A Note on Capability Comparison and Social Evaluation

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1. Introduction

The purpose of this paper is to analyze the possibility of *capability comparison* in the light of social choice aiming "to secure basic capability for all".

According to Sen, *capability* is defined as a set of functionings vectors which are realized by using goods and services. When a shortage relative to the basic capability is recognized for an individual, it must be compensated through social policies. However, what functionings vector actually achieves depends on the individual's choice (Sen, 1985). Based on the conception of capability, we can attempt to construct a social policy aiming to secure individual freedom with regard to their interests, goals and choices. However, even intra-personally, interest, goal and choice often diverge from one another, not to mention that, inter-personally, they must differ largely.

Therefore, in applying the capability approach we have to resolve the following problems: how to construct a social evaluation over possible social states, as a foundation to choose certain social policies, which entails comparing the alternative capabilities which individuals would have under alternative social policies.

In the previous studies, these questions were addressed either as part of a measurement problem of capability, ---how to constitute indices aggregating plural factors among individuals--- or as a philosophical problem relating to capability, ---how to identify the constituents of capability in terms of the 'good life'. In contrast, this paper investigates these problems in the light of social choice theory.

Social choice theory mainly studies aggregation procedures leading to social evaluation based on individual information. Once he had provided the framework of the capability approach, Sen left the determination of the components of an individual

capability or of desirable social policies, to the people who actually live in any given society (Sen, 1999). Moreover, he insists that for social evaluation it is enough to define common criteria of well-being by taking an intersection of plural evaluation orderings (complete or incomplete) (Sen, 1985).

These arguments can be understood in the line with Sen's study on poverty and inequality, such as reconstructing a partial order based on the intersection of different measurements (Sen, 1973, 1997). Moreover, the idea to construct a social evaluation based on individual evaluations, and the idea of understanding the procedure not as a ballot but as a process of public discussion or reasoning suggests exhibiting the reach of Sen's original social choice theory.

This paper inquires into the connection between Sen's ideas of public reasoning as they appear in the capability approach and Sen's insights concerning social choice theory (which he greatly advanced and brought forward since its original Arrovian formulation), by reexamining the meaning of the main conceptions of social choice in the context of the capability approach. To make this problem more concrete, let us focus on the following questions.

"To secure basic capability for all", a public reasoning cannot escape measuring individual capabilities and making inter-personal or inner-personal comparisons. Yet, to what extent should we do so?

Can *public reasoning* respect the aim "to secure basic capability for all" while satisfying the conditions which are familiar to social choice theory such as unrestricted domain, anonymity, neutrality and the Weak Pareto?

The answer of this last question will be positive or negative depending on the answer we give to the first question. We can construct a public reasoning, that respects the aim "to secure basic capability for all" while satisfying the capability-based Pareto condition and the Arrovian conditions with a small modification, and that provides a ---generally incomplete--- social evaluation defined as 'non-contradiction parts' of the three types of disadvantage-based evaluations.

In order to achieve this result, we must, first of all, re-examine the assumptions of interpersonal and intrapersonal comparison in the context the capability approach through an exploration of the conception of individual disadvantage itself.

2. Philosophical Inquiry into the Comparability of Capability
To secure the basic capability for all, let us focus on individuals disadvantaged in their basic capability. We can recognize at least three types of disadvantage which are difficult to compare to each other.

First, a type of disadvantage that cannot be grasped by focusing on the momentary outcome situations of individuals, for example, disadvantages derived from historical injustices (invaded indigenous people, victims of colonial exploitation), social disasters or criminality. These disadvantages strike individuals, who in relation with each other, are in diverse circumstances before suffering this damage and remain different afterwards. In these cases, we should evaluate the disadvantage they suffer over the whole duration of their life through direct research on the evaluation of the disadvantages taking into account their struggles to keep on living in the face of this disadvantage.

Second, a type of disadvantage which are easier to identify through the extra needs a social group or category may have, for example, disadvantages derived from natural or social characteristics such as disability, particular diseases, age, nationality, sex or being a single parent. In these cases, we can approximate individual disadvantages by estimating extra cash or in-kinds needed to live adequately compared to a group without such characteristics.

