Inequality Adjusted HDI

Suman Seth

Vanderbilt University & OPHI

3rd September, 2009
Measuring Human Development

- Multidimensional in nature
Measuring Human Development

- Multidimensional in nature
- Mail challenges
Measuring Human Development

- Multidimensional in nature
- Mail challenges
  - Choice of dimensions
Measuring Human Development

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  - Choice of dimensions
  - Choice of indicators
Measuring Human Development

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  - Choice of appropriate weights
Measuring Human Development

- Multidimensional in nature
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  - Choice of dimensions
  - Choice of indicators
  - Choice of appropriate weights
  - Proper data collection methodology

Choice of normalization method for dimensions

For today’s lecture - it’s incorporating inequality in the measurement of human development

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Measuring Human Development

- Multidimensional in nature
- Mail challenges
  - Choice of dimensions
  - Choice of indicators
  - Choice of appropriate weights
  - Proper data collection methodology
  - Proper aggregation methods incorporating various aspects

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Measuring Human Development

- Multidimensional in nature
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  - Proper aggregation methods incorporating various aspects
    - Choice of normalization method for dimensions
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  - Proper data collection methodology
  - Proper aggregation methods incorporating various aspects
    - Choice of normalization method for dimensions

- For today’s lecture - its incorporating inequality in the measurement of human development
Briefly discuss the current methodology for measuring Human Development Index (HDI)
Outline of Today’s Lecture

- Briefly discuss the current methodology for measuring Human Development Index (HDI)
- Introduce basic framework for today’s discussion
Briefly discuss the current methodology for measuring Human Development Index (HDI)
Introduce basic framework for today’s discussion
Introduce related axioms
Briefly discuss the current methodology for measuring Human Development Index (HDI)

Introduce basic framework for today’s discussion

Introduce related axioms

Discuss various constructive proposals for incorporating inequality into human development
Basic Framework

- HDI is a composite index consisting of three sub-indices:
Basic Framework

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  - Per capits Gross Domestic Product (PGDP)
Basic Framework

- HDI is a composite index consisting of three sub-indices:
  - Per capita Gross Domestic Product (PGDP)
  - Life expectancy (LE) Index
Basic Framework

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  - Per capits Gross Domestic Product (PGDP)
  - Life expectancy (LE) Index
  - Education (E) Index
Basic Framework

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- All indices are normalized between zero and one
HDI is a composite index consisting of three sub-indices:

- Per capits Gross Domestic Product (PGDP)
- Life expectancy (LE) Index
- Education (E) Index

All indices are normalized between zero and one

GDP Index

\[
\text{GDP Index} = \frac{\log (\text{PCGDP}) - \log (\$100)}{\log (\$40,000) - \log (\$100)}
\]
Basic Framework

- HDI is a composite index consisting of three sub-indices:
  - Per capits Gross Domestic Product (PGDP)
  - Life expectancy (LE) Index
  - Education (E) Index

- All indices are normalized between zero and one

- GDP Index
  \[ \text{GDP Index} = \frac{\log (\text{PCGDP}) - \log ($100)}{\log ($40,000) - \log ($100)} \]

- Life Expectancy Index
  \[ \text{LE Index} = \frac{\text{LE} - 25}{85 - 25} \]
Basic Framework

- Education Index

Adult literacy index $\text{AL} = 100$

Gross school enrolment index $\text{GSE} = 100$

Education Index = $\text{AL} + \text{GSE}$

HDI = GDP Index + Life exp Index + Education Index

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Inequality Adjusted HDI

3rd September, 2009
Basic Framework

- Education Index
  - Consists of two sub-indices
Basic Framework

- Education Index
  - Consists of two sub-indices
  - Adult literacy (AL) index
Basic Framework

- Education Index
  - Consists of two sub-indices
    - Adult literacy (AL) index
    - Gross school enrolment (GSE) index
Basic Framework

- Education Index
  - Consists of two sub-indices
    - Adult literacy (AL) index
    - Gross school enrolment (GSE) index
  - Adult literacy index
    \[
    \text{Adult literacy index} = \frac{AL - 0}{100 - 0}
    \]
Basic Framework

- **Education Index**
  - Consists of two sub-indices
  - Adult literacy (AL) index
  - Gross school enrolment (GSE) index

- **Adult literacy index**
  \[
  \frac{AL - 0}{100 - 0}
  \]

- **Gross school enrolment index**
  \[
  \frac{GSE - 0}{100 - 0}
  \]
Basic Framework

- **Education Index**
  - Consists of two sub-indices
  - Adult literacy (AL) index
  - Gross school enrolment (GSE) index

- **Adult literacy index**
  \[ \text{AL} - 0 = \frac{\text{AL} - 0}{100 - 0} \]

- **Gross school enrolment index**
  \[ \text{GSE} - 0 = \frac{\text{GSE} - 0}{100 - 0} \]

- **Education Index**
  \[ \frac{2}{3} \times \text{Adult literacy index} + \frac{1}{3} \times \text{Gross school enrolment index} \]
Basic Framework

- **Education Index**
  - Consists of two sub-indices
    - Adult literacy (AL) index
    - Gross school enrolment (GSE) index
  - **Adult literacy index**
    \[ \text{AL} = \frac{\text{AL} - 0}{100 - 0} \]
  - **Gross school enrolment index**
    \[ \text{GSE} = \frac{\text{GSE} - 0}{100 - 0} \]
  - **Education Index**
    \[ \text{Education Index} = \frac{2}{3} \times \text{Adult literacy index} + \frac{1}{3} \times \text{Gross school enrolment index} \]

- **HDI**
  \[ \text{HDI} = \frac{1}{3} \times \text{GDP Index} + \frac{1}{3} \times \text{Life exp Index} + \frac{1}{3} \times \text{Education Index} \]
### 2004 HDI Table: Top Ten Countries

<table>
<thead>
<tr>
<th>Rank</th>
<th>Country</th>
<th>HDI</th>
<th>LE Index</th>
<th>Edu Index</th>
<th>GDP Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Norway</td>
<td>0.965</td>
<td>0.909</td>
<td>0.993</td>
<td>0.993</td>
</tr>
<tr>
<td>2</td>
<td>Iceland</td>
<td>0.960</td>
<td>0.931</td>
<td>0.981</td>
<td>0.968</td>
</tr>
<tr>
<td>3</td>
<td>Australia</td>
<td>0.957</td>
<td>0.925</td>
<td>0.993</td>
<td>0.954</td>
</tr>
<tr>
<td>4</td>
<td>Ireland</td>
<td>0.956</td>
<td>0.882</td>
<td>0.990</td>
<td>0.995</td>
</tr>
<tr>
<td>5</td>
<td>Sweden</td>
<td>0.951</td>
<td>0.922</td>
<td>0.982</td>
<td>0.949</td>
</tr>
<tr>
<td>6</td>
<td>Canada</td>
<td>0.950</td>
<td>0.919</td>
<td>0.970</td>
<td>0.959</td>
</tr>
<tr>
<td>7</td>
<td>Japan</td>
<td>0.949</td>
<td>0.953</td>
<td>0.945</td>
<td>0.948</td>
</tr>
<tr>
<td>8</td>
<td>United States</td>
<td>0.948</td>
<td>0.875</td>
<td>0.971</td>
<td>0.999</td>
</tr>
<tr>
<td>9</td>
<td>Switzerland</td>
<td>0.947</td>
<td>0.928</td>
<td>0.946</td>
<td>0.968</td>
</tr>
<tr>
<td>10</td>
<td>Netherlands</td>
<td>0.947</td>
<td>0.892</td>
<td>0.987</td>
<td>0.962</td>
</tr>
</tbody>
</table>
HDI: Sensitive to Inequality?

