\ \underline{2} \\ \underline{4} \\ 1 \end{array}
\qquad \text{Persons}
$$

Note

Includes both union and intersection

Especially useful when number of dimensions is large

Union becomes too large, intersection too small

Identification – Dual Cutoff Approach

Q/ Who is poor?

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$$g^0 = \begin{array}{cc} \text{Domains} & c \\ \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} & \begin{array}{c} \mathbf{0} \\ \underline{\mathbf{2}} \\ \underline{\mathbf{4}} \\ \mathbf{1} \end{array} \end{array} \qquad \text{Persons}$$

Note

Includes both union and intersection

Especially useful when number of dimensions is large

Union becomes too large, intersection too small

Next step

How to aggregate into an overall measure of poverty

Aggregation

$$\mathbf{g}^{0} = \begin{array}{ccc} \text{Domains} & c & \\ \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} & \begin{array}{c} \mathbf{0} \\ \underline{\mathbf{2}} \\ \underline{\mathbf{4}} \\ \mathbf{1} \end{array} & \text{Persons} \end{array}$$

Aggregation

Censor data of nonpoor

$$g^0 = \begin{array}{cc} \text{Domains} & c \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 1 \end{matrix} \end{array} \quad \text{Persons}$$

Aggregation

Censor data of nonpoor

$$\begin{array}{cccc} & \text{Domains} & c(k) & \\ g^{0}(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} & \text{Persons} \end{array}$$

Aggregation

Censor data of nonpoor

$$\mathbf{g^{0}(\mathit{k})} = \begin{array}{ccc} \text{Domains} & c(k) & \\ \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} & \begin{array}{c} \mathbf{0} \\ \underline{\mathbf{2}} \\ \underline{\mathbf{4}} \\ \mathbf{0} \end{array} & \text{Persons} \end{array}$$

Similarly for $g^1(k)$, etc

Aggregation – Headcount Ratio

$$\begin{array}{ccccc} & \text{Domains} & & c(k) & \\ g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \text{Persons} \end{array}$$

Aggregation – Headcount Ratio

$$
\begin{array}{rccl}
 & \text{Domains} & c(k) & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \text{Persons}
\end{array}
$$

Two poor persons out of four: **H = 1/2**

Critique

Suppose the number of deprivations rises for person 2

$$\begin{array}{cccc} & \text{Domains} & c(k) & \\ g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \text{Persons} \end{array}$$

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Two poor persons out of four: **H = 1/2**

Critique

Suppose the number of deprivations rises for person 2

$$
\begin{array}{cc}
 & \begin{array}{c}\text{Domains}\end{array} \qquad c(k) \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{array} \quad \text{Persons}
\end{array}
$$

Two poor persons out of four: **H = 1/2**

No change!

Critique

Suppose the number of deprivations rises for person 2

Domains $\quad c(k)$

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{array} \quad \text{Persons}$$

Two poor persons out of four: **H = 1/2**

No change!

Violates ‘dimensional monotonicity’

Aggregation

Return to the original matrix

$$g^{0}(k) = \begin{array}{cc} \text{Domains} & c(k) \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{matrix} \end{array} \qquad \text{Persons}$$

Aggregation

Return to the original matrix

$$\begin{array}{cccc} & \text{Domains} & c(k) & \\ g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \text{Persons} \end{array}$$

Aggregation

Need to augment information

$$
g^0(k) = \begin{array}{c} \text{Domains} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{c} c(k) \\ 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} \qquad \text{Persons}
$$

Aggregation

Need to augment information

deprivation shares among poor

$$
\begin{array}{ccc}
\text{Domains} & c(k) & c(k)/d \\
g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \begin{matrix} \\ 2/4 \\ 4/4 \\ \\ \end{matrix}
\end{array}
\quad \text{Persons}
$$

Aggregation

Need to augment information

deprivation shares among poor

$$\begin{array}{ccccc} & \text{Domains} & c(k) & c(k)/d & \\ g^0(k) = & \begin{bmatrix}0&0&0&0\\0&1&0&1\\1&1&1&1\\0&0&0&0\end{bmatrix} & \begin{matrix}0\\ \underline{2}\\ \underline{4}\\ 0\end{matrix} & \begin{matrix} \\ 2/4 \\ 4/4 \\ \\ \end{matrix} & \text{Persons}\end{array}$$

