Oxford Poverty & Human Development Initiative (OPHI)
Oxford Department of International Development
Queen Elizabeth House (QEH), University of Oxford

OPHI WORKING PAPER NO. 126

The Role of Inequality in Poverty Measurement

Sabina Alkire* and James Foster**

July 2019

Abstract
The adjusted headcount ratio, or MPI, is widely used by countries and international organizations to track multidimensional poverty and coordinate policy. Several characteristics have encouraged its rapid diffusion: applicability to ordinal data, ease of communication, a practical identification of the poor based on multiple deprivations, and a dimensional breakdown that informs and coordinates policy. Sen (1976) and others have argued that poverty should also be sensitive to *inequality* among the poor. This paper provides a new axiom that embodies this perspective in the multidimensional context and defines an *M*-gamma family containing a range of measures satisfying the axiom. Like the FGT or *P*-alpha class of monetary measures, it has three main members: the headcount ratio to evaluate the prevalence of poverty, the adjusted headcount ratio to account for its intensity, and the "squared count" measure that reflects severity and inequality among the poor. We note that any inequality sensitive measure must violate the dimensional breakdown axiom and investigate Shapley decomposition methods as an alternative. Unfortunately, these methods can yield counterintuitive result; however, the squared count measure avoids this critique and its Shapley breakdown reduces to an easy to compute formula that supplements the traditional breakdown for the MPI with information relevant to inequality among the poor. An example from Cameroon illustrates our method of using *M*-gamma measures in tandem to evaluate multidimensional poverty while accounting for inequality and dimensional contributions.

Keywords: poverty measurement, multidimensional poverty, inequality, Shapley decomposition, transfer axiom, dimensional breakdown, FGT measures, decomposability, ordinal variables.

JEL classification: I3, I32, D63, O1, H1

* Oxford Poverty and Human Development Initiative (OPHI), University of Oxford. Email: sabina.alkire@qeh.ox.ac.uk.
** George Washington University and Oxford Poverty and Human Development Initiative (OPHI), University of Oxford.

This study has been prepared within the OPHI theme on multidimensional measurement.
ISSN 2040-8188 ISBN 978-1-91229-117-5

Acknowledgements
We are grateful to Suman Seth, Bryan Stuart and Nicolai Suppa for helpful comments, and to participants in the International Economic Association World Congress in Mexico City, the Royal Economic Society Annual Conference in Sussex, and others in workshops and seminars in CORE Louvain la Neuve, GWU, Oxford University, Princeton University, University of Manchester, IIT Kanpur, and UNU-WIDER for their reflections.

Funding information
We acknowledge financial support from Magdalen College, Oxford, and from Mansfield College, Oxford, each of which provided Prof. Foster with a visiting fellowship, and to the University of Manchester, which provided support to Prof. Foster as part of the Hallsworth Visiting Professorship. We also are grateful for support from ESRC-DFID grants ES/I032827/1 and ES/N01457X/1.

Citation: Alkire, S. and Foster, J. (2019). 'The role of inequality in poverty measurement', OPHI Working Paper 126, University of Oxford.

The Oxford Poverty and Human Development Initiative (OPHI) is a research centre within the Oxford Department of International Development, Queen Elizabeth House, at the University of Oxford. Led by Sabina Alkire, OPHI aspires to build and advance a more systematic methodological and economic framework for reducing multidimensional poverty, grounded in people's experiences and values.

Oxford Poverty and Human Development Initiative (OPHI)
Oxford Department of International Development
Queen Elizabeth House (QEH), University of Oxford
3 Mansfield Road, Oxford OX1 3TB, UK
Tel. +44 (0)1865 271915 Fax +44 (0)1865 281801
ophi@qeh.ox.ac.uk http://www.ophi.org.uk

1. Introduction

Multidimensional poverty measures are increasingly being applied in academic studies and policy analyses. The global Multidimensional Poverty Index (MPI) of the United Nations Development Programme, is now entering its tenth year of monitoring acute poverty in over 100 countries in developing regions and was recently revised and relaunched.[1] The Atkinson Commission Report *Monitoring Global Poverty* (World Bank 2017) has called upon the World Bank to construct a multidimensional poverty measure for monitoring the non-monetary dimensions of global poverty. Dozens of official, national statistics on multidimensional poverty are now being produced by countries in line with Goal 1.2 of the Sustainable Development Goals.[2] The measures have been adapted to evaluate women's empowerment, decent work, gross national happiness and other multidimensional concepts.[3]

The methodology underlying these examples – the adjusted headcount ratio M_0 or MPI approach – has several characteristics that greatly facilitate its use in policy. First, it can be meaningfully applied to the types of data typically encountered when measuring multidimensional poverty, including ordinal variables and groups of indicators lacking a common measuring rod. Previous multidimensional measures require variables that have the cardinality and comparability properties of income, and hence are far less applicable. They violate an Ordinality axiom (defined below) embodying these data-implied constraints, while M_0 satisfies it.

Second, its novel 'dual cutoff' method of identifying the poor preserves the utility of the traditional headcount ratio H and augments it with a second component A (the average breadth of deprivation) to obtain $M_0 = HA$. In contrast, the 'union' approach (anyone deprived in a single indicator is poor) and the

[1] See UNDP and OPHI (2019a), Alkire and Santos (2014), Alkire and Jahan (2018).

[2] For example Mexico (CONEVAL 2009), Colombia (CONPES 2012), Bhutan (RGOB-NSB 2012), Chile (Government of Chile 2015) and others as listed in UNDP and OPHI (2019b) or on www.mppn.org.

[3] See respectively Alkire Meinzen-Dick *et al.*. (2013), IDB (2017), Ura et al (2012), Cameron et al (2019).

'intersection' approach (a person must be deprived in all dimensions to be poor) render the headcount ratio H useless for prioritizing and targeting the poor, especially when there are many indicators.

Third, M_0 can be broken down by population and dimension to obtain subindices that facilitate policy analysis. The Subgroup Decomposability axiom allows the assessment of subgroup contributions to overall poverty, thus facilitating regional analysis and targeting; the Dimensional Breakdown axiom (formalized below) allows the assessment of dimensional contributions to overall poverty, thus facilitating policy analysis and coordination.[4] The result is an information platform comprising the headline poverty index M_0 and a coordinated collection of subindices that reveal the contributions of populations and dimensions to overall poverty.

Concern regarding inequality is now widespread.[5] A natural question to ask is whether inequality can be incorporated into this form of poverty measurement and, if so, at what cost. The desire for measures that reflect inequality among the poor – powerfully voiced by Sen (1976) – has been reiterated in the multidimensional context with increasing frequency and may seem implicit in the Sustainable Development Goals' priority to 'leave no one behind' UNGA (2015). But what sort of inequality is not being measured with M_0?

Inequality's role in unidimensional poverty measurement has traditionally been represented by a transfer principle requiring poverty to fall as a result of a progressive transfer among the poor. This in turn has led to an array of distribution-sensitive unidimensional poverty measures that satisfy this property.[6] In the multidimensional setting, there are two competing notions of inequality, leading to two distinct ways of

4 Subgroup Decomposability (or Decomposability) is defined for unidimensional measures in Foster, Greer, and Thorbecke (1984) and for multidimensional measures in Chakravarty, Mukherjee, and Renade (1998), Tsui (2002), and Bourguignon and Chakravarty (2003). Dimensional Breakdown and Ordinality were intuitively introduced in Atkinson (2003) and outlined in Alkire and Foster (2011a); they are both defined formally below.

5 See for example Piketty (2014), Atkinson (2015).

6 See Sen (1976), Clark, Hemming, and Ulph (1981), and Foster, Greer, and Thorbecke (1984) among others. It should be noted that a property properly depends on both the identification and aggregation steps. In unidimensional measurement, identification usually has a standard format, so we often say that the poverty measure satisfies a given property without explicitly specifying the identification method. Multidimensional measurement requires a more precise specification of identification, which when set, allows the use of a similar shorthand.

conceiving of inequality in multidimensional poverty. The first, linked most closely to Kolm (1977), generalizes the notion of a progressive transfer (or more broadly a Lorenz comparison) to the multidimensional setting by applying the same bistochastic matrix to every variable.[7] This results in a coordinated "smoothing" of the distributions that preserves their means. The associated transfer principle for poverty measures requires the level of poverty to fall, or at least not to rise, when such a smoothing is applied among the poor.

The second form of multidimensional inequality is linked to the work of Atkinson and Bourguignon (1982) and relies on patterns of achievements across dimensions. Imagine a case where one person initially has more of everything than another person and the two persons switch achievements in a single dimension. This can be interpreted as a progressive transfer that preserves the marginal distribution of each dimensional variable and lowers inequality by relaxing the positive association across variables. The resulting transfer principle specifies conditions under which this form of progressive transfer among the poor should lower poverty, or at least not raise it.

Many multidimensional poverty methodologies satisfy one or both of these transfer principles.[8] In particular, Alkire and Foster (2011a) show that the adjusted FGT measure M_{α}, when used with a dual cutoff method of identification, satisfies the first type of transfer principle for $\alpha \geq 1$ and the second type for $\alpha \geq 0$. Note, though, that transfer properties in the multidimensional poverty literature have usually been 'weak' in that they allow poverty to remain unchanged in the face of a progressive transfer.[9] It is possible to define strict versions that require poverty to fall as a result of a suitably strict progressive

7 A bistochastic matrix is a weighted average of different permutation matrices (each of which switches achievements among people). When applied to an income distribution it ensures that each person's transformed income is a weighted average of all the original incomes. See Foster and Sen (1997) or Alkire et al. (2015).

