Decomposing the adjusted headcount ratio

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- In this class we will explore the nice decomposability properties of $\Delta\%M0$ and its components: $\Delta\%H$ and $\Delta\%A$.
- This material is based on Apablaza, Ocampo and Yalonetzky (2010).

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- Then we will show some results from Apablaza, Ocampo and Yalonetzky (2010).
- Then we will discuss one method to produce confidence intervals for these changes.
- We will finish with some remarks on comparability from Apablaza, Ocampo and Yalonetzky (2010)

Basic Notation

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$$g_{nd}(k) = \frac{z_d - x_{nd}}{z_d} \text{ if } z_d > x_{nd} \wedge c_n \geq k$$

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Multidimensional poverty headcount:

$$H(X^t; Z) \equiv \frac{1}{N^t} \sum_{n=1}^{N^t} [\sum_{d=1}^{D} w_d g_{nd}(k)]^0 = \frac{1}{N^t} \sum_{n=1}^{N^t} I(c_n \geq k)$$

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$$A(X^t; Z) \equiv \frac{\sum_{n=1}^{N^t} \sum_{d=1}^{D} w_d [g_{nd}(k)]^0}{D \sum_{n=1}^{N^t} [\sum_{d=1}^{D} w_d g_{nd}(k)]^0}$$

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The M0, or MPI:

$$M^0(X^t; Z) \equiv H^t A^t$$

$$\Delta\%_a M^0(t) \equiv \frac{M^0(X^t; Z) - M^0(X^{t-a}; Z)}{M^0(X^{t-a}; Z)}$$

Basic decomposition of M0

$$\Delta\%_a M^0(t) = \Delta\%_a H(t) + \Delta\%_a A(t) + \Delta\%_a H(t)\Delta\%_a A(t)$$

- $\Delta\%_a H(t)$ and $\Delta\%_a A(t)$ are not generally independent, but sometimes a change in one may not produce a change in the other.

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- or if $k < D$ it is possible that $\Delta\%_a H(t) = 0$ and $\Delta\%_a A(t) \neq 0$.
- As k goes from 1 to D, H decreases and A increases "mechanically". Hence as k increases toward D, it is more likely to find higher $\Delta\%_a H(t)$ and lower $\Delta\%_a A(t)$.

Basic decomposition of H

H is decomposable into the multidimensional headcounts of subgroups:

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In turn:

$$H^i(X_i^t, Z) = \frac{1}{N_i^t} \sum_{n=1}^{N} [\sum_{d=1}^{D} w_d g_{nd}(k)]^0 I(n \in i)$$

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Then:

$$\Delta\%_a H(t) = \sum_{i=1}^{G} r_i(t-a)[\Delta\%_a \psi_i^t + \Delta\%_a H^i(X_i^t, Z) + \Delta\%_a \psi_i^t \Delta\%_a H^i(X_i^t, Z)]$$

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And $A_d(X^t, Z) \equiv \frac{\sum_{n=1}^{N^t} I(g_{nd}>0)}{H(t)}$

Then:

$$\Delta\%_a A(t) = \sum_{d=1}^{D} s_d(t-a)[\Delta\%_a \theta_d A_d(X^t, Z)] = \sum_{d=1}^{D} s_d(t-a)[\Delta\%_a A_d(X^t, Z)]$$

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Because, by construction, $\Delta\%_a \theta_d = 0$

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In practical comparisons, we divide the changes by the year-gaps to improve comparability.