Third, a type of disadvantage for which it is difficult to identify the specific causes or particular needs but where individuals actually suffer here and now. In this case, it is appropriate to adopt a social policy that compensates their shortage of basic capability compared to the minimum level of resources including income, assets and other personal resources by utilizing the list of functionings which are recognized to be crucial to live well in the given society and price information of the goods and services which are effective to promote those functionings.

These three types of disadvantage can be seen as corresponding to three different conceptions of justice that underlay the reason why and how a society should compensate individuals' disadvantages.

The first corresponds to a conception called by Aristotle "justice as redress". It is based on the recognizing the causes of the suffered disadvantage as an injustice that needs to be redressed and where the responsibility of society as a whole is engaged. Compensation declares Society's resolution not to repeat such injustice in the future[1]. It requires arithmetic equity between the estimation of damages, widely understood as its human consequences and the compensation which is deemed necessary to support individuals' struggles to keep living in spite of the damages suffered.

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[1]Even when the one who is directly responsible for the damage can be identified, if he is unable to compensate, society must supplement his compensation, as long as society has a responsibility not only not to harm individuals negatively but also to protect them positively. Of course, in this case, the substitution effects must be considered as usual.

The second one corresponds to a conception of "justice as compensation". Injustice is not in the natural or social characteristics of individuals but in social and economic institutions which treat individuals who have some natural or social characteristics in specific ways[2]. Institutions which fail to protect those who are vulnerable to discriminations, or institutions that leave without help those who are vulnerable to certain forms of natural selection may be recognized unjust through public reasoning and discussions. It requires equity between the extra needs that occur which are publicly recognized and statistically measured for a particular group.

The third corresponds to "justice as protection", which considers unjust a society that leaves an individual with less than basic capability and requires a form of outcome-equality that brings every individual to this reference capability.

In consequence, the diversity of disadvantage and forms of justice make the conception of "to secure *basic capability for all"* itself plural. For example, in the case of individuals who have suffered disadvantage as a result of having been victim of the atomic bomb, this has totally changed their life plan and goal, and they have decided to live as witnesses of this social disaster to prevent if from ever being repeated at any place or time. In their case of them, air tickets to fly to the New York, where will be held the "no more Hiroshima/Nagasaki congress" or a grant for publishing their memoirs as a pamphlet may be counted as a necessity for securing their basic capability. It suggests that under the common concept of "basic" or "capability", concrete conceptions must be re-defined regarding the special needs relative to the different disadvantage types as well as the general necessity for people to live well in society.

3. Capability Comparison: To What Extent and For What Purpose?

This short philosophical inquiry makes us realize that a complete comparison of disadvantages which is independent of the difference of types is not only difficult but actually inappropriate either interpersonally or intra-personally.

In this context, the original Arrovian social choice theory, because it is oriented towards completely general model-building with rigorous methodological individualism, and assumes full intrapersonal comparability (completeness) and full interpersonal non-comparability is, unsuitable[3].

However, we can and should make a partial comparison both intra-personally and interpersonally among individuals belonging to the same disadvantage type to the

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[2] As Rawls says, nature itself is not just or unjust (Rawls, 1971, p.)
[3] The weakest version of intrapersonal level comparability is the Arrovian one. However, the point is that it assumes completeness, and therefore too strong to follow.

extent that makes it possible to construct a reasonable social procedure which yields a partial social evaluation.[4]

To examine this possibility, let us borrow a slightly modified version of Sen's framework of the *social welfare functional,* which explicitly permits the variety of informational basis, easily to be scrutinized, and which permits individual welfare to be interpreted not as an individual evaluation but as an observable individual situation (Sen, 1970, p.126)[5]. Furthermore, it permits a social evaluation to be interpreted not as a function of an individual element but as a function of non-individualized information[6]. Let us define it formally.