8 See, for example, Chakravarty, Mukherjee, and Renade (1998), Tsui (2002), Bourguignon and Chakravarty (2003), Chakravarty and D'Ambrosio (2006), Maasoumi and Lugo (2008), Alkire and Foster (2011a), Bossert, Chakravarty, and D'Ambrosio (2013), Rippin (2013, 2017), Silber and Yalonetzky (2013), Aaberge and Brandolini (2015), Dhongde et al (2016), Datt (2019), and Bérenger (2017).

9 See, for example, the related axioms of Tsui (2002), Atkinson (2003), Bourguignon and Chakravarty (2003), Chakravarty and D'Ambrosio (2006), Chakravarty (2009), Alkire and Foster (2011a), and Rippin (2017), and in the context of inequality, Gajdos and Weymark (2005). For a clarifying discussion theoretically, see Alkire et al. (2015), and empirically, Bérenger (2017). Dhongde et al (2016) formulates a strict version that relies on binary data for each variable. Datt (2019) formulates a strict version that does not apply when ordinal variables have more than one deprived level. See the discussion in section 3.1.

transfer. Indeed, one could show that M_α satisfies a strict version of the first transfer principle for $\alpha > 1$, while for $\alpha > 0$ it is easily transformed into a *new* measure satisfying a strict version of the second.[10] However, each of these distribution-sensitive measures violates Ordinality, thus severely limiting their applicability. This leads to the following natural questions: Is it possible to formulate a strict version of distribution sensitivity – by which greater inequality among the poor strictly raises poverty – that is applicable to multidimensional poverty methodologies satisfying Ordinality? And can we find poverty measures satisfying this requirement as well as the other properties that have proved to be so useful in practice?

This paper considers the possibility of constructing multidimensional poverty measures satisfying key properties and a strict form of distribution sensitivity called *Dimensional Transfer*. The axiom uses an Atkinson-Bourguignon multidimensional transfer between the poor, but with the additional proviso that in one of the switched dimensions the poorer person is deprived while the other poor person is not – so that a deprivation is effectively transferred in the process. Given a dual cutoff approach to identification, we generalize the adjusted headcount ratio M_0 to a parametric class M_0^γ – called here the *M*-gamma class – that has a subclass satisfying Dimensional Transfer. The *M*-gamma class contains three main measures: the headcount ratio $M_0^0 = H$, which provides information on the prevalence of poverty but violates the key axioms of Dimensional Breakdown and Dimensional Monotonicity (defined below); the adjusted headcount ratio $M_0^1 = M_0$, which satisfies these axioms but just violates Dimensional Transfer since it is neutral with respect to its defining transfer; and a *squared count* measure M_0^2, which satisfies Dimensional Transfer but not Dimensional Breakdown.[11] Each of these three *M*-gamma measures focuses on a different aspect of poverty in a manner reminiscent of the traditional *P*-alpha or FGT monetary measures.

10 This argument is outlined in Alkire and Foster (2011a, p. 485) where the individual poverty function $M_\alpha(y_i; z)$ is replaced with $[M_\alpha(y_i; z)]^\gamma$ for some $\gamma > 0$ and averaged across the population.

11 Chakravarty and D'Ambrosio (2006) present measures of social exclusion that correspond to M_0^2 and other members of this class in the special case of union identification.

Unfortunately, no measure in the *M*-gamma class simultaneously satisfies Dimensional Transfer and Dimensional Breakdown, and we confirm that this 'impossibility' extends to *all* multidimensional poverty methodologies. Consequently, the property of Dimensional Transfer carries with it a significant opportunity cost: the loss of Dimensional Breakdown. One might avoid this trade off by using multiple measures, an approach which has precedence in the way the *P*-alpha measures typically are used. For example, M_0^1 could be used to assess dimensional contributions, while some M_0^γ for $\gamma > 1$ is used to provide information on inequality in poverty. But without a meaningful breakdown for the latter measures, the role of inequality would be limited to aggregate comparisons of poverty.

Datt (2019) has suggested employing the Shapley decomposition methods of Shorrocks (2013) to illuminate the contribution of dimensions to overall poverty. The Shapley value's theoretical meaning and axiomatic characterization have contributed to its wide adoption in many distributional contexts.[12] Yet Shapley values are computationally complex and far less intuitive than standard decompositions. Could Shapley methods provide understandable, policy relevant information about dimensional contributions to overall poverty? To address this question, we apply them to the *M*-gamma measures for a specific example having hierarchical variables, the nested weighting structure, and the identification method of the global MPI, and find that they can lead to counterintuitive results in this canonical case. We conclude that the Shapley value approach does not offer a universal solution, nor a genuine alternative to M_0's dimensional breakdown.

Exactly one distribution-sensitive measure in the *M*-gamma class manages to escape this critique: the squared count measure M_0^2. We explain why and use this insight to derive a formula for its Shapley breakdown that is unexpectedly tractable and intuitive. The formula builds upon the standard breakdown expression for M_0 but with additional 'censored intensity' terms to reflect the average deprivation score among those who are both poor and deprived in a given dimension. When all censored intensity levels are

12 See Ravallion and Huippi (1991) and Shorrocks (2013).

the same, the relative contributions revert to the original M_0 levels; a relatively higher censored intensity will tend to increase a dimension's relative contribution. While the squared count measure violates Dimensional Breakdown (as do all the measures satisfying Dimensional Transfer), its Shapley formula is intuitive and explicitly accounts for inequality in its poverty breakdown. An empirical example from Cameroon illustrates how using the three main *M*-gamma members in tandem can inform poverty analysis, with the headcount ratio M_0^0 providing information on the incidence of poverty, the adjusted headcount ratio M_0^1 adding intensity to the mix and providing the core breakdown formula, and the squared count M_0^2 adding inequality and a Shapley breakdown that deviates from the core formula in informative ways.

The definitions and notation used in the paper are given in Section 2, while Section 3 presents formal definitions for three key axioms of the paper including one that proposes a role for inequality in poverty measurement. Section 4 presents the *M*-gamma class and establishes the impossibility result. Section 5 explores Shapley decomposition methods and derives the breakdown formula for the squared count measure. Section 6 illustrates these new techniques, while Section 7 concludes.

2. Notation and Definitions

We begin with the notation and definitions needed for the subsequent analysis. Let $|x|$ denote the sum of all elements in any given vector x of real numbers and let $\mu(x)$ signify the mean of x, or $|x|$ divided by the total number of elements in x. Where x and x' are vectors having the same number of entries, let $x > x'$ denote the case where x vector dominates x' (so that each coordinate of x is as large as the respective coordinate of x', while $x \neq x'$).

In what follows, we consider allocations of dimensional achievements across populations. The number of dimensions is assumed to be a fixed integer $d \geq 2$, where the typical dimension is $j = 1,2, \ldots, d$. The population size is any integer $n \geq 1$, where n is permitted to range across the positive integers, and $i = 1,2, \ldots, n$ denotes the typical person. Let $y = [y_{ij}]$ be an $n \times d$ matrix of achievements belonging to the domain $Y = \{y \epsilon R_{+}^{nd}: n \geq 1\}$ of nonnegative real matrices.[13] The typical entry in y is $y_{ij} \geq 0$. We use y_i to signify the row vector of individual *i*'s achievements, while $y_{.j}$ is the column vector that provides the distribution of dimension *j*'s achievements across people. A *deprivation cutoff* $z_j > 0$ for dimension j is compared to achievement level y_{ij} to determine when person *i* is deprived in *j*, namely, when $y_{ij} < z_j$. The row vector of dimension-specific deprivation cutoffs is denoted by z.

Poverty measurement has an identification step and an aggregation step. An *identification function* $\rho: R_{+}^{d} \times R_{++}^{d} \rightarrow \{0,1\}$ is used to identify whether person *i* is poor, where $\rho(y_i; z)$ takes the value 1 if person *i* is poor, and the value 0 otherwise, and is weakly decreasing in each y_{ij} (lowering achievements does not bring a poor person out of poverty).[14] The *poverty status vector* associated with y is the column vector r whose *i*th entry is $\rho(y_i; z)$. An *index* or *measure of multidimensional poverty* $M: Y \times R_{++}^{d} \rightarrow R_{+}$ aggregates the data into an overall level $M(y;\ z)$ of poverty in y given z and the identification function ρ. The resulting *methodology* for measuring multidimensional poverty is given by $\mathcal{M} = (\rho, M)$. For any given dimension *j*, let $Y_{.j}$ be the set of all column vectors $y_{.j}$ of *j*th dimensional achievements. It will sometimes be useful to focus on y and $y_{.j}$ that are consistent with a given poverty status vector r. Let Y_r denote the set of all y having r as its poverty status vector and let Y_{rj} denote the set of all $y_{.j}$ that are derived from an achievement matrix y found in Y_r.

Multidimensional poverty is identified and measured with the help of a vector $w = (w_1, \ldots, w_d)$ of *dimensional weights* satisfying $|w| = 1$ and $w_j > 0$ for all *j*, and a *poverty cutoff* k satisfying $0 < k \leq 1$. For any person *i*, the *deprivation score* (or *weighted count*) c_i is the sum of weights w_j across all dimensions in which *i* is deprived.[15] The *dual cutoff identification function* ρ_k is defined by $\rho_k(y_i; z) = 1$ whenever $c_i \geq k$, and

13 We follow Alkire and Foster (2011a) in assuming that achievements are represented as nonnegative real numbers, while deprivation cutoffs are strictly positive. Other assumptions are clearly possible but are not explicitly covered here.

14 While not included in our previous work, this requirement would seem to be a reasonable restriction on ρ and the orientation of achievements.

15 This notation expresses weights, poverty cutoff, and counts relative to the magnitude of d. An alternative but equivalent notation $w' = wd$, $k' = kd$ and $c' = cd$ was used in Alkire and Foster (2011a).