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA

$$\begin{array}{ccccc} & \text{Domains} & & & \\ g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} c(k) \\ 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} & \begin{array}{c} c(k)/d \\ \\ 2/4 \\ 4/4 \\ \\ \end{array} & \text{Persons} \end{array}$$

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\mu(g^0(k))$

$$
\begin{array}{cccc}
 & \text{Domains} & c(k) & c(k)/d \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \begin{matrix} \\ 2/4 \\ 4/4 \\ \end{matrix} & \text{Persons}
\end{array}
$$

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\boldsymbol{\mu(g^0(k))}$ **= 6/16 = .375**

$$
\begin{array}{cccc}
 & \text{Domains} & c(k) & c(k)/d \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} & \begin{array}{c} \\ 2/4 \\ 4/4 \\ \\ \end{array}
\end{array}
\quad \text{Persons}
$$

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\boldsymbol{\mu(g^0(k))}$ **= 6/16 = .375**

$$
\begin{array}{cccc}
 & \text{Domains} & c(k) & c(k)/d & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \begin{matrix} \\ 2/4 \\ 4/4 \\ \\ \end{matrix} & \text{Persons}
\end{array}
$$

A = average deprivation share among poor = 3/4

Note: if person 2 has an additional deprivation, M_0 rises

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\boldsymbol{\mu(g^0(k))}$ **= 6/16 = .375**

$$
\begin{array}{cccc}
 & \text{Domains} & c(k) & c(k)/d & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \begin{matrix} \\ 2/4 \\ 4/4 \\ \\ \end{matrix} & \text{Persons}
\end{array}
$$

A = average deprivation share among poor = 3/4

Note: if person 2 has an additional deprivation, M_0 rises

Satisfies dimensional monotonicity

Aggregation – Adjusted Headcount Ratio

Observations

Uses ordinal data

Similar to traditional gap P_1 = HI

- HI = per capita poverty gap = total income gap of poor/total pop
- HA = per capita deprivation = total deprivations of poor/total pop

Can be broken down across dimensions

- $M_0 = \sum_j H_j/d$

Axioms: Replication Invariance, Symmetry, Poverty Focus, Deprviation Focus, (Weak) Monotonicity, Dimensional Monotonicity, Non-triviality, Normalisation, Weak Transfer, Weak Rearrangement

Characterization via freedom – Pattanaik and Xu 1990.

Note: If cardinal variables, can go further

Pattanaik and Xu 1990 and M_0

- Freedom = the number of elements in a set.
- But does not consider the *value* of elements
- If dimensions are of intrinsic value and are usually valued in practice, then *every deprivation* can be interpreted as a shortfall of something that is valued
- the (weighted) sum of deprivations can be interpreted as the unfreedoms of each person
- Adjusted Headcount can be interpreted as a measure of unfreedoms across a population.

Aggregation: Adjusted Poverty Gap

Can augment information of M_0 Use normalized gaps

$$\begin{array}{cc} & \text{Domains} \\ g^1(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{Persons} \end{array}$$

Aggregation: Adjusted Poverty Gap

Need to augment information of M_0 Use normalized gaps

$$
\begin{array}{cc}
 & \text{Domains} \\
g^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \text{Persons}
\end{array}
$$

Average **gap** across all deprived dimensions of the poor:

G = (0.04+0.42+0.17+0.67+1+1)/6

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = $M_1 = M_0G$ = HAG

$$
\begin{array}{cc}
 & \text{Domains} \\
g^1(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{Persons}
\end{array}
$$

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Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = M_1 = M_0G = HAG = $\mu(g^1(k))$

$$\begin{array}{cc} & \text{Domains} \\ g^1(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ Persons} \end{array}$$

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Obviously, if in a deprived dimension, a poor person becomes even more deprived, then M_1 will rise.

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = $M_1 = M_0G$ = HAG = **$\mu(g^1(k))$**

$$\begin{array}{cc} & \text{Domains} \\ g^1(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{Persons} \end{array}$$

Obviously, if in a deprived dimension, a poor person becomes even more deprived, then M_1 will rise.