Consider a society with the population $N=\{1,2,...,n\}$, where $2 \leq n \leq +\infty$. Let us denote a social state, a state of resource distribution for each individual under certain social policies by x, and the set of all possible social states by X, where $3 \leq X \leq +\infty$. Let us assume that *1,2, ...n* representing the members of the population include information of her personal features in relation to transforming resources into capability, and denote $C(x,i)$ for an appraisal of individual i's capability, where C denotes a *capability function*[7] which correlates a pair of resource distribution and personal features into an appraisal of capability[8]. Let the universal set of capability functions be K.

Based on the appraisal of individuals' capabilities, we assume, society makes an evaluation of alternative social policies. Let $x R y$ represent a binary relation between x and y, e.g., the evaluation of "x is at least as good as y", while let $x P(R) y$ represent a strict preference and $x I(R) y$ represent indifference. Through this paper, we assume R satisfies reflexivity and *acyclicity* but not necessarily *completeness*.

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[4] Intrapersonal comparison is also partial, because we cannot compare which is better for a first type disadvantaged taking a support for elderly or taking a general assistance, even if we can compare that a grant for publishing is better than both of these supports.

[5] This paper follows d'Aspremont, C. & L.Gevers (2002), which gives excellent survey on the social welfare functionals approach including Sen's original ideas, in adopting the term of *social evaluation* in stead of *social welfare*.

[6] Sen seems to be opposed not only to a purely subjective model but also to a purely individualistic model in the context of capability comparison. Refer to "there is, in none of these cases, no obvious analogue of the *inter-utility-functional* comparisons in the case of valuation of well-being"(Sen, 1985, p.57)

[7] *Capability function* is similar to but distinguished from the *capability correspondence* in Gotoh, Suzumura and Yoshihara (2005) in that the former is not assumed to be necessarily defined on the real number spaces.

[8] This paper assumes that an appraisal of capability function is undertaken through the process of making evaluation over social states not prior to this process. Furthermore, though we assume type-based evaluations, we do not necessarily assume that they are made in each type, but can be made publicly focusing on information relevant to each type.

[Social Welfare Functional]

A *social welfare functional* is a functional relation that specifies a *social evaluation R* over *X*, for any *capability function* $C \in K$ which is defined over $X \times N$ with generic image $R_C = F(C)$.

Next, let us consider the informational basis of a social welfare functional. According to Sen, the informational basis is expressed by an "invariance requirement" which states that two profiles of welfare function that belong to the same informational set should be treated in the same way by a social welfare functional.

If individual capability is fully observable and commensurable, we can assume full comparability on *capability function* and represent it using numerical indicators. Yet, as we saw before, we cannot and should not compare individuals' capabilities across different disadvantage types, while it is partially possible within a type. Thus, we impose the following *invariance requirement*.

First, identify three non-empty subgroups of the population belonging to the three types of disadvantage: T^1, T^2, $T^3 \subset N$, where individuals who belong to *N* but do not belong to either T^1, T^2 or T^3 are non-disadvantaged[9] and denote the set of capability appraisals for all social states X and individuals belonging to T^l ($l=1,2,3$) by $C(X, T^l)$. Then, we assume that for each $C(X, T^l)$ ($l=1,2,3$) the following partial "level comparability"[10] is satisfied.

(1) For each type, for each social state, individuals whose capabilities never dominate others' can be identified as the *least advantaged* of type *l*. Formally, the *least advantaged* of type *l* in *x* is defined as an individual $i \in T^l$ ($l=1,2,3$) such that for each T^l ($l=1,2,3$), for each social state $x \in X$, $C(x,j) \geq C(x,i)$ for all $j \in T^l$.

(2) T^l ($l=1,2,3$), for each social state $x \in X$, the capability of the *least advantaged* defined as (1) can be partially compared with the *basic capability* of type l[11].

However, for all domains which include different types of disadvantaged or

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[9] For simplicity, we assume further that $T^1 \bigcap T^2 = \phi$, $T^2 \bigcap T^3 = \phi$, $T^1 \bigcap T^3 = \phi$ and $N \backslash T^1 \bigcup T^2 \bigcup T^3 \neq \phi$.