$\rho_k(y_i; z) = 0$ whenever $c_i < k$. In other words, ρ_k identifies person *i* as poor when the deprivation score c_i is at least k; otherwise, *i* is not poor. At one extreme, when $k = 1$, the function ρ_k becomes *intersection* identification, in which a person must be deprived in all dimensions to be poor. When $0 < k \leq \min_j w_j$, it becomes *union* identification, in which a person need be deprived in only one dimension to be identified as poor. Thus, while our emphasis is on the intermediate cases, ρ_k includes the two limiting identification methods as well.

It is helpful to construct a matrix of deprivations from the matrix of achievements, making use of the deprivation cutoffs. Let $g^0 = [g_{ij}^0]$ denote the *deprivation matrix* whose typical element is given by $g_{ij}^0 = 1$ when $y_{ij} < z$, and $g_{ij}^0 = 0$ otherwise. In words, when person *i* is deprived in the *j*th dimension, the associated entry g_{ij}^0 is 1; otherwise it is 0. The column vector $g_{\cdot j}^0$ of g^0 indicates all the persons deprived in dimension *j*; the row vector $g_i^0 = (g_{i1}^0, \dots, g_{id}^0)$ lists the deprivations of person *i* and is called *i*'s *deprivation status vector*. The deprivation score of person *i* can then be written as $c_i = w_1 g_{i1}^0 + \cdots + w_d g_{id}^0$ which, in turn, generates the column vector c that contains the distribution of deprivation scores across all persons, whether poor or nonpoor. The *censored deprivation matrix* $g^0(k)$, defined by $g_{ij}^0(k) = g_{ij}^0\ \rho_k(y_i; z)$ for all *i* and *j*, uses the identification function to replace the data of the nonpoor with zeros.[16] The censored versions of the associated vectors $g_{\cdot j}^0(k), g_i^0(k)$, and $c(k)$ are then analogously defined. For example, the vector of censored deprivation scores $c(k)$ has $c_i(k) = c_i\ \rho_k(y_i; z)$ as its *i*th entry.

The adjusted headcount ratio $M_0 = M_0(y; z)$ of Alkire and Foster (2011a) is defined as

$$M_0 = \mu\big(c(k)\big) = HA \tag{1}$$

where $\mu\big(c(k)\big) = |c(k)|/n$ is the mean censored deprivation score across the entire population, $A = |c(k)|/q$ is the average intensity (or deprivation score) among the poor, and $H = q/n$ is the headcount

[16] Note that in the case of union identification, the censored and original versions are identical.

ratio, where $q = \sum_{i=1}^{n} \rho_k(y_i; z)$ is the number of the poor. Notice that (1) sums horizontally across the entries of $g^0(k)$, and then vertically, yielding a decomposition across persons. Reversing the order yields the *dimensional breakdown formula* for M_0 as follows

$$M_0 = w_1 H_1 + \cdots + w_d H_d \quad (2)$$

where $H_j = \mu(g_{\cdot j}^0(k))$ is the *censored headcount ratio*, or the percentage of the population that is both poor and deprived in dimension *j*. In words, (2) expresses the adjusted headcount ratio as the weighted average of the censored headcount ratios. The associated *breakdown vector* $b = (w_1 H_1, \ldots, w_d H_d)$ lists the dimensional contributions to poverty, while $b_j / |b| = w_j H_j / M_0$ are the relative contributions.

Other measures defined in Alkire and Foster (2011a) require each variable *j* to be cardinally meaningful in order to gauge the depth of deprivation using the normalized gap $(z_j - y_{ij})/z_j$. The censored deprivation matrix $g^0(k)$ is replaced with matrix $g^\alpha(k)$ having as its typical entry $g_{ij}^\alpha(k) = g_{ij}^0(k)((z_j - y_{ij})/z_j)^\alpha$ for a given $\alpha > 0$. Adding up the entries of $g^\alpha(k)$ and dividing by nd generates the family M_α for $\alpha > 0$, which contains a measure M_1 sensitive to the depth of deprivations, another measure M_2 which emphasizing the largest gaps and is sensitive to the Kolm type of multidimensional inequality in the distribution of achievements. However, since multidimensional poverty analyses typically entail noncardinal variables, these measures like so many others requiring cardinal variables are effectively ruled out. Instead, we focus here on measures like M_0 that satisfy Ordinality and can be implemented with real world data.

3. Properties

The properties of a poverty measure specify the patterns in the underlying data the measure should ignore, the aspects it should highlight, and the kinds of policy questions it can be used to answer. This section presents properties for multidimensional poverty measures, focusing first on the traditional properties satisfied by M_0 or, more precisely, by the methodology (ρ_k, M_0) since properties are, in fact, joint restrictions on identification and aggregation. Only brief descriptions of these properties are provided here; precise definitions and verifications can be found in Alkire and Foster (2011a). Two additional properties of M_0 that were previously discussed, but have not yet received a rigorous treatment, will be defined: Ordinality, which ensures that the measure can be meaningfully applied to ordinal data, and Dimensional Breakdown, which allows poverty to be broken down by dimension after identification. We conclude with a new property – Dimensional Transfer – that ensures that poverty is sensitive to a form of Atkinson-Bourguignon inequality among the poor.

The properties of multidimensional poverty measures can be divided into the categories of invariance, subgroup, and dominance properties. Invariance properties isolate aspects of the data that should *not* be measured. They include *Symmetry* (invariance to permutations of achievement vectors across people), *Replication Invariance* (invariance to replications of achievement vectors across people), *Deprivation Focus* (invariance to an increment in a nondeprived achievement), and *Poverty Focus* (invariance to an increment in an achievement of a nonpoor person).

Next are the subgroup properties that connect overall poverty to levels obtained from data broken down by population subgroup or by dimension. Two of the key properties here are *Subgroup Consistency* (if poverty rises in a population subgroup and stays constant in the remaining population, while subgroup population sizes are unchanged, then overall poverty must rise) and *Subgroup Decomposability* (overall poverty is a population-weighted sum of the poverty levels in population subgroups).

Dominance properties focus on aspects of the data that should be measured and ensure that poverty levels respond appropriately to changes in achievements. They include *Weak Monotonicity* (an increment in a single achievement cannot increase poverty), *Dimensional Monotonicity* (a dimensional decrement among the poor – which lowers an achievement of a poor person from a nondeprived to a deprived level – must increase overall poverty), and *Weak Rearrangement* (a progressive transfer among the poor arising from an association-decreasing rearrangement cannot increase poverty).[17]

This paper takes all four invariance properties, the two subgroup properties, one dominance property (Weak Monotonicity) as its set of *Basic Axioms* for multidimensional poverty measures. We now present the three additional properties – an invariance property, a subgroup property, and a dominance property – that are the special focus of this paper.

3.1 Ordinality

The basic data used to construct the achievement matrix are typically derived from circumstances and conditions that are easy to describe and understand but have no natural metric in which to be measured. The numbers assigned to the various achievement levels (and deprivation cutoffs) in this domain are in a real sense simply placeholders designed to convey information about underlying conditions and, in particular, whether they are conditions of deprivation.[18] Note that this general line of argument may be true even for the cases where the variable has an 'in-built' representation such as income or years of schooling, since the cardinalization that comes with the variable may not be the right one for reckoning gains and losses in a given context.[19]

We say that $(y'; z')$ is obtained from $(y; z)$ as an *equivalent representation* if there exist increasing functions $f_j: R_+ \to R_+$ for $j = 1,\ldots,d$ such that $y'_{ij} = f_j(y_{ij})$ and $z'_j = f_j(z_j)$ for all $i = 1,\ldots,n$ and every $j = 1,\ldots,d$. In other words, an equivalent representation assigns a different set of numbers to the same underlying basic data while preserving the original order. The methodology (ρ_k, M_0) satisfies the following invariance

17 See Alkire and Foster (2011a) for more precise definitions of these properties. Another dominance property of Weak Transfer, which requires the application of the same bistochastic matrix to each dimension, is not well suited for ordinal variables and will not be considered here.

18 In fact, categorical information is all that is necessary in the present context. Even if the deprived achievements cannot be ranked one against the other, and the same is true for the achievements in the non-deprived category, one could use any numerical assignment that would correctly separate achievements into the deprived or non-deprived categories, with the deprivation cutoff being set at an appropriate value in between. The functions used below in the definition of equivalent representation need only preserve the categorical allocations.

19 For a fuller treatment of scales and measurement see Stevens (1946), Sen (1973, 1997), Alkire et al. (2015), and the references therein.

property, which embodies the concern that the measure should be independent of the way the underlying data are represented.[20]

Ordinality: Suppose that $(y';\ z')$ is obtained from $(y;\ z)$ as an equivalent representation. Then the methodology $\mathcal{M} = (\rho, M)$ satisfies $\rho(y_i'; z') = \rho(y_i; z)$, for all *i*, and $M(y'; z') = M(y; z)$.

Suppose that a dual cutoff method ρ_k is used for identification. To see that M_0 satisfies this property, note that the dimensions in which person *i* is deprived are unchanged between $(y'; z')$ and $(y; z)$, since the monotonic transformation ensures that $y_{ij}' < z_j'$ whenever $y_{ij} < z_j$. Consequently, the deprivation score is unchanged, which ensures that $\rho_k(y_i'; z') = \rho_k(y_i; z)$ for all *i*. It follows that the associated censored deprivation matrices are identical, so that their means are the same, and hence $M_0(y'; z') = M_0(y; z)$. The headcount measure H likewise satisfies Ordinality. But since normalized gaps can be very different for equivalent representations, Ordinality is violated by M_α for $\alpha > 0$ and any other measure making use of cardinal information on the depth of deprivations.[21]

3.2 Dimensional Breakdown

Multidimensional poverty by definition has multiple origins, and it is useful for policy purposes to have a method of gauging how each dimension contributes to overall poverty. For example, information on contributions of dimensional deprivations helps in the allocation of resources across sectors and the design of specific or multisectoral policies to address poverty, while monitoring progress dimension by dimension helps clarify the sources of progress.[22] A thoroughgoing decomposition of poverty by dimension would

20 We might imagine a weaker ordinality requirement that would only require the ordering, and not necessarily the measured level of poverty, to be preserved by equivalent representations.