Satisfies monotonicity

Aggregation: Adjusted FGT

Consider the matrix of squared gaps

$$
\begin{array}{cc}
 & \text{Domains} \\
g^1(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{Persons}
\end{array}
$$

Aggregation: Adjusted FGT

Consider the matrix of squared gaps

$$
\begin{array}{rcl}
 & \text{Domains} & \\
g^2(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \text{Persons}
\end{array}
$$

Aggregation: Adjusted FGT

Adjusted FGT is $M_2 = \boldsymbol{\mu(g^2(k))}$

$$\begin{array}{cc} & \text{Domains} \\ g^2(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Persons} \end{array}$$

Aggregation: Adjusted FGT

Adjusted FGT is $M_2 = \boldsymbol{\mu(g^2(k))}$

$$g^2(k) = \begin{array}{cc} \text{Domains} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \text{Persons} \end{array}$$

Satisfies transfer axiom

Aggregation: Adjusted FGT Family

Adjusted FGT is $M_\alpha = \mu(g^\alpha(\tau))$ for $\alpha \geq 0$

$$
\begin{array}{cc}
 & \text{Domains} \\
g^{\alpha}(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^{\alpha} & 0 & 1^{\alpha} \\ 0.04^{\alpha} & 0.17^{\alpha} & 0.67^{\alpha} & 1^{\alpha} \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ Persons}
\end{array}
$$

Properties

- In the multidimensional context, the axioms for poverty measures are actually *joint restrictions* on the identification and aggregation methods.
- Our methodology satisfies a number of typical properties of multidimensional poverty measures (suitably extended):
- *Symmetry,* *Scale invariance*
  Normalization *Replication invariance*
  Focus (Poverty & Depriv) *Weak Monotonicity*
  Weak Re-arrangement

- M_0, M_1 and M_2 satisfy *Dimensional Monotonicity, Decomposability*

- M_1 and M_2 satisfy *Monotonicity* (for $\alpha > 0$) – that is, they are sensitive to changes in the depth of deprivation in all domains with cardinal data.

- M_2 satisfies *Weak Transfer* (for $\alpha > 1$).

Extension

- Modifying for weights
  - Weighted identification
    - Weight on income: 50%
    - Weight on education, health: 25%
  - Cutoff = 0.50
    - Poor if income poor, or suffer two or more deprivations
  - Cutoff = 0.60
    - Poor if income poor and suffer one or more other deprivations
      - Nolan, Brian and Christopher T. Whelan, Resources, Deprivation and Poverty, 1996
  - Weighted aggregation

Extension

Modifying for weights: identification and aggregation (technically weights need not be the same, but conceptually probably should be)

- Use the g_0 or g_1 matrix
- Choose relative weights for each dimension w_d
- Important: *weights must sum to the number of dimensions*
- Apply the weights (sum = d) to the matrix
- c_k now reflects the *weighted sum* of the dimensions.
- Set cutoff k across the weighted sum.
- Censor data as before to create $g_0(k)$ or $g_1(k)$
- Measures are still the mean of the matrix.

Illustration: USA

- **Data Source:** National Health Interview Survey, 2004, *United States Department of Health and Human Services. National Center for Health Statistics* - ICPSR 4349.
- **Tables Generated By:** Suman Seth.
- **Unit of Analysis:** Individual.
- **Number of Observations:** 46009.
- **Variables:**
  - (1) income measured in poverty line increments and grouped into 15 categories
  - (2) self-reported health
  - (3) health insurance
  - (4) years of schooling.

