Problem Set on Multidimensional Poverty Measures
ANSWER KEY

A) Paper-Based Problems:

Given the following matrix of distribution of three dimensions (income, self rated health, and years of education):

$$X = \begin{bmatrix} 4 & 1 & 5 \\ 8 & 4 & 6 \\ 12 & 1 & 11 \\ 3 & 4 & 6 \\ 15 & 1 & 9 \\ 12 & 5 & 12 \end{bmatrix} \qquad z = \begin{bmatrix} 10 & 3 & 8 \end{bmatrix}$$

$$g^0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad c = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

Then using k=2: $g^0(k) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$g^1(k) = \begin{bmatrix} 0.6 & 0.66 & 0.375 \\ 0.2 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0.7 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad g^2(k) = \begin{bmatrix} 0.36 & 0.44 & 0.14 \\ 0.04 & 0 & 0.0625 \\ 0 & 0 & 0 \\ 0.49 & 0 & 0.0625 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

I. Then: H=1/20.50

Mo=7/18=0.39

M1=3.0375/18=0.17

M2=1.595/18=0.088

II. Contribution of each dimension:

Contribution of Income=[(3/6)*(1/3)]/0.39=0.43

Contribution of Health=[(1/6)*(1/3)]/0.39=0.14

Contribution of Education=[(3/6)*(1/3)]/0.39=0.43

This means that deprivation in income accounts for 43% of overall multidimensional poverty. Same for education, whereas deprivation in health accounts for 14% of overall M0.

III. If individual 2 reports a health status of 2 instead of 4, it becomes deprived in one additional dimension. This refers to dimensional monotonicity. H will not change, but Mo, M1 and M2 will increase.

b) Using nested weights: $w = [2 \quad 0.5 \quad 0.5]$ Now we compute the weighted g0 matrix:

$$g^0 = \begin{bmatrix} 2 & 0.5 & 0.5 \\ 2 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 2 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad c = \begin{bmatrix} 3 \\ 2.5 \\ 0.5 \\ 2.5 \\ 0.5 \\ 0 \end{bmatrix}$$

Then using k=2, the same people will be identified as multidimensionally poor as before, <u>but this may not always be the case</u>.

$$g^0 = \begin{bmatrix} 2 & 0.5 & 0.5 \\ 2 & 0 & 0.5 \\ 0 & 0 & 0 \\ 2 & 0 & 0.5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$g^1(k) = \begin{bmatrix} 2(0.6) & 0.5(0.66) & 0.5(0.375) \\ 2(0.2) & 0 & 0.5(0.25) \\ 0 & 0 & 0 \\ 2(0.7) & 0 & 0.5(0.25) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$g^2(k) = \begin{bmatrix} 2(0.36) & 0.5(0.44) & 0.5(0.14) \\ 2(0.04) & 0 & 0.5(0.0625) \\ 0 & 0 & 0 \\ 2(0.49) & 0 & 0.5(0.0625) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then:

H=1/2

M0=8/18=0.44

M1=3.77/18=0.21

M2=2.13/18=0.118

B) Computer-Based Problems (Using Stata):

See the do file attached.

a) The overall sample, with the different k values and distinguishing between urban and rural areas

	Multidimensional H			Mo		
k	Total	Rural	Urban	Total	Rural	Urban
1	0.64	0.75	0.34	0.26	0.33	0.08
2	0.38	0.49	0.053	0.21	0.28	0.022
3	0.20	0.27	0.004	0.14	0.19	0.002
4	0.077	0.105	0	0.064	0.09	0
5	0.013	0.018	0	0.013	0.018	0

For k=2, H=0.38, M0=0.21, so A=0.55, meaning that on average the multidimensional poor are deprived in 2.75 dimensions.

c) What is the percentage of people deprived in each dimension? How are these percentages in rural and urban areas? Which dimensions present the highest levels of deprivation?

We see for example that there are 23% of the population deprived in income, 32% deprived in education, 36% deprived in room, 31% deprived in electricity and 9% in water. These % are higher in rural areas when compared with urban ones.

```
sum INC_Deprived EDU_Deprived ROOM_Deprived ELECTR_Deprived WATER_Deprived

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
INC_Deprived |     24786    .2300896    .4208985          0          1
EDU_Deprived |     24786     .321149    .4669273          0          1
ROOM_Depri~d |     24786    .3601226    .4800454          0          1
ELECTR_Dep~d |     24786    .3140886    .4641612          0          1
WATER_Depr~d |     24786    .0955781    .2940178          0          1
```

```
. sort area

.  by  area:  sum  INC_Deprived  EDU_Deprived  ROOM_Deprived  ELECTR_Deprived
WATER_Deprived

-------------------------------------------------------------------------------
-> area = urban

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
INC_Deprived |      6508    .0145974    .1199439          0          1
EDU_Deprived |      6508    .1401352    .3471539          0          1
ROOM_Depri~d |      6508    .2212661    .4151312          0          1
ELECTR_Dep~d |      6508    .0127535    .1122177          0          1
WATER_Depr~d |      6508    .0061463    .0781629          0          1

------------------------------------------
-> area = rural

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
INC_Deprived |     18278    .3068169    .4611854          0          1
EDU_Deprived |     18278    .3856002    .4867501          0          1
ROOM_Depri~d |     18278    .4095634    .4917667          0          1
ELECTR_Dep~d |     18278    .4213809    .4937939          0          1
WATER_Depr~d |     18278    .1274209     .333453          0          1
```

c) How do the measures change as k increases? How is M0 with respect to H? Why? For k=2, in the estimates of the whole sample, what is the average deprivation share among the poor (A)? How do you interpret this?

As k increase, M0 decreases, because the identification criterion becomes more and more requiring, moving from a union approach to an intersection approach. M0 is always lower than H, because it adjusts the percentage of the multidimensionally poor by the average deprivation share among the poor. For k=2, A=M0/H=0.21/0.38=0.55.