21 As does Datt (2019) for $\alpha > 0$ and $\beta > 1$, which corresponds to indices we mentioned in 2011. Datt (2019) has recently proposed an alternative transfer axiom that, strictly speaking, is violated by the squared count measure (and the other *M*-gamma measures satisfying our Dimensional Transfer axiom) when deprivation cutoffs are high enough to include two or more deprived levels. The axiom, and indeed the entire paper, seems oriented to measures requiring cardinal data, whereas our focus is measures satisfying Ordinality.

22 Each of these has been used in practice: for example, in Colombia (CONPES 2012, Angulo 2016) and in Costa Rica (Government of Costa Rica INEC 2015). Naturally the translation from measure to policy response requires additional analysis.

require overall poverty to be a weighted average of dimensional components, each of which is a function of that dimension's distribution of achievements only (without reference to achievements in the other dimensions). For example, Chakravarty, Mukherjee, and Ranade (1998) propose the following property for measures M identified with a union approach:[23]

Factor Decomposability: There exist $v_j > 0$ summing to one, and component functions $m_j: Y_j \times R_{++} \rightarrow R_+$ for $j = 1,2,\ldots,d$ such that

$$M(y;z) = v_1 m_1(y_{\cdot 1};z_1) + \cdots + v_d m_d(y_{\cdot d};z_d) \text{ for } y \text{ in } Y$$

Two alterations must be made to this property to fit the present purpose. First, Factor Decomposability was constructed for methodologies using a union identification approach, so that by definition every deprivation is a poor person's deprivation. With an intermediate dual cutoff approach to identification, being deprived in a dimension does not automatically mean that a person is poor. Additional deprivations in other achievements may also be needed. And since a given deprivation only contributes to poverty when the deprived person is poor, we must consider a form of breakdown by dimension that explicitly permits the component functions to depend on information on who is poor. In our breakdown property, overall poverty is expressible as a weighted sum of dimensional components, but only after identification has taken place and the domain has been limited to the fixed set Y_r of achievement matrices with the same poverty status vector.[24]

Second, while Factor Decomposability places no constraints on the component functions, one could argue that in order for a breakdown to be policy relevant, the component functions should reflect basic descriptive facts. The contribution of a dimension to overall poverty should intuitively be zero if no poor persons are deprived in that dimension; while if one or more persons are both poor and deprived in the

[23] Chakravarty, Mukherjee, and Ranade (1998) also require the component functions to be identical, which seems a bit restrictive and that requirement is relaxed here. See also Chakravarty and Silber (2008) and Chakravarty (2009).

[24] This property allows the functional form of the breakdown to vary for every set of distributions having a different set of the poor – a less stringent and more general assumption than a full dimensional decomposition that requires the same functional form across all the subsets.

dimension, then the contribution should be positive. We say that $m_j(y_{\cdot j}; z_j)$ is *normalized* if $m_j(y_{\cdot j}; z_j) = 0$ whenever $y_{ij} \geq z_j$ holds for every poor person *i*, while $m_j(y_{\cdot j}; z_j) > 0$ whenever $y_{ij} < z_j$ holds for some poor person *i*.

Dimensional Breakdown: For any given poverty status vector r, there exist $v_j > 0$ summing to one and normalized component functions $m_j: Y_{rj} \times R_{++} \to R_+$ for $j = 1, \ldots, d$, such that

$$M(y; z) = v_1 m_1(y_{\cdot 1}; z_1) + \cdots + v m_d(y_{\cdot d}; z_d) \qquad \text{for } y \text{ in } Y_r \qquad (3)$$

In words, after identification has taken place and the poverty status of each person has been fixed, multidimensional poverty can be expressed as a weighted sum of dimensional components.

For the adjusted headcount ratio M_0, the weights are given by $v_j = w_j$ and the normalized component functions are $m_j(y_{\cdot j}; z) = H_j$, and hence by expression (2) the measure satisfies dimensional breakdown. Note that the censored headcount ratio $H_j = \mu(g_{\cdot j}^0(k))$ depends on the distribution of the other dimensional achievements, since all achievement levels are needed to determine who is poor. However, since we have restricted consideration to distributions y having the same poverty status vector r, each r_i is effectively a constant and the entries in column $g_{\cdot j}^0(k)$ can be expressed as $g_{ij}^0(k) = g_{ij}^0 \rho_k(y_i; z) = g_{ij}^0 r_i$ without reference to other dimensions. Consequently H_j depends only on $y_{\cdot j}$ and z over this domain as required by Dimensional Breakdown. The measures M_α for $\alpha > 0$ also satisfy Dimensional Breakdown. On the other hand, the multidimensional headcount ratio H violates the property for any ρ_k excluding intersection identification. Clearly, for every such ρ_k there is at least one dimension *j* in which a person could be nondeprived and yet remain poor. If everyone went from being deprived in all dimensions to being deprived in all but *j*, the normalized component function $m_j(y_{\cdot j}; z)$ would have to fall to 0 from a positive value. Since $w_j > 0$, the overall measure would have to fall, and yet H remains unchanged, violating Dimensional Breakdown. In the special case of intersection identification, intensity A is 1 and so the multidimensional headcount ratio H becomes M_0 and satisfies Dimensional Breakdown.

3.3 Dimensional Transfer

Transfer properties are motivated by the idea that poverty should be sensitive to the level of inequality among the poor, with greater inequality being associated with a higher (or at least no lower) level of poverty.[25] But which notion of inequality should be used in the multidimensional context? As noted in the introduction, there are two concepts in common use, one linked most closely to Kolm (1977) and another from Atkinson and Bourguignon (1982). The first is based on a definition of a progressive transfer as a 'common smoothing', whereby each dimensional distribution is transformed using the same bistochastic matrix. However, for this form of inequality to be meaningful, each dimensional variable would need to exhibit properties that are at odds with Ordinality.[26]

The second inequality concept is based on a specialized transfer called a rearrangement, in which two persons switch achievements in certain dimensions. The role of a progressive transfer in this context is played by an *association-decreasing rearrangement*, in which the achievement vectors of the two persons are initially ranked by vector dominance (so that one person has no less in each dimension than the other person and more in one) and then after the rearrangement their achievement vectors cannot be ranked (so that one person has more in one dimension and the other has more in a second dimension). In symbols, we say that y' is obtained from y by an association-decreasing rearrangement if (a) both y' and y have the same population size; (b) there exist persons u and i such that for each $j = 1,..., d$ we have $\{y'_{uj}, y'_{ij}\} = \{y_{uj}, y_{ij}\}$, while the achievements for all other persons are unchanged; and (c) $y_i > y_u$, while neither $y'_i > y'_u$ nor $y'_u > y'_i$. This transformation can be interpreted as a progressive transfer in that it transforms an initial 'spread' between two persons – a spread represented by the dominance between achievement vectors – into a moderated situation where neither person has unambiguously more than the other. The

[25] See Sen (1976), Foster and Sen (1997), and Alkire et al. (2015).

[26] The transformed achievement levels in a dimension are weighted averages of initial levels and hence depend on the specific cardinal representation of variables. H and M_0 are independent of this form of inequality among the poor.

overall achievement levels in society are unchanged, but the correlation between them (and hence inequality) has been reduced.[27]

Since this form of transfer involves a rearrangement and not an algebraic averaging of two persons' dimensional achievements, it can be applied to ordinal data and is in principle consistent with the Ordinality property. The resulting transfer axiom for multidimensional poverty measures typically specifies that the persons involved in the rearrangement are poor. For example, the Weak Rearrangement axiom requires poverty not to rise as a result of an association-decreasing rearrangement among the poor. Note also that this axiom – like virtually all related axioms in the literature – is weak in that it does not ensure that poverty must strictly fall. It rejects the most problematic measures for which poverty can be 'alleviated' by increasing inequality among the poor but at the same time allows measures to be entirely insensitive to progressive rearrangements among the poor.[28]

A natural question to ask is how to formulate a version of this transfer axiom that would, in certain circumstances, require poverty to strictly fall in response to a decline in inequality among the poor. One minimalist approach is to restrict consideration to cases where the association-decreasing rearrangement among the poor involves achievement levels that are on either side of deprivation cutoffs – thus affecting the distribution of deprivations as well. This would yield an association-decreasing rearrangement among the poor in achievements that is simultaneously an association-decreasing rearrangement *in deprivations*. Recalling the definition of an association-decreasing rearrangement, we say that y' is obtained from y by a *dimensional rearrangement among the poor* if for poor persons *u* and *i* it satisfies (a) – (c) above, plus (d) $g_u^0 > g_i^0$ while neither $g_i^{0\prime} > g_u^{0\prime}$ nor $g_u^{0\prime} > g_i^{0\prime}$. In other words, the initial deprivation status vectors (and achievement vectors) are ranked by vector dominance, while the final deprivation status vectors (and

[27] Note that in order for an association-decreasing rearrangement among the poor to exist, there must be at least three dimensions; otherwise, it would be impossible to have vector dominance initially and the absence of vector dominance subsequently.

[28] Such is the case of the headcount ratio *H* (since the number of poor persons is unchanged) and the adjusted headcount ratio M_0 (since the number of deprivations among the poor is also unchanged).

achievement vectors) are not.[29] The extra condition ensures that the person with lower achievement levels is actually deprived in two or more dimensions for which the other person is not and that, through the rearrangement, one or more of these deprivations (but not all) are traded for non-deprived levels. The following transfer property for multidimensional poverty measures requires poverty to decrease when there is a dimensional rearrangement among the poor.