Illustration: USA

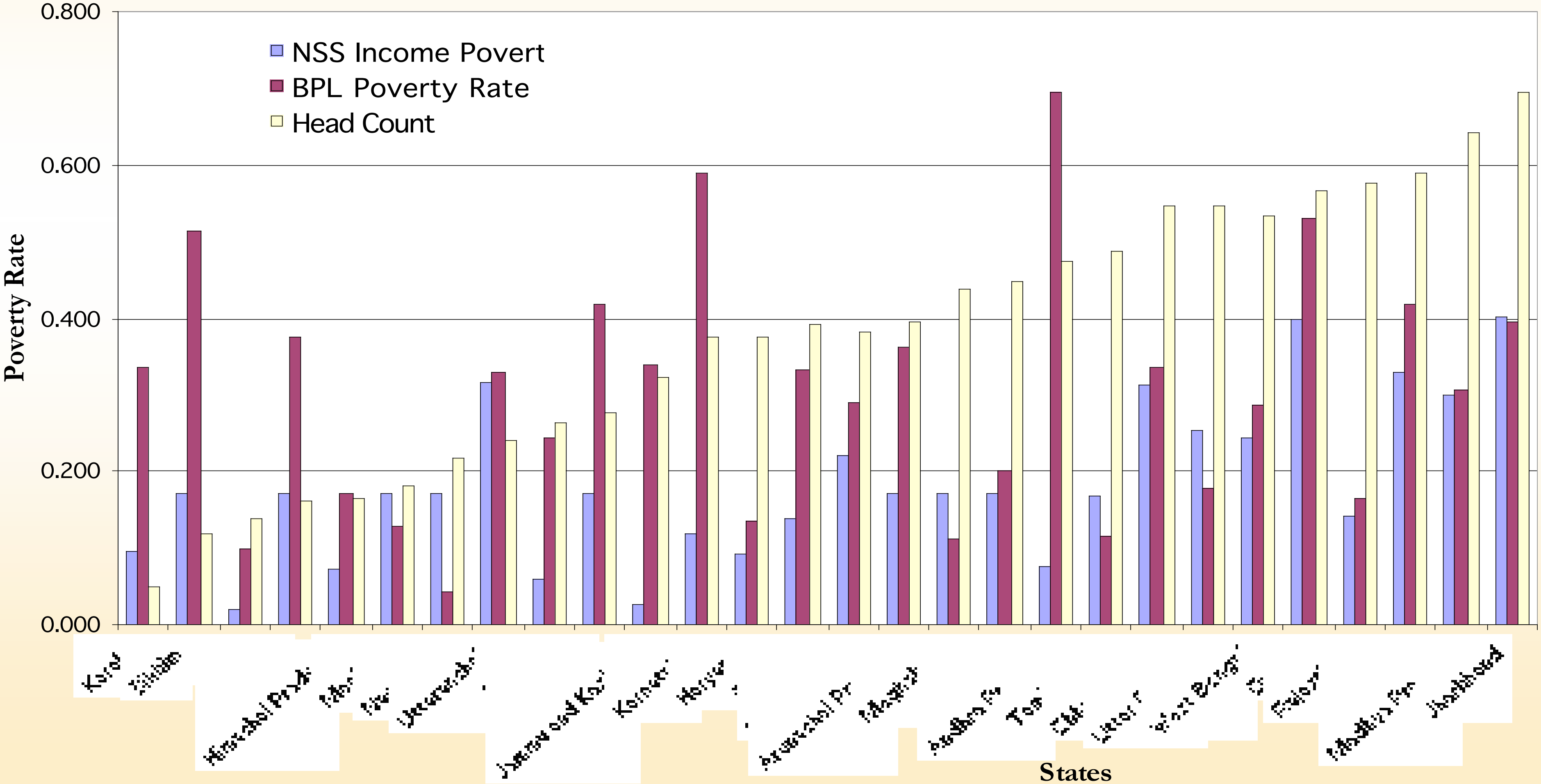
1	2	3	4	5	6	7	8	9
Ethnicity	**Population**	**Percentage Contributn**	**Income Poverty Headcount Ratio**	**Percentage Contributn**	***H***	**Percentage Contributn**	**M_0**	**Percentage Contributn**
Hispanic	9100	19.8%	0.23	37.5%	0.39	46.6%	0.229	47.8%
White	29184	63.6%	0.07	39.1%	0.09	34.4%	0.050	33.3%
Black	5742	12.5%	0.19	20.0%	0.21	16.0%	0.122	16.1%
Others	1858	4.1%	0.10	3.5%	0.12	3.0%	0.067	2.8%
Total	**45884**	**100.0%**	**0.12**	**100.0%**	**0.16**	**100.0%**	**0.09**	**100.0%**