The countries

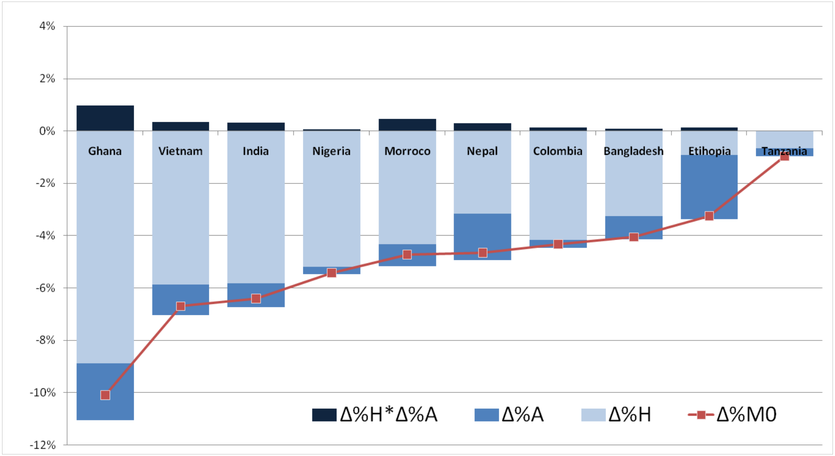
Country	Years
Bangladesh	2004-2007
Colombia	1995-2005
Ethiopia	2000-2005
Ghana	2003-2008
India	1999-2005
Morocco	1992-2004
Nepal	2001-2006
Nigeria	1999-2003
Tanzania	2005-2008
Vietnam	1997-2002

The variables

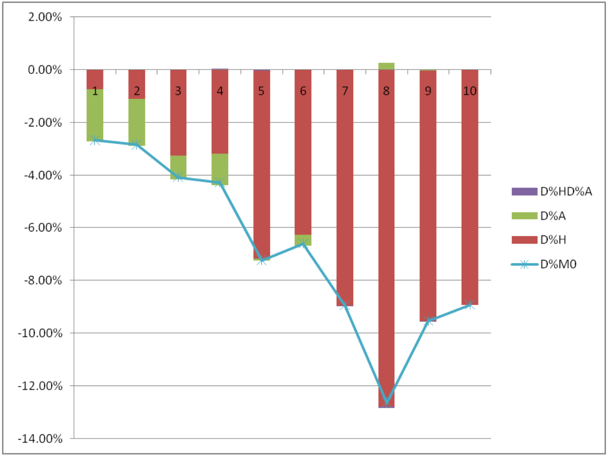
Variable	B	C	E	G	I	M	Ne	Ni	T	V
Years school	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Enrollment	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Child mortality	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Nutrition	✓	✓	✓	✓	✓	✓	✓	✓	x	x
Electricity	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Toilet	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Water	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Floor	✓	✓	✓	✓	x	✓	✓	✓	✓	✓
Cooking	✓	✓	✓	✓	✓	x	✓	x	✓	x
Asset	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

B=Bangladesh; C=Colombia; E=Ethiopia; G=Ghana; I=India
M=Morocco; Ne=Nepal; Ni=Nigeria; T=Tanzania; V=Vietnam

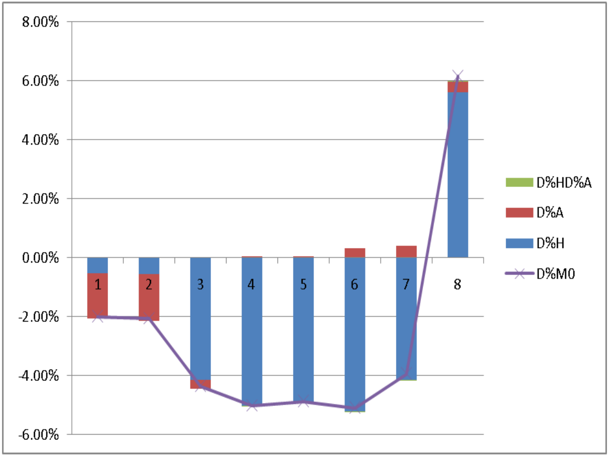
Decomposition of M0 for 10 countries and k=3