Dimensional Transfer: If y' is obtained from y by a dimensional rearrangement among the poor, then $M(y';\, z) < M(y;\, z)$.

The axiom does not apply to cases where the association-decreasing rearrangement leaves deprivations unaffected; instead, it requires the two persons to switch deprivations as well as achievements.[30] This additional requirement is analogous to the one found in Dimensional Monotonicity. Its *dimensional decrement among the poor* goes beyond an ordinary decrease in one achievement by requiring that the affected poor person must gain a deprivation in the process. Clearly, the headcount ratio methodology H violates both Dimensional Monotonicity and Dimensional Transfer since neither transformation affects the number of poor persons.[31] In contrast, a dimensional decrement among the poor increases the average intensity of poverty A and hence $M_0 = HA$, which ensures that the adjusted headcount ratio satisfies Dimensional Monotonicity. But since a dimensional rearrangement among the poor leaves both H and A unchanged, M_0 just fails to satisfy Dimensional Transfer.[32] The next section presents a parametric class containing H, M_0 and a range of measures satisfying Dimensional Transfer.

[29] The vector dominance in deprivations is converse to the vector dominance in achievements so that the person with lower achievements has more deprivations. In order to construct such a rearrangement, it must be possible to remove (or add) a deprivation from some poor person without altering his or her poverty status; this rules out intersection identification, for example, since a person must be deprived in all dimensions to be poor.

[30] While it is possible to formulate a strict property based only on (a) – (c), constructing a meaningful poverty measure that satisfies it might prove to be a challenge if indicators have no inherent cardinal content upon which to rely. Adding (d) allows deprivations to function as the measuring rod for practical measures.

[31] This of course assumes that such a transformation exists. For example, a dimensional decrement among the poor requires there to be at least two dimensions and a way for a poor person to add a deprivation, which rules out intersection identification.

[32] Again, assuming a dimensional rearrangement exists. Since A is the average share of deprivations poor people experience, a rearrangement of the same set of deprivations among the same set of poor persons does not change A.

4. The *M*-Gamma Class

Seth (2013) has shown how a simple transformation at the individual level can produce measures that are sensitive to Atkinson-Bourguignon inequality.[33] The approach is now applied to M_0. For any $\gamma \geq 0$, let $c^{\gamma}(k)$ be the vector obtained from the vector of censored deprivation scores $c(k)$ by raising each positive entry to the power γ and leaving the zero entries unchanged, so that $c_i^{\gamma}(k) = (c_i)^{\gamma}$ when person *i* is poor and $c_i^{\gamma}(k) = 0$ otherwise. For example, when $\gamma = 0$ we obtain $c^0(k) = r$, the poverty status vector, while $\gamma = 1$ yields $c^1(k) = c(k)$, the vector of censored deprivation scores, and with $\gamma = 2$ we obtain vector of squared censored deprivation scores $c^2(k)$. The *M-gamma* family of measures is defined by

$$M_0^{\gamma} = \mu(c^{\gamma}(k)) \qquad \text{for } \gamma \geq 0.$$

Note that $M_0^0 = H$ is the headcount ratio that measures the incidence of multidimensional poverty, $M_0^1 = M_0$ is the adjusted headcount ratio that includes the breadth of deprivation, while M_0^2, which might be called the *squared count* measure, emphasizes the severity of deprivation by squaring each person's deprivation score.[34] The properties of this family are given in the following result.

Theorem 1. The methodology (ρ_k, M_0^{γ}) satisfies: The Basic Axioms and Ordinality for $\gamma \geq 0$, Dimensional Monotonicity for $\gamma > 0$, Weak Rearrangement for $\gamma \geq 1$, and Dimensional Transfer for $\gamma > 1$.

Proof: See Appendix.

The verification of Dimensional Transfer shows how a dimensional rearrangement among the poor across two achievement matrices becomes a progressive transfer among the poor across the censored deprivation

[33] This also follows Alkire and Foster (2011a p. 485) who applied the transformation to M_α for $\alpha > 0$ and obtained measures requiring cardinal data.

[34] For the case of union identification and equal weights, M_0^2 corresponds to a measure of social exclusion proposed in Chakravarty and D'Ambrosio (2006), which in turn is used by Jayaraj and Subramanian (2010) for equal weights and our dual cutoff approach. See also Silber and Yalonetzky (2013), Aaberge and Brandolini (2015), Rippin (2017), Bérenger (2017), Datt (2019), and Pattanaik and Xu (2018).

score vectors, which lowers $\mu(c^\gamma(k))$ and hence poverty for $\gamma > 1$. A person's censored deprivation score resembles the normalized poverty gap in the unidimensional world, so that the *M*-gamma measure M_0^γ has a form analogous to an FGT or *P*-alpha index, with γ playing the role of α in the unidimensional measures.[35] And, like P_2, the squared count measure M_0^2 is directly linked to standard measures of dispersion and inequality:

$$M_0^2 = (M_0)^2 + V = H\left(A^2 + V_p\right) \tag{4}$$

where V denotes the variance of the censored distribution $c(y)$, while V_p is the variance applied to the distribution of deprivation scores among the poor.[36] Dividing each variance in (4) by its associated mean yields a restatement

$$M_0^2 = (M_0)^2(1 + C^2) = HA^2\left(1 + C_p^2\right) \tag{4'}$$

in terms of relative inequality, where C^2 (or C_p^2) is the squared coefficient of variation. Comparable expressions can also be derived for inequalities in the censored *attainment* distribution $a(k)$, defined by $a_i(k) = 1 - c_i(k)$ for all i. This change of variable has no effect on the variance and hence expression (4) continues to apply. However, the means are now $1 - M_0$ and $1 - A$, respectively, thus altering the squared coefficients of variance and expression (4'). In particular, where C_{pa}^2 is the squared coefficient of variation applied to the attainment distribution of the poor, we obtain

$$M_0^2 = H[A^2 + (1 - A^2)C_{pa}^2]$$

which mirrors the expression for P_2 found in Foster et al (1984).

[35] In a companion paper (Alkire and Foster, 2016) we show how *any* unidimensional poverty measure satisfying a traditional transfer axiom can generate a multidimensional measure satisfying Dimensional Transfer.

[36] The verification is elementary. See also Chakravarty and D'Ambrosio (2006) and Seth and Alkire (2017). V_p represents (absolute) inequality among the poor. V can be decomposed across the poor and nonpoor groups into a within group term HV_p and a between group term in which the variance is applied to a 'smoothed' distribution where each person is assigned the group mean. Thus, it also measures inequality across poor and nonpoor.

While all *M*-gamma measures in the range $\gamma > 1$ satisfy Dimensional Transfer, it can be seen that none of them satisfies Dimensional Breakdown. Intuitively, the strict convexity of c_i^γ ensures that the marginal value of an additional deprivation is increasing in the overall deprivation score of a poor person, while Dimensional Breakdown requires the marginal value to be independent of the other dimensions (at least over the restricted domain Y_r). This conflict between Dimensional Transfer and Dimensional Breakdown holds more generally, as is shown in the following result.

Theorem 2. Let methodology $\mathcal{M} = (\rho, M)$ satisfy Symmetry and suppose that $\mathcal{M}$ has at least one dimensional rearrangement among the poor. Then $\mathcal{M}$ cannot simultaneously satisfy Dimensional Breakdown and Dimensional Transfer.

Proof: See Appendix.

Theorem 2 shows that the conflict identified for the *M*-gamma class (given ρ_k) extends to general multidimensional poverty methodologies. Of course, if no dimensional rearrangement were to exist – which could be the case for certain methodologies – Dimensional Transfer would technically hold but would be empty in practice. When there is at least one situation in which the axiom applies, the conflict must follow. Previous results in the literature assume full factor decomposability and a union identification, which diminishes their relevance in the present context.[37] Theorem 2 invokes Dimensional Breakdown, which constrains the measure *within* each Y_r and *not* across domains and allows for *any* identification method including ρ_k.

One response to Theorem 2 would be to use two measures – one satisfying Dimensional Breakdown and another satisfying Dimensional Transfer – instead of seeking one measure satisfying both. This would give inequality a role in comparing poverty levels, but not in understanding them, which seems less than

[37] For example, see the impossibility result of Pattanaik, Reddy, and Xu (2012), which uses an ordinal version of full factor decomposability. The main intuition underlying all of these results can be found in the earlier work of Bourguignon and Chakravarty (2003), or Gajdos and Weymark (2005) in the context of inequality orderings. Rippin (2017, table 3.1 cf Burchi *et al.* 2018) claims incorrectly that she has found a measure satisfying both full factor decomposability and sensitivity to inequality; in fact, it violates full factor decomposability.

satisfactory. Another response would be to alter one or the other of the competing axioms. For example, dropping the requirement (d) from the definition of an association-decreasing rearrangement among the poor changes Dimensional Transfer – but in the *wrong* direction. An axiom based on (a) – (c) (or fewer restrictions) *broadens* its applicability and maintains the conclusions of Theorem 2: either there is no such rearrangement (and so the axiom is effectively inapplicable) or there is at least one such rearrangement and impossibility ensues.[38] As for Dimensional Breakdown, which is already a weakening of Dimensional Decomposition, it is unclear how a specific replacement axiom might be constructed and, more importantly, justified.