[10] Formally, it is written as follows. For any $C, C^* \in K$, for any type T^l ($l=1,2,3$) and for any $x \in Z$ (where $Z \subset X$), for every increasing function φ defined on $C(Z, T^l)$:

$R_C = R_{C^*}$ if for any $(x,i) \in Z \times T^1$, $C(x,i) = \varphi(C^*(x,i))$.

[11] To guarantee the existence of a social evaluation we will later make a minimum assumption on the comparability, which seems natural in terms of possible social states.

non-disadvantaged, we assume interpersonal non-comparability.

4. From Type-Based Evaluation to Social Evaluation: Conditions on its Procedure

Based on this invariance requirement, this section explores the possibility of a social welfare functional which can specify a (quasi-ordering) social evaluation that ranks social states partially from the viewpoint of "to secure basic capability for all". First of all, we impose the Basic Capability condition defined as follows on *F*.

[Basic Capability Condition]

For any capability function *C* and any pair of social states *x, y,* society must say "*x* is better than *y*", if

(1) for social state *x* the *least advantaged* of a type *l* is at least as good as *basic capability* in type *l* and for social state *y* the *least advantaged* of type *l* is worse off or cannot be compared to *basic capability*, or

(2) for social state *y* the *least advantaged* of type l is worse than *basic capability* and for social state *x* the *least advantaged* of type *l* is at least as good as *basic capability* or cannot be compared with *basic capability*.

Formally, it is written as follows. Let $L^l(x), L^l(y) \subset T^l (l = 1,2,3)$ be the least advantaged of type *l* in each *x, y* and the basic capability of type *l* be BC^l. Then, for any $x, y \in X$, for any $l \in \{1,2,3\}$ and for any $C \in K,$ $x\, P(R_C)\, y$ if (1) C

$(x, L^l(x)) \geq BC^l$ and { $BC^l > C(y, L^l(y))$ or "$C(y, L^l(y))$ and BC^l is incomparable"}, or

(2) "$C(x, L^l(x))$ and BC^l is incomparable" and $BC^l > C(y, L^l(y))$.

Note, first, that by definition, since a social state that secures the *basic capability* for the least advantaged in a type also secures the basic capability for others who belong to the same type and individuals who belong to neither type are non-disadvantaged, we can say that a social state where every least disadvantaged is at least as good as the basic capability satisfies a social goal "to secure the basic capability for all"[12].

Note, second, that according to the three types of disadvantage and the corresponding ideas of *basic capability* we have three types of evaluation which satisfy the "Basic Capability condition" with three types of the *least advantaged.* Let us call

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[12] There might be a case of a non-disadvantaged individual become disadvantaged of type 3 after for example paying tax. In this case, after including him into type 3, F will be applied again. I thank Paul Dumouchel who pointed out this problem to me.

each *type l-based evaluation* (l =1,2,3) and denote it by R^l. We assume that R^l satisfies *reflexivity* and *acyclicity* and the Basic Capability condition but not completeness. It is a local evaluation in the sense that for any $C \in K$, R_C^l is specified based on the reduced space, $C(X, L(T^l))$, where $L(T^l)$ denotes the set of the least advantaged of type l in each and every $x \in X$, e.g., $L(T^l) = \{ L^l(x) \mid x \in X \}$.

Then, the question arises: Can we make a global quasi-ordering evaluation combining these three *type l-based evaluations*, resolving conflicts, not only crucial but also non-crucial disagreements, between different types of least advantaged? The first step to resolve this problem is to restrict the power of *type l-based evaluation* to avoid disagreements which are not crucial to type *l*.

[Refrain Condition]

For any pair of social states which *type l* (l =1,2,3)*-based evaluation* cannot compare based on the Basic Capability Condition, it should refrain from making a judgment. Formally, for any $l \in \{1,2,3\}$, for any $C \in K$, for any pair $(x, y) \in X \times X$ such that (1) $C(x, L^l(x)) \geq BC^l$ and $C(y, L^l(y)) \geq BC^l$, or (2) $BC^l >C (x, L^l(x))$ and $BC^l >C$

$(y, L^l(y))$, (3) neither $(x, L^l(x)) \geq BC^l$, $(y, L^l(y)) \geq BC^l$, $C (x, L^l(x)) > BC^l$ nor C

$(y, L^l(y)) > BC^l$, $x R_C^l y$ nor $y R_C^l x$.