- No!
HDI: Sensitive to Inequality?

- No!
- Three proposals
HDI: Sensitive to Inequality?

- No!
- Three proposals
  - Hicks (1997)
HDI: Sensitive to Inequality?

- No!
- Three proposals
  - Hicks (1997)
  - Foster, López-Calva, Székely (2005)
HDI: Sensitive to Inequality?

- No!
- Three proposals
  - Hicks (1997)
  - Foster, López-Calva, Székely (2005)
  - Harttgen, Klasen, and Misselhorn (2008)
No!

Three proposals

- Hicks (1997)
- Foster, López-Calva, Székely (2005)
- Harttgen, Klasen, and Misselhorn (2008)

Finally, incorporate the concept of inter-dimensional correlation into HDI
No!

Three proposals
- Hicks (1997)
- Foster, López-Calva, Székely (2005)
- Harttgen, Klasen, and Misselhorn (2008)

Finally, incorporate the concept of inter-dimensional correlation into HDI
- Seth (2009)
### Table 5. Country rankings by HDI and IAHDI

<table>
<thead>
<tr>
<th>Country</th>
<th>HDI</th>
<th>IAHDI</th>
<th>Change in ranks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hong Kong</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>2</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>Korea (Rep.)</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Chile</td>
<td>4</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Venezuela</td>
<td>5</td>
<td>7</td>
<td>-2</td>
</tr>
<tr>
<td>Panama</td>
<td>6</td>
<td>8</td>
<td>-2</td>
</tr>
<tr>
<td>Mexico</td>
<td>7</td>
<td>10</td>
<td>-3</td>
</tr>
<tr>
<td>Colombia</td>
<td>8</td>
<td>9</td>
<td>-1</td>
</tr>
<tr>
<td>Thailand</td>
<td>9</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Malaysia</td>
<td>10</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Brazil</td>
<td>11</td>
<td>12</td>
<td>-1</td>
</tr>
<tr>
<td>Peru</td>
<td>12</td>
<td>14</td>
<td>-2</td>
</tr>
<tr>
<td>Sri Lanka</td>
<td>14</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>Philippines</td>
<td>15</td>
<td>13</td>
<td>2</td>
</tr>
<tr>
<td>Nicaragua</td>
<td>16</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>Guatemala</td>
<td>17</td>
<td>19</td>
<td>-2</td>
</tr>
<tr>
<td>Honduras</td>
<td>18</td>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>Zimbabwe</td>
<td>19</td>
<td>18</td>
<td>1</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>20</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>State</td>
<td>HDI-GM</td>
<td>Ranking</td>
<td>HDI-GM</td>
</tr>
<tr>
<td>---------------------</td>
<td>----------</td>
<td>---------</td>
<td>----------</td>
</tr>
<tr>
<td>Aguascalientes</td>
<td>0.7001</td>
<td>5</td>
<td>0.5811</td>
</tr>
<tr>
<td>Baja California</td>
<td>0.7176</td>
<td>2</td>
<td>0.6150</td>
</tr>
<tr>
<td>Baja California Sur</td>
<td>0.7038</td>
<td>3</td>
<td>0.5787</td>
</tr>
<tr>
<td>Campeche</td>
<td>0.6734</td>
<td>15</td>
<td>0.5473</td>
</tr>
<tr>
<td>Chiapas</td>
<td>0.5735</td>
<td>32</td>
<td>0.3797</td>
</tr>
<tr>
<td>Chihuahua</td>
<td>0.6739</td>
<td>14</td>
<td>0.5069</td>
</tr>
<tr>
<td>Coahuila</td>
<td>0.6957</td>
<td>6</td>
<td>0.5637</td>
</tr>
<tr>
<td>Colima</td>
<td>0.6884</td>
<td>7</td>
<td>0.5428</td>
</tr>
<tr>
<td>Distrito Federal</td>
<td>0.7403</td>
<td>1</td>
<td>0.6376</td>
</tr>
<tr>
<td>Durango</td>
<td>0.6608</td>
<td>20</td>
<td>0.4708</td>
</tr>
<tr>
<td>Estado de México</td>
<td>0.6824</td>
<td>9</td>
<td>0.5185</td>
</tr>
<tr>
<td>Guanajuato</td>
<td>0.6546</td>
<td>22</td>
<td>0.4937</td>
</tr>
<tr>
<td>Guerrero</td>
<td>0.5968</td>
<td>30</td>
<td>0.3995</td>
</tr>
<tr>
<td>Hidalgo</td>
<td>0.6449</td>
<td>24</td>
<td>0.4784</td>
</tr>
<tr>
<td>Jalisco</td>
<td>0.6772</td>
<td>12</td>
<td>0.5246</td>
</tr>
<tr>
<td>Michoacán</td>
<td>0.6363</td>
<td>26</td>
<td>0.4509</td>
</tr>
<tr>
<td>Morelos</td>
<td>0.6691</td>
<td>16</td>
<td>0.5139</td>
</tr>
<tr>
<td>Nayarit</td>
<td>0.6638</td>
<td>18</td>
<td>0.4898</td>
</tr>
<tr>
<td>Nuevo León</td>
<td>0.7021</td>
<td>4</td>
<td>0.5783</td>
</tr>
</tbody>
</table>
We follow the approach of Foster, López-Calva, Székely (2005)
Normalizing Dimensions at the Individual Level

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- They do not aggregate first across observations and then normalize (like the traditional HDI)
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- They do not aggregate first across observations and then normalize (like the traditional HDI)
- Rather, first normalizes and then aggregates
  - Income varies across individuals
  - Enrolment rates and literacy rates varies across households
  - Infant survival rate (health variable) varies across municipalities
Basic Theoretical Framework

- $N$ persons and $D$ dimensions

Normalized achievement vector: $X = x_1 \times x_2 \times \ldots \times x_N$ where $x_{nd}$ is the achievement of the $n$th person in the $d$th dimension.

Denote: $x_n = (x_{n1}, \ldots, x_{nD})$ and $x_d = (x_{1d}, \ldots, x_{Nd})$.

$x_n$ is the achievement vector of the $n$th person.