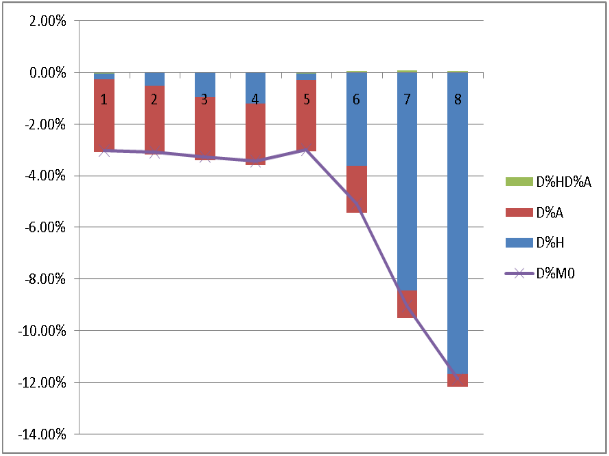
The impact of the choice of k: the case of Bangladesh



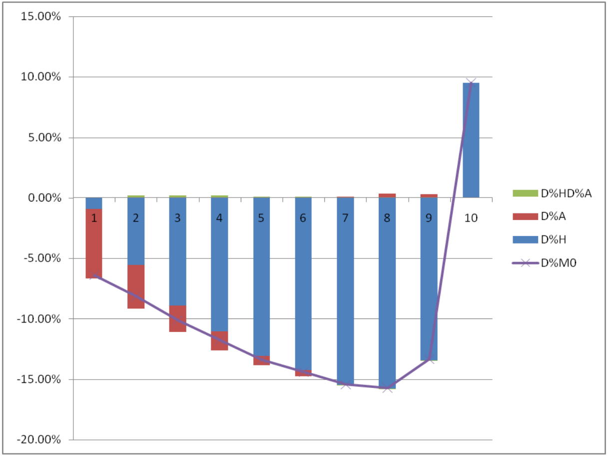
The impact of the choice of k: the case of Colombia



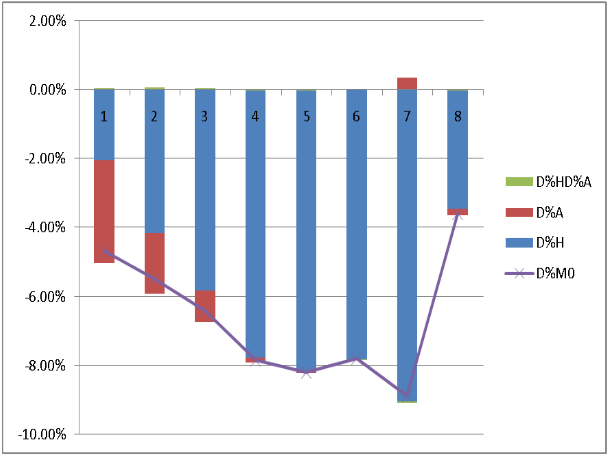
The impact of the choice of k: the case of Ethiopia



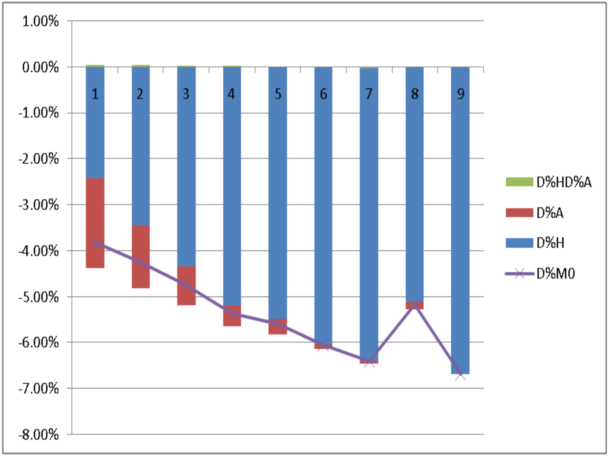
The impact of the choice of k: the case of Ghana