Illustration: USA

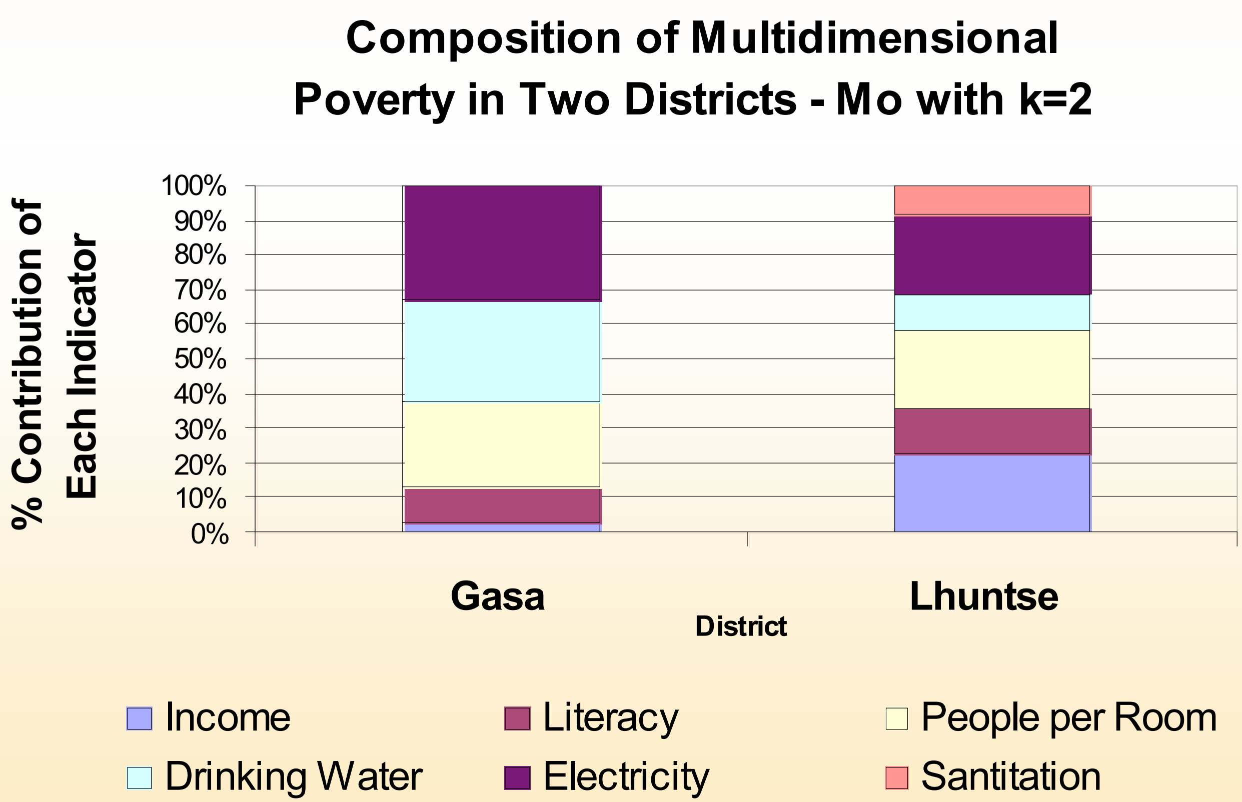
1	2	3	4	5	6
Ethnicity	***H_1* Income**	***H_2* Health**	***H_3* H. Insurance**	***H_4* Schooling**	***M_0***
Hispanic	0.200	0.116	0.274	0.324	0.229
Percentage Contribution	21.8%	12.7%	30.0%	35.5%	100%
White	0.045	0.053	0.043	0.057	0.050
Percentage Contribution	22.9%	26.9%	21.5%	28.7%	100%
Black	0.142	0.112	0.095	0.138	0.122
Percentage Contribution	29.1%	23.0%	19.5%	28.4%	100%
Others	0.065	0.053	0.071	0.078	0.067
Percentage Contribution	24.2%	20.0%	26.5%	29.3%	100%

India: We can vary the dimensions to match existing policy interests.