$x_d$ is the achievement vector in the $d$th dimension.

A Human Development Index is $W(H) : H \rightarrow [0, \infty)$.
Basic Theoretical Framework

- $N$ persons and $D$ dimensions

- Normalized achievement vector: $X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1D} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{ND} \end{bmatrix}$
Basic Theoretical Framework

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- $x_{nd}$ is the achievement of the $n^{th}$ person in the $d^{th}$ dimension
Basic Theoretical Framework

- $N$ persons and $D$ dimensions
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- $x_{nd}$ is the achievement of the $n^{th}$ person in the $d^{th}$ dimension
- $x_{nd} > 0 \ \forall n, d$
$N$ persons and $D$ dimensions

Normalized achievement vector: $X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1D} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{ND} \end{bmatrix}$

$x_{nd}$ is the achievement of the $n^{th}$ person in the $d^{th}$ dimension

$x_{nd} > 0 \ \forall n, d$

Denote: $x_{n}^{*} = (x_{n1}, \ldots, x_{nD}) \forall n$ and $x_{d}^{*} = (x_{1d}, \ldots, x_{Nd}) \forall d$
Basic Theoretical Framework

- $N$ persons and $D$ dimensions

- Normalized achievement vector: $X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1D} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{ND} \end{bmatrix}$

- $x_{nd}$ is the achievement of the $n^{th}$ person in the $d^{th}$ dimension

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Basic Theoretical Framework

- $N$ persons and $D$ dimensions

- Normalized achievement vector: $X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1D} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{ND} \end{bmatrix}$

- $x_{nd}$ is the achievement of the $n^{\text{th}}$ person in the $d^{\text{th}}$ dimension

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Basic Theoretical Framework

- $N$ persons and $D$ dimensions
- Normalized achievement vector: $X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1D} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{ND} \end{bmatrix}$
- $x_{nd}$ is the achievement of the $n^{th}$ person in the $d^{th}$ dimension
- $x_{nd} > 0 \quad \forall n, d$
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- $x_{n^*}$ is the achievement vector of the $n^{th}$ person
- $x_{d^*}$ is the achievement vector in the $d^{th}$ dimension
- A Human Development Index is $W(H) : H \rightarrow [0, \infty]$
For a vector $\mathbf{h} = (h_1, ..., h_M)$, Generalized mean of order $\gamma$ is

$$\mu_\gamma(\mathbf{h}) = \frac{h_1^\gamma + ... + h_M^\gamma}{\gamma M}$$

for $\gamma \neq 0$ and

$$\mu_0(\mathbf{h}) = \left(\frac{h_1 + ... + h_M}{M}\right)^{1/M}$$

for $\gamma = 0$.

If $\gamma = 1$, then

$$\mu_1(\mathbf{h}) = \frac{h_1 + ... + h_M}{M}$$

is the Arithmetic Mean.

If $\gamma = \infty$, then

$$\mu_\infty(\mathbf{h}) = \min(h_1, ..., h_M)$$

is the Harmonic Mean.

As $\gamma$ falls, more emphasis is given on lower values.

If $\gamma = \infty$, then

$$\mu_\infty(\mathbf{h}) = \max(h_1, ..., h_M)$$

is the Harmonic Mean.

As $\gamma$ rises, more emphasis is given on higher values.
For a vector $\mathbf{h} = (h_1, ..., h_M)$, Generalized mean of order $\gamma$ is

- $\mu_\gamma (\mathbf{h}) = \left[ \frac{1}{M} (h_1^\gamma + ... + h_M^\gamma) \right]^{1/\gamma}$ for $\gamma \neq 0$ and
- $\mu_\gamma (\mathbf{h}) = (h_1 \times ... \times h_M)^{1/M}$ for $\gamma = 0$
For a vector $\mathbf{h} = (h_1, ..., h_M)$, Generalized mean of order $\gamma$ is

- $\mu_\gamma (\mathbf{h}) = \left[ \frac{1}{M} (h_1^\gamma + ... + h_M^\gamma) \right]^{1/\gamma}$ for $\gamma \neq 0$ and $\mu_\gamma (\mathbf{h}) = (h_1 \times ... \times h_M)^{1/M}$ for $\gamma = 0$
- If $\gamma = 1$, then $\mu_1 (\mathbf{h}) = \frac{h_1 + ... + h_M}{M} \rightarrow$ Arithmetic Mean
For a vector \( \mathbf{h} = (h_1, \ldots, h_M) \), Generalized mean of order \( \gamma \) is

- \( \mu_{\gamma}(\mathbf{h}) = \left[ \frac{1}{M} (h_1^\gamma + \ldots + h_M^\gamma) \right]^{1/\gamma} \) for \( \gamma \neq 0 \) and
  \[ \mu_{\gamma}(\mathbf{h}) = (h_1 \times \ldots \times h_M)^{1/M} \] for \( \gamma = 0 \)

- If \( \gamma = 1 \), then \( \mu_1(\mathbf{h}) = \frac{h_1 + \ldots + h_M}{M} \rightarrow \text{Arithmetic Mean} \)

- If \( \gamma = 0 \), then \( \mu_0(\mathbf{h}) = (h_1 \times \ldots \times h_M)^{1/M} \rightarrow \text{Geometric Mean} \)
For a vector \( h = (h_1, ..., h_M) \), Generalized mean of order \( \gamma \) is

- \( \mu_\gamma (h) = \left[ \frac{1}{M} (h_1^\gamma + ... + h_M^\gamma) \right]^{1/\gamma} \) for \( \gamma \neq 0 \) and
  \[ \mu_\gamma (h) = (h_1 \times ... \times h_M)^{1/M} \] for \( \gamma = 0 \)
- If \( \gamma = 1 \), then \( \mu_1 (h) = \frac{h_1 + ... + h_M}{M} \to \text{Arithmetic Mean} \)
- If \( \gamma = 0 \), then \( \mu_0 (h) = (h_1 \times ... \times h_M)^{1/M} \to \text{Geometric Mean} \)
- If \( \gamma = -1 \), then \( \mu_{-1} (h) = \frac{M}{h_1^{-1} + ... + h_M^{-1}} \to \text{Harmonic Mean} \)
For a vector $\mathbf{h} = (h_1, ..., h_M)$, Generalized mean of order $\gamma$ is

- $\mu_\gamma(\mathbf{h}) = \left[ \frac{1}{M} (h_1^\gamma + ... + h_M^\gamma) \right]^{1/\gamma}$ for $\gamma \neq 0$ and $\mu_\gamma(\mathbf{h}) = (h_1 \times ... \times h_M)^{1/M}$ for $\gamma = 0$
- If $\gamma = 1$, then $\mu_1(\mathbf{h}) = \frac{h_1 + ... + h_M}{M}$ → Arithmetic Mean
- If $\gamma = 0$, then $\mu_0(\mathbf{h}) = (h_1 \times ... \times h_M)^{1/M}$ → Geometric Mean
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As $\gamma$ falls, more emphasis is given on lower values
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For a vector \( h = (h_1, \ldots, h_M) \), Generalized mean of order \( \gamma \) is