5. The Shapley Breakdown

Datt (2019) proposed using the Shapley value decomposition methods advanced by Shorrocks (2013) to obtain a vector of dimensional contributions adding up to the poverty level. Indeed, the Shapley value $\phi = (\phi_1, \dots, \phi_d)$ provides an axiomatically justified technique for allocating a given quantity M (in this case the poverty level in an achievement matrix) across *d* many constitutive variables in such a way that $|\phi| = M$. Each ϕ_j is based on the notion of the marginal impact of *j* 's deprivations on poverty, which (without Dimensional Breakdown) depends on the presence or absence of the other dimensions' deprivations and hence the order in which dimensional deprivations are serially suppressed to calculate each marginal impact. The Shapley approach computes a separate marginal impact for each ordering or permutation of dimensions, and then defines ϕ_j as the average marginal impact (where each ordering is assumed to have equal weight).

We should note that there are practical and conceptual barriers to applying Shapley methods. One challenge is the computational complexity of deriving the Shapley value $\phi = (\phi_1, \dots, \phi_d)$, each entry of which entails $d!$ many individual calculations (or their combinatoric equivalents). A second barrier is the

[38] Datt's (2019) "strong rearrangement axiom" is of this type, for example.

absence of a summary formula for understanding its meaning. After the calculations are completed and ϕ has been derived, we are essentially left with a vector of *d* numbers. Without an explicit formula to help understand these numbers, their relevance in policy applications can be limited. Finding dimensional components that add up to total poverty is easy; finding components that add up and have a tangible, readily conveyed meaning for policy applications is far more challenging.

The adjusted headcount ratio is one measure for which the Shapley method produces a useful breakdown—namely, the traditional breakdown introduced in (2). Given union identification, it is immediate that $\phi = b$ is its breakdown vector, with components $b_j = w_j H_j$ that are easily understood. This conclusion follows from the linearity of M_0, which ensures that the marginal contribution of dimension *j* is always b_j, no matter the order of removing or adding dimensions. And the result continues to hold for *any* dual cutoff identification ρ_k, so long as the analysis uses what we call here the *Shapley breakdown* approach, which follows Dimensional Breakdown by conducting its entire analysis post identification.[39] Are there other measures for which the Shapley breakdown approach might provide a workable alternative?

We explore this question for the *M*-gamma class M_0^γ using a simple example based on the structure of the global MPI of the UNDP as described in Alkire and Jahan (2018). The global MPI has three dimensions with ten dimensional indicators allocated among them: two indicators with weights of $1/6$ in the first dimension (health), two with weights of $1/6$ in the second (education), and six with weights of $1/18$ in the third (living standards), resulting in equal weights of $1/3$ for each dimension. Its overall poverty cutoff is $k = 1/3$ so that a person is poor if deprived in the equivalent of a dimension or more.

[39] In other words, the analysis takes the censored deprivation matrix $g^0(k)$ as given and serially replaces columns with a column vector of zeros in the order determined by a permutation. The change in poverty from zeroing dimension *j*'s deprivations is its marginal impact for that permutation. The Shapley component ϕ_j is the average of the $d!$ many marginal impacts for *j*. Note that this approach is inapplicable to H if $H > 0$, since all marginal impacts would be 0 and the requirement $|\phi| = H$ fails to hold. An alternative 'pre-identification' Shapley approach *does* work for H, but it has certain drawbacks as discussed below.

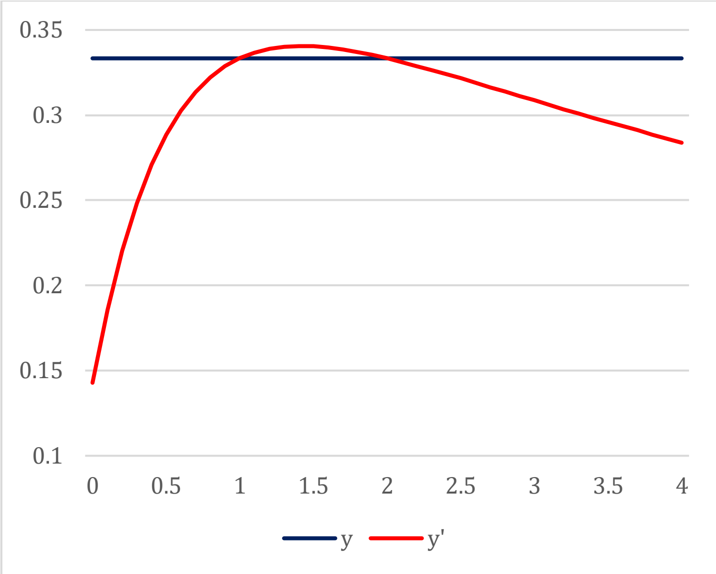
The example compares two single person achievement matrices y and y', whose health indicators are identical but have differing education and living standards. While both are deprived in the first health indicator (nutrition) and not deprived in the second, y is deprived in every education indicator and no living standards indicator while y' is deprived in every living standards indicator and no education indicator.[40] Their deprivation scores of 0.50 exceed k, and hence both persons are poor. For any given $\gamma > 0$ the associated M-gamma poverty levels are $M_0^\gamma(y; z) = M_0^\gamma(y'; z) = (0.50)^\gamma$, and so by definition the Shapley breakdown vectors ϕ and ϕ' will satisfy $|\phi| = |\phi'| = (0.50)^\gamma$. Our expectation is that the relative contribution of indicator 1 to overall poverty, namely $\phi_1/|\phi|$ and $\phi_1'/|\phi'|$, should be the same since they are identical in health and are fully deprived in exactly one other (equally weighted) dimension. In other words, the relative contribution of indicator 1 should not depend on whether it is education or living standards in which the person is fully deprived.

Figure 1, which depicts the relative contributions $\phi_1/|\phi|$ and $\phi_1'/|\phi'|$ over a range of γ values, shows how widely the two can depart.[41] For y, the relative contribution of indicator 1 is always $\phi_1/|\phi| = 1/3$. For y' the value of $\phi_1'/|\phi'|$ begins near 1/7, rises above 1/3 then returns to 1/7 as gamma tends to infinity. The departure between the two illustrates that the Shapley method can produce unintuitive results in the context of multidimensional poverty and should not be applied without careful evaluation. This kind of conceptual problem has been noted by Shorrocks (2013, p. 109), who warned that the Shapley method may not work well with hierarchical variables. As most implementations of multidimensional poverty measures entail hierarchical variables in a nested weighting structure, we conclude that Shapley methods do not in general provide a workable solution.

[40] In symbols, y has $g_{1j}^0 = 1$ for *j* = 1,3,4, and $g_{1j}^0 = 0$ otherwise; while y' yields $g_{1j}^0 = 1$ for *j* = 1,5,…,10 and $g_{1j}^0 = 0$ otherwise.

[41] This calculation entailed a rather extensive accounting of all marginal contributions for all variables across all permutations of the ten variables; it is available from the authors. The example would be unchanged if a union identification were used, or if union and a pre-identification Shapley approach (discussed below) were used.

Figure 1. Relative Contribution of Nutrition as Gamma Varies



One aspect of this example, however, invites further scrutiny. Figure 1 reveals that for two values of γ, namely $\gamma = 1$ and $\gamma = 2$, the relative contribution of indicator 1 *is* the same for y and y'. In other words, both measures ensure that the relative contribution is independent of whether the deprived dimension is education or living standards. Why this holds for $\gamma = 1$ (and hence $M_0^1 = M_0$) is straightforward: Since its Shapley breakdown vector ϕ and its dimensional breakdown vector b are the same, we know that $\phi_1 = b_1 = 1/6$ and $|\phi| = |b| = 0.50$ for y, and similarly for y', so that

$$\frac{\phi_1}{|\phi|} = \frac{b_1}{|b|} = \frac{1}{3} = \frac{b_1'}{|b'|} = \frac{\phi_1'}{|\phi'|}$$

In the case of $\gamma = 2$, however, although we know that $|\phi| = |\phi'| = (0.50)^\gamma$, the associated Shapley formulas are so complex that they do not immediately reveal why $\phi_1 = \phi_1'$. The following result provides the explanation.

Lemma 1. Let y be an achievement matrix with $n = 1$. Then the Shapley breakdown vector ϕ for $M_0^2(y;z)$ is given by $\phi = b|b|$, where $b = (w_1 H_1, \ldots, w_d H_d)$ is the dimensional breakdown vector for $M_0^1(y;z)$.

Proof: See Appendix.

From Lemma 1, it is apparent why the two curves in Figure 1 intersect at $\gamma = 2$. The Shapley breakdowns for measure M_0^2 yield the following relative contributions for indicator 1:

$$\frac{\phi_1}{|\phi|} = \frac{b_1|b|}{|b||b|} = \frac{1}{3} = \frac{b_1'|b'|}{|b'||b'|} = \frac{\phi_1'}{|\phi'|}$$

Of course, the formula from Lemma 1 only applies to the case where $n = 1$. However, as we shall now see, it leads directly to an unexpectedly intuitive formula for the Shapley breakdown of $M_0^2(y; z)$ for any y with an arbitrary population size.

Recall that for any y with $n \geq 1$, the Shapley breakdown vector of $M_0^1(y; z)$ has $b_j = w_j H_j$ as its *j*th dimensional component, where $H_j = q_j/n$ is the censored headcount ratio for dimension *j* and $q_j = |g_{\cdot j}^0(k)|$ is the number of poor persons deprived in *j*. The next result makes use of two additional definitions. The *censored intensity* for dimension *j* is defined by

$$A_j = \frac{g_{1j}^0(k)c_1(k) + \cdots + g_{nj}^0(k)c_n(k)}{q_j}$$

This gives the average deprivation score across all poor persons *i* deprived in a given *j* (i.e., satisfying $g_{ij}^0(k) = 1$). The *censored adjusted headcount ratio* for dimension *j* is defined by

$$M_{0j} = H_j A_j$$

It takes the average across all n persons, where the censored persons (those not poor or not deprived in *j*) are assigned a deprivation score of zero. We have the following useful result.