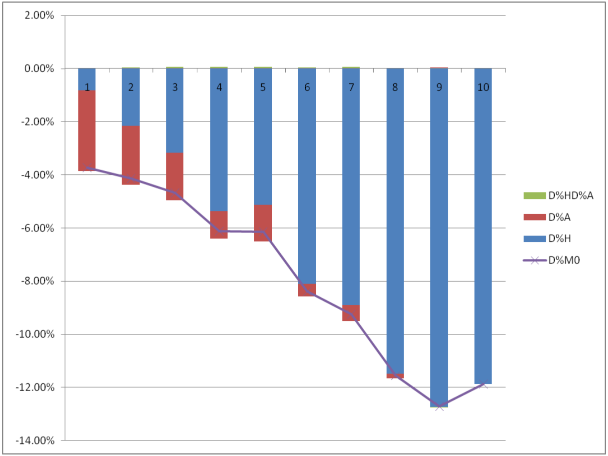
The impact of the choice of k: the case of India



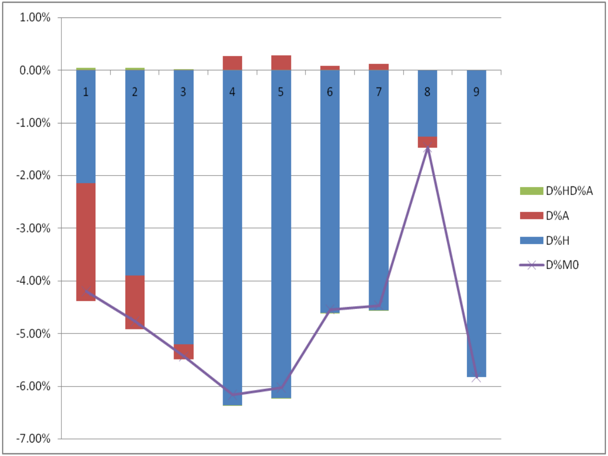
The impact of the choice of k: the case of Morocco



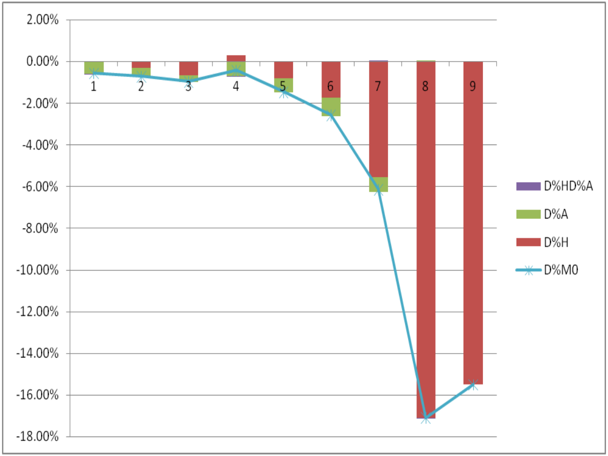
The impact of the choice of k: the case of Nepal



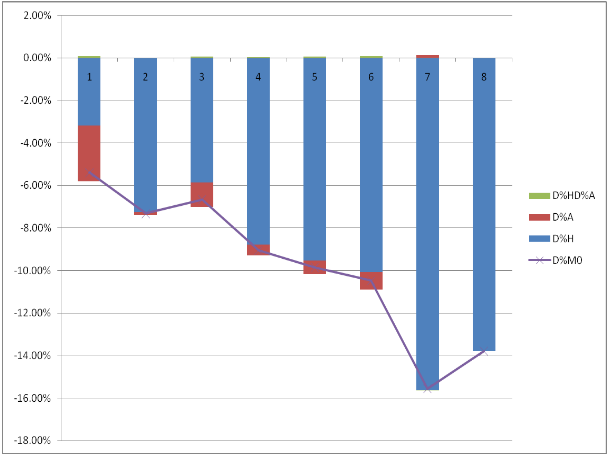
The impact of the choice of k: the case of Nigeria