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\mu_\gamma (h) = \left[ \frac{1}{M} (h_1^\gamma + \ldots + h_M^\gamma) \right]^{1/\gamma} \quad \text{for } \gamma \neq 0 \quad \text{and}
\]
\[
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Basic Axioms Satisfied by the HDI

- Normalization (NORM). If \( x_{nd} = \delta \) for all \( n, d \), then \( W(X) = \delta \)
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HDI: The Traditional Approach ($W_A$)

- The simple average of the whole matrix $H$
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- The simple average of the whole matrix $H$
  - First stage: simple average across persons. Second stage: simple average across dimensions
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Example: 3 persons and 3 dimensions

\[
\begin{array}{ccc}
\text{Income} & \text{Education} & \text{Health} \\
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\end{array}
\]

First stage: average across persons yields $(0.5, 0.5, 0.5)$. Second stage: average across dimensions yields $0.5$. Thus, $W_A = 0.5$.

Both sequences of aggregation yield the same result.

Path Independence (PATHIN) - sequence of aggregation is not important (Foster, López-Calva, Székely (2005))
HDI: The Traditional Approach (WA)

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Policy Exercise

- Given achievement matrix

Given achievement matrix

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\end{bmatrix} \]

Policy maker's budget - one indivisible unit of assistance

Suppose the assistance increases achievement in any dimension by 0.1 units and the policy maker is a human development maximizer. Let human development be calculated by applying \( W \).

Question: Where should the assistance be made?

Answer: Anywhere in the matrix. Insensitive to inequality.

Evaluation of \( W \): \( W \) satisfies NORM, LHOM, ANON, MON, POPRI, SUBCON, CONT, PATHIN

\( W \) is not sensitive to inequality across persons.

Suman Seth (Vanderbilt University & OPHI)

Inequality Adjusted HDI

3rd September, 2009
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Given achievement matrix

\[ X = \]

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Two Forms of Multidimensional Inequality

- The **first**: distribution sensitive inequality (Kolm 1977)

Decrease in the spread of the distribution increases human development.

Uniform Majorization (UM): $W(BX) > W(X)$

$B$ is a bistochastic matrix.

Remember: This is a variation of Uniform Pigou Dalton Transfer.

What does transfer imply for non-transferable dimensions such as income and health?

The other form of inequality will be introduced later.
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Indices Sensitive to Inequality

- Indices sensitive to the first form of inequality across persons

\[ S = \mu(1) \left[ 1 - G(\mu) \right] \]

First stage: aggregates across persons by using Sen welfare standard

Second stage: uses simple average across dimensions

Example: \( X = 2 \quad 4 \quad 0 \quad 8 \quad 0 \quad 8 \quad 3 \quad 0 \quad 4 \quad 0 \quad 8 \quad 3 \quad 4 \quad 3 \quad 5 \)
Indices Sensitive to Inequality

- Indices sensitive to the first form of inequality across persons
  - Hicks (1997) Index ($W_H$)

Example: $X = [2.40, 0.80, 0.30, 0.40, 0.8]$.

The first stage average across persons yields $(0.50, 0.50, 0.50)$. The Gini vector is $(0.22, 0.22, 0.22)$. The first stage achievement vector is $(0.39, 0.39, 0.39)$.

The second stage average yields $\bar{\mu}(0.39, 0.39, 0.39) = 0.39$. 
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Inequality Adjusted HDI  
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    - The first stage average across persons yields $(0.5, 0.5, 0.5)$. The Gini vector is $(0.22, 0.22, 0.22)$. The first stage achievement vector is $(0.39, 0.39, 0.39)$.
    - The second stage average yields \[ \mu_1(0.39, 0.39, 0.39) = 0.39. \]
Indices Sensitive to Inequality Across Persons

Example: \( \bar{X} = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.35 & 0.35 & 0.6 \\ 0.35 & 0.35 & 0.6 \end{bmatrix} \)
Example: $\bar{X} = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.35 & 0.35 & 0.6 \\ 0.35 & 0.35 & 0.6 \end{bmatrix}$

- The first stage average across persons yields $(0.5, 0.5, 0.5)$. The Gini vector is $(0.2, 0.2, 0.13)$. The first stage achievement vector is $(0.4, 0.4, 0.42)$. 

Indices Sensitive to Inequality Across Persons
Example: $\bar{X} = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.35 & 0.35 & 0.6 \\ 0.35 & 0.35 & 0.6 \end{bmatrix}$

- The first stage average across persons yields (0.5, 0.5, 0.5). The Gini vector is (0.2, 0.2, 0.13). The first stage achievement vector is (0.4, 0.4, 0.42).
- The second stage average yields - $\mu_1 (0.4, 0.4, 0.42) = 0.41$. 

Example: \( \bar{X} = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.35 & 0.35 & 0.6 \\ 0.35 & 0.35 & 0.6 \end{bmatrix} \)

- The first stage average across persons yields \((0.5, 0.5, 0.5)\). The Gini vector is \((0.2, 0.2, 0.13)\). The first stage achievement vector is \((0.4, 0.4, 0.42)\).
- The second stage average yields - \( \mu_1 (0.4, 0.4, 0.42) = 0.41 \).
- Thus, \( W_H (X) = 0.39 \) and \( W_H (\bar{X}) = 0.41 \).
Example: \( \bar{X} = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.35 & 0.35 & 0.6 \\ 0.35 & 0.35 & 0.6 \end{bmatrix} \)

- The first stage **average** across persons yields \((0.5, 0.5, 0.5)\). The Gini vector is \((0.2, 0.2, 0.13)\). The first stage achievement vector is \((0.4, 0.4, 0.42)\).
- The second stage average yields \(-\mu_1 (0.4, 0.4, 0.42) = 0.41.\)
- Thus, \(W_H (X) = 0.39\) and \(W_H (\bar{X}) = 0.41\)
- Gini Index - not subgroup consistent
Example: $\bar{X} = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.35 & 0.35 & 0.6 \\ 0.35 & 0.35 & 0.6 \end{bmatrix}$

- The first stage average across persons yields $(0.5, 0.5, 0.5)$. The Gini vector is $(0.2, 0.2, 0.13)$. The first stage achievement vector is $(0.4, 0.4, 0.42)$.
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Indices Sensitive to Inequality Across Persons

- Foster, López-Calva, Székely (2005) Index \((W_F)\)
Foster, López-Calva, Székely (2005) Index ($W_F$)

- First stage: aggregates across persons using $\mu_\alpha(\cdot)$. Second stage: aggregates across dimensions using $\mu_\alpha(\cdot)$; and vice versa. $\alpha \leq 1$
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Example: $X = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.4 & 0.3 & 0.8 \\ 0.3 & 0.4 & 0.4 \end{bmatrix}$
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First Stage: Generalized mean across persons yields $0.4, 0.4, 0.4$. 
Indices Sensitive to Inequality Across Persons