The Refrain Condition requires each *type l-based evaluation* to keep silent as long as two social states are indifferent or incomparable with respect to the basic capability of its type. It prohibits a *type l-based evaluation* from exerting its influence on matters which are irrelevant to it but might be relevant to other types. The judgment of relevance is nothing more than 'satisfying basic capability or not'.

Based on this condition, let us introduce another condition which requires F to make a global quasi-ordering evaluation combining three *type l-based evaluations.*

[Non-Contradiction Condition]

For any $C \in K$, for any $l \in \{1,2,3\}$, for any $x, y \in X$, $x P(R_c) y$ if there is R_C^l such that x

$P(R_C^l) y$ and there is no R_C^l such that $y P(R_C^l) x$.

What this condition says that if there is a social state *x*, where the least advantaged of every type is at least as good as (or worse than) the *basic capability* and a social state *y*, where at least one of the three types' least advantaged capabilities is worse (or better) than basic capability or incompatible with it, since there is a type *l*-based evaluation which says "*x* is better than *y*" and there is no *type l-based evaluation* who says "*y* is better (or worse) than *x*, society must say "*x* is better than *y* (or *y* is better than *x*)*".

This requirement is a stronger version of the *intersection approach* proposed by Sen[13], in the sense that for any social states x and y, if any two of three types evaluation contradicts each other, the global social evaluation reflect neither of them, yet if there is no contradiction, the global social evaluation reflects such an evaluation even if it is not supported unanimously. It can easily be shown that if this condition is satisfied, intersection approach is also satisfied but not vice verse.

Based on this social evaluation, for any capability function $C \in K$, we can have a subset of possible social states *X*, elements of which are not dominated by any other element of *X* with respect to the social function R_c. Let us call this subset the Basic Capability Maximal. Formally, $M(X, R_c) = \{y \mid y \in X \ \& \ for \ no \ x \in X : xP(R_c)y\}$. We can have a result that the Basic Capability Maximal is non empty given the minimum hypothesis that the set of possible social states X is finite and moreover, there is a social state *y* where the all types of least advantaged are worse than basic capability and another social state *x* where at least a type is not.

[Theorem 1]

For any $C \in K$, if there are two social states $x, y \in X$ such that $BC^l > C(y, L^l(y))$ for every type $l \in \{1,2,3\}$, while $C(x, L^l(x)) \geq BC^l$ or "$C(x, L^l(x))$ and BC^l is incomparable" for a type *l*, $M(X, R_C) \neq \phi$.

Next, let us check if *a social welfare functional F* which satisfies the Basic

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[13] The Intersection Approach requires including the intersection of the three *type-based social evaluations*. Formally, as for any $C \in K$, $R_C = \bigcap R_C^l$ (l=1,2,3).

Capability condition, the Refrain condition and the Non-contradiction condition also satisfies the Weak Pareto condition, which is defined with a modification as follows.

[Capability-based Weak Pareto Condition]

For any $C \in K$, for any $x, y \in X$, $\quad xP(R_C)y \quad$ if there is no $i \in N$ such that $C(y, i) \geq C(x, i)$.

The Capability-based Weak Pareto Condition evaluates alternative social states regarding all individuals' capabilities without focusing on the least disadvantaged capability. Note that it can be defined without *completeness*. We can easily ascertain that if the utility-based Weak Pareto condition is redefined as this capability-based one, it does not conflict with the Basic Capability condition. Let us illustrate the essence of the proof.

(1) If in social state x the least advantaged of every type is at least as good as the basic capability but in social state y not all are, and all individuals are better in x than in y, then both the basic capability condition and the Pareto condition entail that "x is better than y".

(2) If in social state x the least advantaged of every type is at least as good as the basic capability but in social state y not all are, while at least one individual is worse in x than in y, then the basic capability condition says "x is better than y", while the Pareto condition remains silent.

(3) If there is no individual whose capability becomes better in x than in z, the Pareto condition says "z is better than x", while the Basic Capability condition keeps silent if in both z and x the least advantaged of every type is already at least as good as the basic capability or if in both z and x the least advantaged of every type is still worse than the basic capability.