Theorem 3. For any y, the Shapley breakdown vector of $M_0^2(y; z)$ is given by $\phi = (\phi_1, \dots, \phi_d)$ where

$$\phi_j = w_j M_{0j} = w_j H_j A_j \qquad \text{for } j = 1, \dots, d \qquad (5)$$

Proof: See Appendix.

Theorem 3 shows that the Shapley breakdown formula for the squared count measure

$$M_0^2 = w_1 M_{01}^1 + \cdots + w_d M_{0d}^1 \qquad (6)$$

is remarkably similar to the dimensional (and Shapley) breakdown formula for the adjusted headcount ratio

$$M_0^1 = w_1 M_{01}^0 + \cdots + w_d M_{0d}^0 \tag{7}$$

where we denote the censored adjusted headcount ratios by $M_{0j}^1 = M_{0j}$ and the censored headcount ratios by $M_{0j}^0 = H_j$ to highlight the parallels between the two. Equation (7) expresses the adjusted headcount ratio M_0^1 as the weighted average of censored headcount ratios M_{0j}^0, while (6) expresses M_0^2 as the weighted average of censored *adjusted* headcount ratios M_{0j}^1. Hence in both cases censored measures with parameter $(\gamma - 1)$ are combined to obtain an *M*-gamma measure with parameter γ. This common structure between the two breakdowns is quite striking and unexpected.

It is also informative to trace out why formulas (6) and (7) are in fact true. To verify (7), pick any *i* and examine the components on the right-hand side of (7). For each dimension *j* in which person *i* is deprived and poor, one of the q_j-many w_j/n terms underlying the weighted censored headcount ratio $w_j M_{0j}^0 = q_j(w_j/n)$ is due to person *i*. Adding up across dimensions yields $c_i(k)/n$, and summing these terms across persons yields $M_0^1 = M_0$. To verify (6), pick any *i* and examine the components on the right-hand side of (6). In each dimension *j* for which *i* is deprived and poor, the term $w_j M_{0j}^1$ contains $w_j c_i(k)/n$ from person *i*, which when added across dimensions yields $c_i^2(k)/n$. Summing across persons yields M_0^2. Clearly the two verifications are quite parallel.

The Shapley breakdown follows the 'post-identification' perspective of Dimensional Breakdown in that it suppresses deprivations *after* it has fixed the poverty status of each person. Alternatively, one could imagine suppressing deprivations *before* the poor have been identified and then using this 'counterfactual' scenario for identifying the poor and measuring marginal impacts.[42] How would such a 'pre-identification' perspective affect the Shapley value? For a single individual and a specific ordering of dimensions, the

[42] See Pérez et al (2015), who applies it to H, and Datt (2019).

marginal impacts are unaffected for all dimensions preceding the 'pivotal' dimension in which the person's poverty status changes; the pivotal dimension itself is then assigned the marginal contributions from all later dimensions, leaving them with zero. For union identification, the pivotal dimension is the *final* dimension in which the person is deprived, which implies that all marginal impacts are unaffected and the original Shapley breakdown vector is obtained. For intersection identification, the marginal contributions all accrue to the *first* deprived dimension of the ordering and hence every deprived dimension is allocated an equal share regardless of its weight or inherent importance. For an intermediate dual cutoff identification, which is the primary concern of this paper, the Shapley value can depend on the combinatoric properties of the specific poverty cutoff and weights in complex, non-transparent ways.[43] Given our desire to be consistent with Dimensional Breakdown, and the potential for arbitrary and erratic results using a pre-identification approach, we have opted for the Shapley breakdown approach.

This section has explored whether Shapley methods can usefully be applied to multidimensional poverty measures – especially those that satisfy Dimensional Transfer and, hence, violate Dimensional Breakdown. We concluded that Shapley methods are untenable for most *M*-gamma measures but lead to an intuitive breakdown formula for M_0^2 that builds upon the canonical breakdown of M_0^1. We suggest using M_0^0, M_0^1, and M_0^2 in tandem to evaluate multidimensional poverty, with a typical analysis including M_0^1, its subgroup decomposition and its canonical dimensional breakdown to understand contributions to poverty; the partial index M_0^0 to add tangible information on the prevalence of poverty; and M_0^2 and its Shapley breakdown to inform how results are affected when accounting for inequality. Indicators A, V, and V_p (and the accompanying relative inequality measures) can supplement the description and help explain the

[43] This erratic behavior is reflected in case of (ρ_k, H), which for $n = 1$ mirrors the structure of a *weighted voting game* -a simple coalitional game having d players, a vector of voting weights w_j, a quota k, and a characteristic function value of 1 for any 'winning' coalition of players whose sum of weights meets or exceeds the quota - and 0 otherwise. Zuckerman et al. (2012), for example, shows how a small change in k can alter Shapley values abruptly and unintuitively, and notes the potential for manipulation by shifting parameters. Similarly, if we apply (ρ_k, H) to y' from our example, the pre-identification Shapley contribution of nutrition rises from 14% to 29% to 49% and back down again as k varies from 0.05 to 0.50.

variations between the three poverty measures; they can also be computed from these measures.[44] Finally, to convert the dimensional breakdown for M_0^1 into the dimensional breakdown for M_0^2 only the censored average intensity levels A_j are needed. If the A_j levels were identical, the two breakdowns would have the same relative contributions; they depart from one in accordance with the variations in A_j. In the next section these tools are applied to recent data from Cameroon.

6. An Illustrative Example: Cameroon

Consider the following example that uses the 2014 Multiple Indicator Cluster Survey to compute multidimensional poverty for Cameroon and its 12 subnational regions following the methods used in the global MPI.[45] Compared to other developing countries, Cameroon has a medium level of multidimensional poverty, with 45.3% of the population living in poverty and an MPI (M_0) of 0.243. To emphasize the relationships among the various indices, we suspend our usual notations for *H* and M_0 and multiply their decimal values by 100 to express their levels in hundredths, so that the headcount ratio for Cameroon is 45.3 and the MPI is 24.3. The disparity in poverty across regions is striking, with the headcount ratio ranging from 3.9 to 80.0 and the MPI ranging from 1.5 to 48.

Table 1 presents the traditional dimensional breakdown of the MPI for the country and each of its regions.[46] The censored headcount ratios reveal a great variety in the composition of poverty across the regions. Consider, for example, Adamaoua and Est, two regions close in population size and with similar levels of MPI (30.7 and 31.5, respectively). From the breakdown, we see that they have marked differences in the composition of poverty, with education deprivations of the poor in Adamaoua being far more

[44] For example, $A = M_0^1/M_0^0$ and $V = M_0^2 - (M_0^1)^2$.

[45] The parameters of the global MPI used in this example are presented in Alkire, Kanagaratnam, and Suppa (2018). Note that from 2019 the MPI Table 1 includes the variance measures of inequality among the poor for every country, and all MPI country briefings include the analysis by intensity bands.

[46] A dimensional breakdown can be depicted in three ways. First, by listing the censored headcount ratios H_j whose weighted average is M_0; second by listing the weighted headcount ratios $b_j = w_j H_j$ that add up to M_0; and third by listing the relative contributions $w_j H_j / M_0$ that add up to 100%. We begin with the first of these.

widespread than in Est, health deprivations of the poor being somewhat lower, and living standards deprivations of the poor are mixed. For example, the censored headcount ratio for school attendance in Adamaoua indicates that 27.0 percent of Adamaoua's population are MPI poor and live in households in which a child is not attending school up to the age at which they should complete class eight; the comparable figure in Est is 19.6. Likewise, poverty in Est shows higher living standards deprivations, with a censored headcount ratio for sanitation and housing at 49.3 and 45.5, respectively, as compared with 40.7 and 35.5 for Adamaoua. The policies needed for responding to poverty in regions with similar MPI levels can vary in terms of allocation and sectoral emphases.

Table 2 lists poverty levels for Cameroon and its regions using the three main *M*-gamma measures, namely, the headcount ratio M_0^0, the adjusted headcount ratio M_0^1 discussed above, and the squared count measure M_0^2. Column 1 gives the headcount ratio for Cameroon (namely $M_0^0 = 45.3$) along with its regional levels. The adjusted headcount ratio for Cameroon and its regions reappears in column 2, beginning with the overall value of $M_0^1 = 24.3$ for the country. It adjusts the headcount ratio M_0^0 by the intensity or the breadth of poverty $A = |c(k)|/q$, which is also computable as $A = M_0^1/M_0^0$. The squared count measure M_0^2 in column 3 adjusts M_0^0 by its own intensity term $A' = |c^2(k)|/q = M_0^2/M_0^0$ which is sensitive to the inequality of deprivations among the poor, yielding a level of $M_0^2 = 14.2$ for Cameroon. In Table 2, regions are ordered according to M_0^1, and the rankings according to the three measures are quite similar. There are in this example no rank reversals when the headcount ratio M_0^0 is used instead of the adjusted headcount measure M_0^1. There is one reversal (Sud-Ouest vs. Ouest) when we move from the adjusted headcount ratio to the squared count measure M_0^2. The difference is the percentage increase in A' is higher than in A, and it is enough to overcome the higher headcount ratio in Ouest.

The final columns of Table 2 present additional information on absolute inequality. Column 6 lists the variance V applied to the entire censored count distribution: it is also the difference between column 3 and the square of column 2. We see the Sud-Ouest has higher variance than Ouest. Column 7 gives the absolute inequality V_p among the poor only. Note that V includes information both on inequality *within* the poor V_p and on inequality *between* the poor and nonpoor groups (which depends on the group means and population sizes). The difference is that the percentage increase in A' is higher than in A, and it is enough to overcome the higher headcount ratio in Ouest, hence the reversal of Sud Ouest and Ouest.