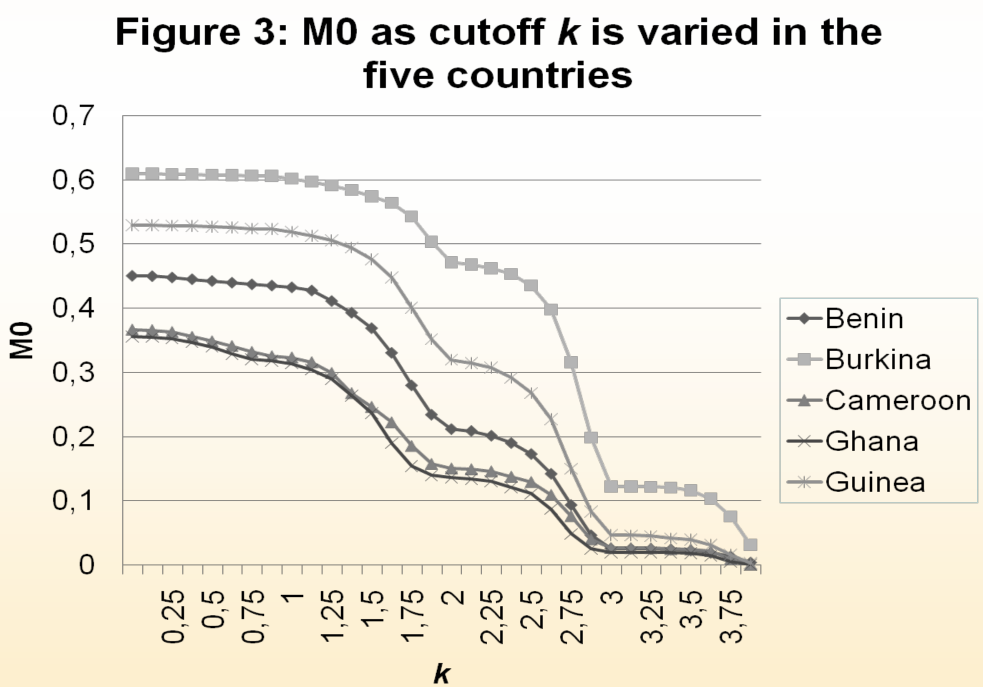
The M0 measure (white) in rural areas (with dimensions that match the Government BPL measure) is in some case strikingly different from income poverty estimates (blue), and from (widely criticised) government programmes to identify those ‘below the poverty line’ (BPL - purple) (Alkire & Seth 2008)



Bhutan: We decompose the measure to see what is driving poverty. In Bhutan the rank of the districts changed. The relatively wealthy state Gasa fell 11 places when ranked by multidimensional poverty rather than income; the state Lhuntse, which was ranked 17/20 by income, rose 9 places. Decomposing M0 by dimension, we see that in Gasa, poverty is driven by a lack of electricity, drinking water and overcrowding; income is hardly visible as a cause of poverty. In Lhuntse, income is a much larger contributor to poverty.



We can test the robustness of k. In **Sub-Saharan Africa**, we compare 5 countries using DHS data and find that Burkina is *always* poorer than Guinea, regardless of whether we count as poor persons who are deprived in only *one* kind of assets (0.25) or *every* dimension (assets, health, education, and empowerment, in this example).



But there are many measures of MD poverty.

Multidimensional Poverty: Identification & Indices

“Counting and Multidimensional Poverty Measurement” by Sabina Alkire and James Foster. Will be *OPHI Working Paper 7***.**

Bourguignon François. and Chakravarty Satya. 2003. “The measurement of multidimensional poverty.” *Journal of Economic Inequality***, 1, p. 25-49.**

Tsui, K. 2002., Multidimensional Poverty Indices. *Social Choice and Welfare,* **vol. 19, pp. 69-93.**

Maasoumi, E. and Lugo, M. A. (2007), 'The Information Basis of Multivariate Poverty Assessments', in N. Kakwani and J. Silber, (eds.), *The Many Dimensions of Poverty,* **Palgrave-MacMillan.**

The MD Focus Axiom

- One of the key properties for a multidimensional poverty measures is that these should not be sensitive to the attainments of those who are not identified as multidimensionally poor. We say that x is obtained from y by a *simple increment to a nonpoor achievement* if there is some dimension d', and a person i' who is not multidimensionally poor in y, such that xid > yid for (i,d) = (i',d') and xid = yid for all (i,d) ≠(i',d'). In other words, the two distributions x and y are only different for a single dimensional achievement for a person who is not multidimensionally poor, and their achievement is larger in x than y.
- *Focus* If x is obtained from y by a simple increment to a nonpoor person *i*'s achievement in any dimension, then M(x;zd,k) = M(y;zd,k). Further, if x is obtained from y by a simple increment to a multidimensionally poor person *i*'s achievement in a *dimension* in which they are non poor, then M(x;zd,k) = M(y;zd,k).
- In other words, if a person is not identified as experiencing MD poverty, then the specific achievements or improvements of that person should not be relevant for the measurement of multidimensional poverty; similarly increments to poor person's achievements in dimensions in which they are non-poor should not affect their poverty measure. Note that this conclusion is intuitive in the case where the achievement in question is above the poverty line. But even when the difference is *below* the poverty line, but the individual is not identified as multidimensionally poor because they are deprived in too few dimensions, multidimensional poverty should not be altered by the change.