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  $$X = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.4 & 0.3 & 0.8 \\ 0.3 & 0.4 & 0.4 \end{bmatrix}$$

- Example: First Stage: Generalized mean across persons yields $(0.4, 0.4, 0.4)$.
- The second stage **generalized mean** of order $-2$ yields $\mu_{-2} (0.4, 0.4, 0.4) = 0.4$. 
Indices Sensitive to Inequality Across Persons

- Foster, López-Calva, Székely (2005) Index ($W_F$)
  - First stage: aggregates across persons using $\mu_\alpha (\cdot)$. Second stage: aggregates across dimensions using $\mu_\alpha (\cdot)$; and vice versa. $\alpha \leq 1$
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- Example: $X = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.4 & 0.3 & 0.8 \\ 0.3 & 0.4 & 0.4 \end{bmatrix}$

- First Stage: Generalized mean across persons yields (0.4, 0.4, 0.4).
- The second stage **generalized mean** of order $-2$ yields $-\mu_{-2} (0.4, 0.4, 0.4) = 0.4$.
- Reversed order of aggregation
Indices Sensitive to Inequality Across Persons

- Foster, López-Calva, Székely (2005) Index ($W_F$)
  - First stage: aggregates across persons using $\mu_\alpha (\cdot)$. Second stage: aggregates across dimensions using $\mu_\alpha (\cdot)$; and vice versa. $\alpha \leq 1$
  - The same power of generalized mean $\rightarrow$ the $W_F$ satisfies path independence (PI)

Example: $X = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.4 & 0.3 & 0.8 \\ 0.3 & 0.4 & 0.4 \end{bmatrix}$

- First Stage: Generalized mean across persons yields $(0.4, 0.4, 0.4)$.
- The second stage **generalized mean** of order $-2$ yields - $\mu_{-2} (0.4, 0.4, 0.4) = 0.4$.
- Reversed order of aggregation
  - The first stage yields - $(0.46, 0.4, 0.36)$ and the second stage yields - $W_F = 0.4$. 
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- The same power of generalized mean → the $W_F$ satisfies path independence (PI)

Example: $X = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.4 & 0.3 & 0.8 \\ 0.3 & 0.4 & 0.4 \end{bmatrix}$

First Stage: Generalized mean across persons yields (0.4, 0.4, 0.4).
- The second stage generalized mean of order $-2$ yields - $\mu_{-2} (0.4, 0.4, 0.4) = 0.4$.
- Reversed order of aggregation
  - The first stage yields - (0.46, 0.4, 0.36) and the second stage yields - $W_F = 0.4$.
- The order of aggregation does not matter.
Example: $\bar{X} = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.35 & 0.35 & 0.6 \\ 0.35 & 0.35 & 0.6 \end{bmatrix}$
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First Stage: Generalized mean across persons yields $(0.41, 0.41, 0.42)$. Therefore, both $W_F$ and $W_H$ are sensitive to inequality across persons.
Indices Sensitive to Inequality Across Persons

- Example: \( \bar{X} = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.35 & 0.35 & 0.6 \\ 0.35 & 0.35 & 0.6 \end{bmatrix} \)

- First Stage: Generalized mean across persons yields \((0.41, 0.41, 0.42)\).

- The second stage **generalized mean** or order \(-2\) yields -
  \[ \mu_{-2}(0.4, 0.4, 0.4) = 0.41. \]
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- Example: \( \bar{X} = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.35 & 0.35 & 0.6 \\ 0.35 & 0.35 & 0.6 \end{bmatrix} \)

- First Stage: Generalized mean across persons yields \((0.41, 0.41, 0.42)\).

- The second stage *generalized mean* or order \(-2\) yields -
  \[ \mu_{-2}(0.4, 0.4, 0.4) = 0.41. \]

- Thus, \( W_F(X) = 0.40 \) and \( W_F(\bar{X}) = 0.41 \)
Indices Sensitive to Inequality Across Persons

Example: \[ \bar{X} = \begin{bmatrix} 0.8 & 0.8 & 0.3 \\ 0.35 & 0.35 & 0.6 \\ 0.35 & 0.35 & 0.6 \end{bmatrix} \]

First Stage: Generalized mean across persons yields \((0.41, 0.41, 0.42)\).

The second stage \textit{generalized mean} or order \(-2\) yields -
\[ \mu_{-2}(0.4, 0.4, 0.4) = 0.41. \]

Thus, \(W_F(X) = 0.40\) and \(W_F(\bar{X}) = 0.41\)

Foster et. al. index satisfies NORM, LHOM, ANON, MON, POPRI, CONT, SUBCON, PATHIN, and UM
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Therefore, both \( W_H \) and \( W_F \) are sensitive to inequality across persons
Motivation
Group-Based HDI

- Motivation
  - Individual/household level data are not always available
Motivation

- Individual/household level data are not always available
- Individual/household level data can not be calculated for every variable
Motivation

- Individual/household level data are not always available
- Individual/household level data cannot be calculated for every variable
  - e.g., life expectancy, enrolment rate etc.
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- Generalized means could be difficult to understand and interpret
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  - Individual/household level data are not always available
  - Individual/household level data can not be calculated for every variable
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- Harttgen, Klasen, and Misselhorn Index ($W_{HKM}$)
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Harttgen, Klasen, and Misselhorn Index ($W_{HKM}$)
- Divide the population by income quintiles
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- Individual/household level data can not be calculated for every variable
  - e.g., life expectancy, enrolment rate etc.
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Harttgen, Klasen, and Misselhorn Index ($W_{HKM}$)

- Divide the population by income quintiles
- Calculate the HDI for each quintile
### Group-Based HDI - Results