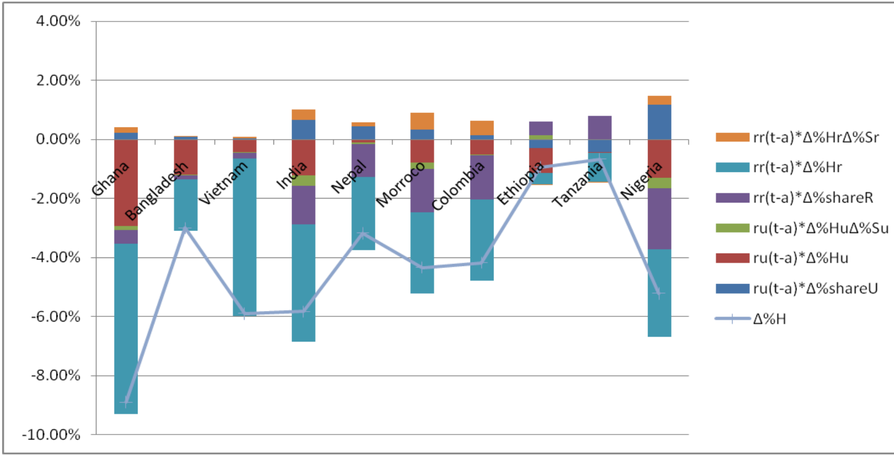
The impact of the choice of k: the case of Tanzania



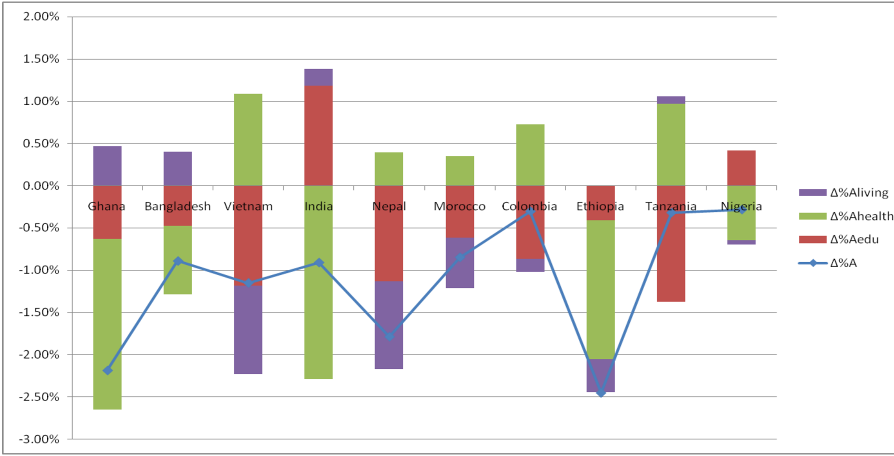
The impact of the choice of k: the case of Vietnam



Decomposition of H for k=3



Decomposition of A for k=3



Producing confidence intervals for the changes

Two ways of producing standard errors (and confidence intervals): analytically derived and resamplings (e.g. bootstrap, jackknife)

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1. Produce a bootstrapped distribution of $M^0(X^t; Z)$ and $M^0(X^{t-a}; Z)$, the same for H and A, and all the other elements, for a a number of resamplings, e.g. 1000.

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3. Such combination should yield one million decomposed changes.

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3. Such combination should yield one million decomposed changes.
4. With these observations one can both estimate a standard error or confidence intervals (e.g. using the percentile method)

Concluding remarks on time comparisons with M0 across countries

When time periods differ three potential problems of comparability arise: (Apablaza, Ocampo and Yalonetzky, 2010)

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- Time spans are different: If the differences are too wild it may be that for one country we observe short-term business cycle fluctuations whereas for the other we observe a medium-term growth trend. Solution: Restrict comparisons to time spans that do not differ too wildly.

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- Time spans are different: If the differences are too wild it may be that for one country we observe short-term business cycle fluctuations whereas for the other we observe a medium-term growth trend. Solution: Restrict comparisons to time spans that do not differ too wildly.
- The years are different: Even when spans are equal, taking year brackets too far apart may affect the meaningfulness of the comparison. (E.g. Kenya in the 1950s with Chile in the 1990s). Solution: Justify your comparisons when the years are different.