New: Dimensional Monotonicity

- This property is a general requirement that the measure be sensitive to the number of dimensions in which a multidimensionally poor person is deprived. We say that x is obtained from y by a *dimensional decrement to a multidimensionally poor person* if there is some dimension d', and a person i' who is multidimensionally poor in y, such that xid < z ≤ yid for (i,d) = (i',d') and xid = yid for all (i,t) ≠(i',t'). In other words, the two distributions x and y are only different for a single dimension of deprivation for a person who is multidimensionally poor. With respect to that dimension the person is not deprived in y, but becomes deprived in x.
- ***Dimensional Monotonicity*** If x is obtained from y by a dimensional decrement to a multidimensionally poor person, then M(x; zd,k) > M(y; zd,k).
- In a situation in which a multidimensionally poor person happens to be *non-deprived* with respect to a particular dimension, if their achievement falls below the dimension-specific poverty line (thus raising the number of dimensions of poverty experienced by this person), then poverty should rise.
- It must be noted that the Headcount Measure H violates dimensional monotonicity, but the other measures in the FGT family satisfy this axiom.

B&C, Tsui: Further MD Axioms

The One Dimensional Transfer Principle (OTP), requires that if there are two poor persons, one less poor than the other with respect to the attribute *j*, and the less-poor of the two *gains* a given amount of the attribute and the poorer of the two *loses* the same amount, the poverty index *should not decrease*.

The Multidimensional Transfer Principle (MTP) extends OTP to a matrix and argues that if a matrix X is obtained by redistributing the attributes of the poor in matrix Y *according to the bistochastic transformation* then X cannot have more poverty than Y. That is because a bistochastic transformation would improve the attribute allocations of all poor individuals (note that MTP imposes proportions on the exchange of attributes). A final criterion in the case of MTP is the

Non-Decreasing Poverty Under Correlation Switch (NDCIS) postulates. If two persons are poor with respect to food and clothing, one with more food and one with more clothing, and then they swap clothing bundles and the person with more food now has more clothing as well, *poverty cannot have decreased.* The converse is the Non-Increasing Poverty Under Correlation Switch postulate (NICIS).

Weak poverty focus makes the poverty index independent of the attribute levels of non-poor individuals only – allows for substitution.

B&C 2002:

higher theta lower subst; theta = 1, perfect substitutes

$$P_{\theta}(X;z) = \frac{1}{n}\sum_{i=1}^{n}\left[\sum_{j=1}^{m} w_j\left(\max\left(1-\frac{x_{ij}}{z_j};0\right)\right)^{\theta}\right]^{\alpha/\theta}$$

Tsui 2002:

$$P_1(X;z)=\frac{1}{n}\sum_{i=1}^{n}\left[\prod_{j=1}^{m}\ln\left(\frac{z_j}{\min(x_{ij};z_j)}\right)^{\delta_j}-1\right]$$

$$P_2(X;z)=\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{m}\delta_j\ln\left(\frac{z_j}{\min(x_{ij};z_j)}\right)$$

Maasoumi & Lugo 2007:

- Employ Information Theory – info fctns and entropy measures (rather than fuzzy set / axiomatic approach)
- The basic measure of divergence between two distributions is the difference between their entropies, or the so called *relative entropy*. Let S_i denote the summary or aggregate function for individual i, based on his/her m attributes (x_{i1}, x_{i2}, …, x_{im}).
- Then consider a weighted average of the relative entropy divergences between (S_1, S_2, …, S_n) and each x_j = (x_{1j}, x_{2j}, …, x_{nj})
- w_j is the weight attached to the Generalized Entropy divergence from each attribute

Maasoumi & Lugo 2007:

A two step approach is to:

1. Define the multi-attribute relative deprivation function as

$$p_i = \max\left(\frac{S_z - S_i}{S_z}; 0\right) = \max\left(1 - \frac{S_i}{S_z}; 0\right)$$

2. Define the following IT multi-attribute poverty measures:

$$P_\alpha(S; z) = \frac{1}{n}\sum_{i=1}^{n}\left[\max\left(1 - \frac{S_i}{S_z}; 0\right)\right]^\alpha = \frac{1}{n}\sum_{i=1}^{n} p_i^\alpha$$

- This is the αth moment FGT poverty index based on the distribution of *S = (S1, S2,…,Sn)*

MD Poverty & Capability Approach

- Focus on *Individuals* as unit of analysis when possible
- Each dimension might be of *intrinsic importance*, whether or not it is also instrumentally effective
- Normative Value Judgments:
  - Choice of dimensions
  - Choice of poverty lines
  - Choice of weights across dimensions