<table>
<thead>
<tr>
<th>Country</th>
<th>$Q = 1$</th>
<th>$Q = 2$</th>
<th>$Q = 3$</th>
<th>$Q = 4$</th>
<th>$Q = 5$</th>
<th>Overall HDI</th>
<th>Ratio Q5/Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Industrialized countries</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USA (2000)</td>
<td>0.837</td>
<td>0.893</td>
<td>0.927</td>
<td>0.957</td>
<td>1.011</td>
<td>0.940</td>
<td>1.208</td>
</tr>
<tr>
<td>Finland (2002)</td>
<td>0.870</td>
<td>0.897</td>
<td>0.919</td>
<td>0.944</td>
<td>0.989</td>
<td>0.930</td>
<td>1.137</td>
</tr>
<tr>
<td><strong>Developing countries</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Columbia (2000/2005)</td>
<td>0.637</td>
<td>0.741</td>
<td>0.800</td>
<td>0.857</td>
<td>0.927</td>
<td>0.790</td>
<td>1.377</td>
</tr>
<tr>
<td>Vietnam (2004/2002)</td>
<td>0.627</td>
<td>0.680</td>
<td>0.718</td>
<td>0.765</td>
<td>0.828</td>
<td>0.713</td>
<td>1.321</td>
</tr>
<tr>
<td>Indonesia (2000/2003)</td>
<td>0.593</td>
<td>0.651</td>
<td>0.700</td>
<td>0.764</td>
<td>0.874</td>
<td>0.701</td>
<td>1.474</td>
</tr>
<tr>
<td>South Africa (2000/1998)</td>
<td>0.561</td>
<td>0.640</td>
<td>0.700</td>
<td>0.743</td>
<td>0.879</td>
<td>0.691</td>
<td>1.567</td>
</tr>
<tr>
<td>Bolivia (2002/2003)</td>
<td>0.550</td>
<td>0.640</td>
<td>0.704</td>
<td>0.741</td>
<td>0.863</td>
<td>0.690</td>
<td>1.570</td>
</tr>
<tr>
<td>Nicaragua (2001/2001)</td>
<td>0.531</td>
<td>0.629</td>
<td>0.678</td>
<td>0.720</td>
<td>0.830</td>
<td>0.667</td>
<td>1.563</td>
</tr>
<tr>
<td>Cameroon (2001/2004)</td>
<td>0.417</td>
<td>0.477</td>
<td>0.529</td>
<td>0.553</td>
<td>0.644</td>
<td>0.523</td>
<td>1.544</td>
</tr>
<tr>
<td>Madagascar (2001/1997)</td>
<td>0.343</td>
<td>0.463</td>
<td>0.496</td>
<td>0.563</td>
<td>0.684</td>
<td>0.488</td>
<td>1.994</td>
</tr>
<tr>
<td>Guinea (1995/1999)</td>
<td>0.340</td>
<td>0.457</td>
<td>0.490</td>
<td>0.594</td>
<td>0.696</td>
<td>0.467</td>
<td>2.047</td>
</tr>
<tr>
<td>Côte d’Ivoire (1998/1999)</td>
<td>0.343</td>
<td>0.416</td>
<td>0.434</td>
<td>0.515</td>
<td>0.561</td>
<td>0.430</td>
<td>1.636</td>
</tr>
<tr>
<td>Zambia (2002/2002)</td>
<td>0.317</td>
<td>0.390</td>
<td>0.431</td>
<td>0.476</td>
<td>0.583</td>
<td>0.426</td>
<td>1.839</td>
</tr>
<tr>
<td>Mozambique (2002/2003)</td>
<td>0.305</td>
<td>0.355</td>
<td>0.380</td>
<td>0.417</td>
<td>0.504</td>
<td>0.387</td>
<td>1.652</td>
</tr>
<tr>
<td>Burkina Faso (2003/2003)</td>
<td>0.257</td>
<td>0.306</td>
<td>0.331</td>
<td>0.365</td>
<td>0.489</td>
<td>0.348</td>
<td>1.903</td>
</tr>
</tbody>
</table>
Group-Based HDI - Pros and Cons

- Pros

- Cons
  - Similar to Kuznet-ratio. Thus, it does not take into account the entire distribution.
  - May not satisfy UM strictly
  - Defends dimensions used in constructing the HDI
  - More and more household level data are available recently

Suman Seth (Vanderbilt University & OPHI)

Inequality Adjusted HDI

3rd September, 2009 23 / 30
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  - Easily applicable

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  - Defends dimensions used in constructing the HDI
  - More and more household level data are available recently.
Pros

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- Easily comprehensible

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Suppose we choose dimensions so that individual level information for each of them is available.

<table>
<thead>
<tr>
<th>Dim 1</th>
<th>Dim 2</th>
<th>Dim 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Person 1</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Person 2</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Person 3</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Let human development be calculated by applying $W_H$ or $W_F$.

Question: Where should the dollar be spent?

Using $W_H$: Answer: Either on dim 1 of person 3, or on dim 2 of person 2, or on dim 3 of person 1.

Using $W_F$: Answer: Either on dim 1 of person 3, or on dim 2 of person 2, or on dim 3 of person 1.
Policy Exercise Revisited - Other Form of Inequality

- Suppose we choose dimensions so that individual level information for each of them is available
- Reconsider the achievement matrix

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Suppose we choose dimensions so that individual level information for each of them is available.

Reconsider the achievement matrix:

\[ X = \begin{array}{c|ccc}
\text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\hline
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\end{array} \]

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Reconsider the achievement matrix

\[ X = \begin{array}{c|ccc}
& \text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\hline
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\end{array} \]

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 & \text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\hline
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
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Let human development be calculated by applying \(W_H\) or \(W_F\).

Question: Where should the dollar be spent?

- Using \(W_H\): Answer: Either on dim 1 of person 3, or on dim 2 of person 2, or on dim 3 of person 1
- Using \(W_F\): Answer: Either on dim 1 of person 3, or on dim 2 of person 2, or on dim 3 of person 1
Policy Exercise

\[ H = \begin{array}{c|c|c|c}
 & \text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\hline
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\end{array} \]

- Where should the assistance be made from an ethical point of view?
Where should the assistance be made from an ethical point of view?

- Suppose, the overall achievement of the \( n^{\text{th}} \) individual is calculated by \( \frac{x_{n1} + x_{n2} + x_{n3}}{3} \)
Policy Exercise

\[
H = \begin{array}{c|ccc}
& \text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\hline
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\end{array}
\]

- Where should the assistance be made from an ethical point of view?
- Suppose, the overall achievement of the \( n^{\text{th}} \) individual is calculated by \( (x_{n1} + x_{n2} + x_{n3}) / 3 \)
- Achievement vector across individuals: \((0.63, 0.5, 0.37)\)
Policy Exercise

\[ H = \begin{array}{c|c|c|c}
 & \text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\hline
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\end{array} \]

- Where should the assistance be made from an ethical point of view?
- Suppose, the overall achievement of the \( n^{th} \) individual is calculated by
  \[ \left( x_{n1} + x_{n2} + x_{n3} \right) / 3 \]
  - Achievement vector across individuals: (0.63, 0.5, 0.37)
- Spend the dollar on dim 1 of person 3
Policy Exercise

\[
H = \begin{bmatrix}
\text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\end{bmatrix}
\]

- Where should the assistance be made from an ethical point of view?
  - Suppose, the overall achievement of the \(n^{th}\) individual is calculated by \((x_{n1} + x_{n2} + x_{n3}) / 3\)
    - Achievement vector across individuals: \((0.63, 0.5, 0.37)\)
  - Spend the dollar on dim 1 of person 3
    - Overall achievement vector: \((0.63, 0.5, 0.4)\)
Policy Exercise

\[ H = \begin{array}{|c|c|c|}
\hline
& \text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\hline
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\hline
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\hline
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\hline
\end{array} \]

- Where should the assistance be made from an ethical point of view?
  - Suppose, the overall achievement of the \( n^{\text{th}} \) individual is calculated by 
    \[ (x_{n1} + x_{n2} + x_{n3}) / 3 \]
    - Achievement vector across individuals: \((0.63, 0.5, 0.37)\)
    - Spend the dollar on dim 1 of person 3
      - Overall achievement vector: \((0.63, 0.5, 0.4)\)
    - Spend the dollar on dim 2 of person 2
**Policy Exercise**

\[
H = \begin{array}{c|ccc}
 & \text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\hline
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\end{array}
\]

- Where should the assistance be made from an ethical point of view?
  - Suppose, the overall achievement of the \( n \)\(^{th} \) individual is calculated by 
    \[
    \left( x_{n1} + x_{n2} + x_{n3} \right) / 3
    \]
  - Achievement vector across individuals: \((0.63, 0.5, 0.37)\)
  - Spend the dollar on dim 1 of person 3
    - Overall achievement vector: \((0.63, 0.5, 0.4)\)
  - Spend the dollar on dim 2 of person 2
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Where should the assistance be made from an ethical point of view?