Thus, we can impose the Weak Pareto Condition without contradicting the Basic Capability condition and succeed in extending the partial ordering which satisfies the Basic Capability condition, the Refrain condition and the Non-Contradiction condition[14]. The appropriateness of imposing the Weak Pareto condition only depends on the normative judgments concerning the condition itself.

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[14] In above case (3), the Refrain condition requires R_C^l to keep silent and the Non-Contradiction condition gives no power to type-based evaluation. Then, the Pareto condition can be influential and bring an evaluation that "z is better than x".

[Theorem2]
There is a social welfare functional *F* which satisfies the Basic Capability condition, the Refrain condition and the Non-contradiction condition, as well as the Weak Pareto condition.

Finally, let us see whether a social welfare functional *F* which satisfies the Basic Capability condition, the Refrain condition and the Non-contradiction condition, as well as the Weak Pareto condition also satisfies other conditions familiar to Arrovian social choice theory, anonymity, neutrality and the unrestricted domain.

First, although the basic capability condition gives to the least disadvantaged of each type a power of dictatorship, given that who is identified as the least advantaged in a type is independent from his name, we can say that the *anonymity* condition is satisfied. Second, as long as the situation of the least advantaged in type *l*, who she is, is evaluated at least as equivalent to basic capability in relation to two different social policies, both social policies similarly reflect to the *type l-based evaluation*, and then they are reflected similarly in the social evaluation, thus the *neutrality* condition is also satisfied.

Third, in principle, whatever individuals' capabilities profile, the "invariance requirement" treats them in a certain way with regard to comparability---partial level comparability inside a type, non-comparability outside a type---, and the same procedures follow. In this sense, unrestricted domain is also satisfied. Let us denote a social welfare functional which satisfies all of these conditions by F^*.

5. Concluding Remarks

Logically, it is impossible that every individual dominates someone, so we can easily verify the existence of the least advantaged in each type, which is defined section 3. Moreover, if there is a social state which is never evaluated *worse* than any other social state, ---this should be understood as a weak requirement in the sense that even if this state is not compatible with basic capability, as long as there is no social state which is at least as good as it, it is satisfied---, we can specify a non-empty subset of the possible social states ("the Basic Capability Maximal") without the assumption of *completeness*.

As can easily be seen the ability of a social evaluation specified by F^* is inevitably restricted by the extent of capability comparison with basic capability. Yet this kind of restriction is not to be confused with a restriction founded on the incommensurability of different conceptions of justice. The former is a technical

problem of research, while the latter a question of intrinsic impossibility. I would like to conclude this essay by stressing this difference.

Reference

Arrow, K. J. (1951/1963): *Social Choice and Individual Values*, 2nd ed., New York: Wiley.

d'Aspremont, C. & L.Gevers (2002) "Social Welfare Functionals and Interpersonal Comparability," Arrow. K., A. K. Sen and Suzumura K. (eds.) *Handbook of Social Choice and Welfare*, vol.1, Amsterdam: North-Holland, 459-541

Gotoh, Suzumura, Yoshihara (2005): Extended Social Ordering Functions for Rationalizing Fair Games a la Rawls, *International Journal of Economic Theory*, Vol.1, 21-42.

Rawls, J. (1971): *A Theory of Justice*, Cambridge, Mass.: Harvard University Press.

Sen, K. A.(1970): *Collective Choice and Social Welfare*, San Francisco: Holden-Day.

Sen, A. K. (1973): *On Economic Inequality*, London: Oxford University Press.

Sen, A. K. (1977):"On Weights and Measures: Informational Constraints in Social Welfare Analysis", *Econometrica* 45

Sen, A. K.(1997):On Economic Inequality, expanded edition with a substantial annex by James E. Foster and Amartya K Sen, Oxford: Clarendon Press.

Sen, A. K. (1985a): *Commodities and Capabilities*, Amsterdam: North-Holland.

Sen, A. K.(2002): *Rationality and Freedom*, Cambridge: Harvard University Press.