- Suppose, the overall achievement of the $n^{th}$ individual is calculated by \((x_{n1} + x_{n2} + x_{n3}) / 3\)

  \begin{itemize}
  \item Achievement vector across individuals: \((0.63, 0.5, 0.37)\)
  \item Spend the dollar on dim 1 of person 3
    \begin{itemize}
    \item Overall achievement vector: \((0.63, 0.5, 0.4)\)
    \end{itemize}
  \item Spend the dollar on dim 2 of person 2
    \begin{itemize}
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    \end{itemize}
  \item Spend the dollar on dim 2 of person 2
  \end{itemize}
Where should the assistance be made from an ethical point of view?

Suppose, the overall achievement of the \( n^{\text{th}} \) individual is calculated by
\[
\frac{x_{n1} + x_{n2} + x_{n3}}{3}
\]
Achievement vector across individuals: \((0.63, 0.5, 0.37)\)

- Spend the dollar on dim 1 of person 3
  - Overall achievement vector: \((0.63, 0.5, 0.4)\)
- Spend the dollar on dim 2 of person 2
  - Overall achievement vector: \((0.63, 0.53, 0.37)\)
- Spend the dollar on dim 2 of person 2
  - Overall achievement vector: \((0.67, 0.5, 0.37)\)
These indices can also not differentiate the following two allocations

\[ H = \begin{pmatrix} 2 & 4 & 0 \\ 0 & 3 & 0 \\ 0 & 4 & 3 \end{pmatrix}, \quad H_0 = \begin{pmatrix} 2 & 4 & 0 \\ 0 & 3 & 0 \\ 0 & 4 & 3 \end{pmatrix} \]
Association Sensitivity

- These indices can also not differentiate the following two allocations

\[
H = \begin{bmatrix}
0.8 & 0.8 & 0.3 \\
0.4 & 0.3 & 0.8 \\
0.3 & 0.4 & 0.4
\end{bmatrix}, \quad H' = \begin{bmatrix}
0.8 & 0.8 & 0.3 \\
0.4 & 0.4 & 0.8 \\
0.3 & 0.3 & 0.4
\end{bmatrix}
\]
These indices can also not differentiate the following two allocations:

\[
\begin{bmatrix}
0.8 & 0.8 & 0.3 \\
0.4 & 0.3 & 0.8 \\
0.3 & 0.4 & 0.4
\end{bmatrix},
\begin{bmatrix}
0.8 & 0.8 & 0.3 \\
0.4 & 0.4 & 0.8 \\
0.3 & 0.3 & 0.4
\end{bmatrix}
\]

\(H'\) is obtained from \(H\) by an association increasing transfer (Atkinson and Bourguignon (1982), Boland and Proschan (1988), Tsui (1995, 1999, 2002), Decancq and Lugo (2008), Seth (2009)).
Association Sensitivity

- These indices can also not differentiate the following two allocations

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H = \begin{bmatrix}
0.8 & 0.8 & 0.3 \\
0.4 & 0.3 & 0.8 \\
0.3 & 0.4 & 0.4 \\
\end{bmatrix}, \quad H' = \begin{bmatrix}
0.8 & 0.8 & 0.3 \\
0.4 & 0.4 & 0.8 \\
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\end{bmatrix}
\]

- \(H'\) is obtained from \(H\) by an association increasing transfer (Atkinson and Bourguignon (1982), Boland and Proschan (1988), Tsui (1995, 1999, 2002), Decancq and Lugo (2008), Seth (2009))

- The second form of inequality across persons - association sensitive inequality
Association Sensitivity

- These indices can also not differentiate the following two allocations:

\[
H = \begin{bmatrix}
0.8 & 0.8 & 0.3 \\
0.4 & 0.3 & 0.8 \\
0.3 & 0.4 & 0.4 \\
\end{bmatrix}, \quad H' = \begin{bmatrix}
0.8 & 0.8 & 0.3 \\
0.4 & 0.4 & 0.8 \\
0.3 & 0.3 & 0.4 \\
\end{bmatrix}
\]

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- The second form of inequality across persons - association sensitive inequality

- Association Sensitivity Axiom
These indices can also not differentiate the following two allocations

\[
H = \begin{bmatrix}
0.8 & 0.8 & 0.3 \\
0.4 & 0.3 & 0.8 \\
0.3 & 0.4 & 0.4
\end{bmatrix}, \quad H' = \begin{bmatrix}
0.8 & 0.8 & 0.3 \\
0.4 & 0.4 & 0.8 \\
0.3 & 0.3 & 0.4
\end{bmatrix}
\]

\(H'\) is obtained from \(H\) by an association increasing transfer (Atkinson and Bourguignon (1982), Boland and Proschan (1988), Tsui (1995, 1999, 2002), Decancq and Lugo (2008), Seth (2009))

The second form of inequality across persons - association sensitive inequality

Association Sensitivity Axiom

- Strictly decreasing in increasing association (SDIA) - \(W(H') < W(H)\)
These indices can also not differentiate the following two allocations

\[
H = \begin{bmatrix}
0.8 & 0.8 & 0.3 \\
0.4 & 0.3 & 0.8 \\
0.3 & 0.4 & 0.4
\end{bmatrix}, \quad H' = \begin{bmatrix}
0.8 & 0.8 & 0.3 \\
0.4 & 0.4 & 0.8 \\
0.3 & 0.3 & 0.4
\end{bmatrix}
\]


The second form of inequality across persons - association sensitive inequality

**Association Sensitivity Axiom**

- Strictly decreasing in increasing association (SDIA) - W(H′) < W(H)
- H′ is obtained from H by a sequence of association increasing transfers
Association Sensitivity

- These indices can also not differentiate the following two allocations
  \[
  H = \begin{bmatrix}
  0.8 & 0.8 & 0.3 \\
  0.4 & 0.3 & 0.8 \\
  0.3 & 0.4 & 0.4 \\
  \end{bmatrix}, \quad H' = \begin{bmatrix}
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  0.4 & 0.4 & 0.8 \\
  0.3 & 0.3 & 0.4 \\
  \end{bmatrix}
  \]

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- The second form of inequality across persons - association sensitive inequality

- Association Sensitivity Axiom
  - Strictly decreasing in increasing association (SDIA) - \(W(H') < W(H)\)
  - \(H'\) is obtained from \(H\) by a sequence of association increasing transfers

- **Proposition:** A human development index that aggregates across persons first and then across dimensions is not sensitive to association among dimensions
**Corollary:** No path independent human development index is sensitive to association among dimensions
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To be association sensitive the aggregation must take place across dimensions first and then across persons
**Corollary:** No path independent human development index is sensitive to association among dimensions

To be association sensitive the aggregation must take place across dimensions first and then across persons

Possible association sensitive human development Index ($W$):

$$W(X) = \mu_\alpha \mu_\beta (x_1, \ldots, x_N),$$
**Corollary:** No path independent human development index is sensitive to association among dimensions

To be association sensitive the aggregation must take place across dimensions first and then across persons

Possible association sensitive human development Index ($W$):

First stage: aggregates across dimensions by $\mu_{\beta}(\cdot)$. Second stage: aggregates across persons by $\mu_{\alpha}(\cdot)$
**Corollary:** No path independent human development index is sensitive to association among dimensions

To be association sensitive the aggregation must take place across dimensions first and then across persons

Possible association sensitive human development Index ($W$):

- First stage: aggregates across dimensions by $\mu_\beta(\cdot)$. Second stage: aggregates across persons by $\mu_\alpha(\cdot)$
- $W(X) = \mu_\alpha(\mu_\beta(x_1), \ldots, \mu_\beta(x_N))$
**Corollary:** No path independent human development index is sensitive to association among dimensions

To be association sensitive the aggregation must take place across dimensions first and then across persons

Possible association sensitive human development Index ($W$):

- First stage: aggregates across dimensions by $\mu_\beta (\cdot)$. Second stage: aggregates across persons by $\mu_\alpha (\cdot)$

  $W(X) = \mu_\alpha \left( \mu_\beta (x_{1^*}), \ldots, \mu_\beta (x_{N^*}) \right)$

$W$ satisfies NORM, LHOM, ANON, MON, POPRI, CONT, SUBCON, UM, and
Corollary: No path independent human development index is sensitive to association among dimensions

To be association sensitive the aggregation must take place across dimensions first and then across persons

Possible association sensitive human development Index ($\mathcal{W}$):

- First stage: aggregates across dimensions by $\mu_\beta (\cdot)$. Second stage: aggregates across persons by $\mu_\alpha (\cdot)$

$$\mathcal{W} (X) = \mu_\alpha \left( \mu_\beta (x_{1*}) , \ldots , \mu_\beta (x_{N*}) \right)$$

$\mathcal{W}$ satisfies NORM, LHOM, ANON, MON, POPRI, CONT, SUBCON, UM, and

- SDIA if and only if $\alpha < \beta \leq 1$
Where should the dollar be spent according to $W$?
Policy Exercise

\[
H = \begin{bmatrix}
\text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\end{bmatrix}
\]

- Where should the dollar be spent according to \( W \)?
- Suppose, \( \alpha = -2 \) and \( \beta = 0.1 \)
### Policy Exercise

\[ H = \begin{array}{ccc}
\text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\hline
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\end{array} \]

- Where should the dollar be spent according to \( W \)?
  - Suppose, \( \alpha = -2 \) and \( \beta = 0.1 \)
  - Spend the dollar on dim 1 of person 3
Where should the dollar be spent according to $\mathcal{W}$?

- Suppose, $\alpha = -2$ and $\beta = 0.1$
- Spend the dollar on dim 1 of person 3
  - Level of human development is $= 0.465$
Policy Exercise

\[ H = \begin{array}{|c|c|c|}
\hline
& \text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\hline
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\hline
\end{array} \]

- Where should the dollar be spent according to \( W \)?
  - Suppose, \( \alpha = -2 \) and \( \beta = 0.1 \)
  - Spend the dollar on dim 1 of person 3
    - Level of human development is \( = 0.465 \)
  - Spend the dollar on dim 2 of person 2
Where should the dollar be spent according to $\mathcal{W}$?

- Suppose, $\alpha = -2$ and $\beta = 0.1$
- Spend the dollar on dim 1 of person 3
  - Level of human development is $= 0.465$
- Spend the dollar on dim 2 of person 2
  - Level of human development is $= 0.456$
Policy Exercise

\[
H = \begin{array}{ccc}
\text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\hline
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\end{array}
\]

Where should the dollar be spent according to \( \mathcal{W} \)?

- Suppose, \( \alpha = -2 \) and \( \beta = 0.1 \)
- Spend the dollar on dim 1 of person 3
  - Level of human development is \( = 0.465 \)
- Spend the dollar on dim 2 of person 2
  - Level of human development is \( = 0.456 \)
- Spend the dollar on dim 3 of person 1
Policy Exercise

\[
\begin{array}{|c|c|c|c|}
\hline
& \text{Dim 1} & \text{Dim 2} & \text{Dim 3} \\
\hline
\text{Person 1} & 0.8 & 0.8 & 0.3 \\
\hline
\text{Person 2} & 0.4 & 0.3 & 0.8 \\
\hline
\text{Person 3} & 0.3 & 0.4 & 0.4 \\
\hline
\end{array}
\]

- Where should the dollar be spent according to \( \mathcal{W} \)?
  - Suppose, \( \alpha = -2 \) and \( \beta = 0.1 \)
  - Spend the dollar on dim 1 of person 3
    - Level of human development is \( = 0.465 \)
  - Spend the dollar on dim 2 of person 2
    - Level of human development is \( = 0.456 \)
  - Spend the dollar on dim 3 of person 1
    - Level of human development is \( = 0.452 \)
### Application to Mexico (Income, Education, and Health)

<table>
<thead>
<tr>
<th>State</th>
<th>HDI ($W_A$)</th>
<th>$WF$ (\alpha = -2)</th>
<th>$\mathcal{W}$ (\beta = -1) (\alpha = -3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Luis Potosí</td>
<td>0.716 (24)</td>
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</tr>
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- **A sequence of association increasing transfers for Tabasco**
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We treated dimensions symmetrically; we could also apply weighted generalized mean
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  - Trade-off?
Summary

- Traditional human development indices are not sensitive to inequality across persons
- Two forms of inequality
- The first form is distribution sensitive inequality
  - Measures incorporating inequality internally
    - Hicks Index
    - Foster, Lopez-Calva, Szekely Index
  - Measures incorporating inequality externally
    - Harttgen, Klasen, and Misselhorn Index
- The second form of inequality - dimensional interactions are important
- Aggregation should take place across dimensions first, and then across persons
  - Trade-off?
- We treated dimensions symmetrically; we could also apply weighted generalized mean