Table 3 divides poor people into categories according to their deprivation scores. Columns 2 to 8 provide an indicative overview of the percentage of poor persons who are deprived in a range of weighted indicators – in different intensity bands. Such descriptive information can be easily constructed and provides additional insight into inequality.[47]

Table 4 compares the Shapley breakdown of the squared count measure to the dimensional breakdown of the adjusted headcount ratio for Cameroon. Columns 1, 2, and 6 provide three views of the traditional dimensional breakdown with column 1 listing the censored headcount ratios whose weighted average is $MPI = M_0^1$, column 2 containing the breakdown terms that sum to M_0^1, and column 6 providing the relative contributions that sum to 100%. Column 1 shows that five of the six living standards indicators (excluding assets) have the highest incidence of deprivation among the poor, with nutrition not far behind. Accounting for weights in columns 2 and 6, we see that the nutrition and years of schooling indicators contribute most to multidimensional poverty with the school attendance and fuel indicators next in line. This reflects both the high censored headcount ratio of fuel (44.6) and the higher relative weights on the health and education indicators (1/6 rather than 1/18).

The censored intensities A_j in column 3 indicate the average deprivation score for poor people deprived in dimensional indicator *j*. The respective dimensional breakdown entries for M_0^1 are multiplied by A_j to obtain the Shapley breakdown information for M_0^2. Column 4 lists the censored adjusted headcount ratios whose weighted average is M_0^2, while column 5 contains the Shapley breakdown terms that sum to M_0^2 and column 7 provides the relative contribution terms summing to 100%. From Column 3 we see that

[47] Country briefings related to the global MPI include these intensity bands for each country and are available on https://ophi.org.uk/multidimensional-poverty-index/mpi-country-briefings/.

persons who are deprived in school attendance, followed by child mortality and years of schooling, have, on average, the highest deprivation scores across all dimensions and hence A_j values. These indicators have higher relative contributions for M_0^2 than M_0^1, reflecting their higher A_j values and increased importance in the squared count measure. For the indicators with the lowest censored intensities – cooking fuel and sanitation – we see that the relative contributions are also lower and by larger margins (column 8). This example illustrates how M_0^2 supplements the headcount ratio (M_0^0) and adjusted headcount ratio (M_0^1) by evaluating inequality among the poor. The Shapley breakdown of the squared count measure, when interpreted though its subcomponents, provides interesting and new information about multidimensional poverty. Note, though, for this example, that the main conclusions from the traditional dimensional breakdown of M_0^1 are largely echoed by the Shapley breakdown of M_0^2.

7. Conclusions

This paper began with the formalization of two properties, Ordinality and Dimensional Breakdown, that significantly contribute to the practical usefulness of the MPI, and then introduced the Dimensional Transfer axiom, which requires multidimensional poverty measures to be sensitive to inequality among the poor. A class of multidimensional poverty measures, which we call the *M*-gamma measures, was also introduced, of which three main measures were highlighted: the traditional headcount ratio, which provides tangible evidence on poverty (its prevalence) but violates several key axioms; the adjusted headcount ratio, which satisfies many desirable axioms including Dimensional Breakdown but not Dimensional Transfer; and a squared count measure, which satisfies Dimensional Transfer but violates Dimensional Breakdown. An impossibility result showed that Dimensional Transfer inevitably conflicts with Dimensional Breakdown, so that by adopting the former, one must forego the latter.

The Shapley value from cooperative game theory was explored as an alternative path to breaking down poverty by dimension. An example drawn from the global MPI suggested that Shapley methods cannot be seen as a general replacement for the traditional breakdown formula. However, when applied to the particular case of the squared count measure, the Shapley breakdown generates coherent results and has an explicit formula that intuitively builds upon the traditional dimensional breakdown formula for the adjusted headcount ratio. The result is a remarkably transparent, ready to use toolkit of three *M*-gamma measures, with the headcount ratio providing tangible information on the prevalence of multidimensional poverty, the standard adjusted headcount ratio adding information on poverty's intensity and its breakdown by dimension, and the squared count introducing inequality into the analysis. The ease with which this technology can be used, and the clarity of its results, were illustrated using Global MPI data from Cameroon.

We conclude by making two additional points in the broader area of poverty analysis. While our method of identifying the poor includes as extreme cases the union approach (anyone deprived in any dimension is poor) and the intersection approach (only those deprived in all dimensions are poor), we would like to stress the practical importance of using an intermediate identification between the two extremes. Poverty analysis entails a separation of the poor from the nonpoor in order to direct policy, and this in turn imbues the headcount ratio with a salience that is fundamental to poverty analysis, not to mention its role in communication. However, these aims are typically thwarted by restricting consideration to an extreme identification. For example, the union approach typically identifies nearly everyone as poor – 93.6 percent in the Cameroon example – dulling prioritization and rendering the headcount ratio ineffective for policy and communication purposes.[48] We must be mindful of the needs and expectations of the eventual users of new tools if we want the tools to affect behavior and produce improved outcomes.

The second point elaborates on a main theme of this paper: the role of inequality in poverty. A key motivation has been the idea that inequality should have a significant, if limited, role in the measurement of multidimensional poverty. A careful look at the Dimensional Transfer axiom reveals how restrictive

[48] Recent papers advocating union identification include Datt (2019), although his justification of union identification relies on cardinal variables and is not relevant to the present case. See also Pattanaik and Xu's (2018) advocacy in the case of cardinal variables. Note that adopting union identification could also skew the choice of indicators in an attempt to limit the size of the headcount ratio to a reasonable magnitude. See Alkire and Foster (2011b).

this role actually is. A dimensional rearrangement does not permit the number of poor to change, and hence M_0^0 is fixed. It preserves the aggregate collection of deprivations among the poor, and so M_0^1 is fixed. After fixing M_0^0 and M_0^1, it reduces the positive association across achievements and across deprivations, thereby reducing multidimensional inequality. The Dimensional Transfer axiom thus gives inequality among the poor a *contingent* priority. Indeed, lower inequality unconstrained by incidence and breadth is not necessarily a good thing: a world in which all persons are maximally deprived in all dimensions achieves lowest inequality and worst poverty. Constraining the role of inequality ensures that, as inequality falls, the conditions of a relatively *poorer* poor person are improving as a result of the dimensional rearrangement. The axiom asserts that this is favorable for overall poverty even though the improvement comes at the expense of a better off poor person.

Since Dimensional Transfer is based on transformations that impose deprivations on one poor person (to benefit another), the axiom may be criticized as being too extreme.[49] It is not our intention to suggest that poverty should be addressed only through this route. Nonetheless, in future work, it may be useful to see whether sensitivity to inequality can be defined without invoking a transformation that 'improves' poverty while depriving a poor person even more. On the other hand, it could be argued that Dimensional Transfer is not extreme or progressive enough as it does not consider rearrangements involving a nonpoor giver. An alternative axiom might require poverty to fall, or at least not rise, whenever a poor person appropriately switches achievements and deprivations with a nonpoor person. This is already covered by dimensional monotonicity apart from the case where the nonpoor person becomes poor as a result. If the goal is to eradicate poverty, though, it is not entirely clear why the forced impoverishment of a nonpoor person necessarily represents a clear-cut improvement, especially if the designation of being poor is seen as salient. And, accordingly, for each *M*-gamma measure, it is easy to construct an example where measured poverty *increases* as result of an association-decreasing rearrangement between a nonpoor and a poor

[49] We have benefited from discussions with Nora Lustig on this point.

person, because the increase in the headcount ratio overwhelms the decrease in average intensity.[50] While the ultimate goal is a world without poverty, different treatments of inequality in poverty can impact tradeoffs among incidence, breadth and severity, and hence policy priorities. The role of inequality in poverty measurement is indeed complex.

References

Aaberge, R. and Brandolini, A. (2015). 'Multidimensional poverty and inequality', in (Anthony B. Atkinson and Francois Bourguignon, eds.), *Handbook of Income Distribution*. Volume 2A, pp. 141–216 Amsterdam: North-Holland.

Alkire, S. and Foster, J. (2011a). 'Counting and multidimensional poverty measurement', *Journal of Public Economics*, vol. 95(7–8), pp. 476–487.

Alkire, S. and Foster, J. (2011b). 'Understandings and misunderstandings of multidimensional poverty measurement', *Journal of Economic Inequality*, vol. 9(2), pp. 289–314.

Alkire, S. and Foster, J. (2016). 'Dimensional and distributional contributions to multidimensional poverty', OPHI Working Paper 100, University of Oxford.

Alkire, S. and Jahan, S. (2018). 'The New Global MPI 2018: Aligning with the Sustainable Development Goals', OPHI Working Paper 121, University of Oxford.

Alkire, S., Kanagaratnam, U. and Suppa, N. (2018). 'The global Multidimensional Poverty Index (MPI): 2018 revision', OPHI MPI Methodological Note 46, Oxford Poverty and Human Development Initiative, University of Oxford.

Alkire, S., Meinzen-Dick, R., Peterman, A., Quisumbing, A., Seymour, G. and Vaz, A. (2013). 'The Women's Empowerment in Agriculture Index', *World Development*, vol. 53, pp. 71–91.

Alkire, S. and Santos, M.E. (2014). 'Measuring acute poverty in the developing world: Robustness and scope of the Multidimensional Poverty Index', *World Development*, vol. 59, pp. 251–274.

Alkire, S., Foster, J., Seth, S., Santos, M.E., Roche, J.M. and Ballon, P. (2015). *Multidimensional Poverty Measurement and Analysis: A Counting Approach*, Oxford: Oxford University Press.

Angulo, R. (2016). 'From multidimensional poverty measurement to multisector public policy for poverty reduction: Lessons from the Colombian case', OPHI Working Paper 102, University of Oxford.

Atkinson, A.B. (2003). 'Multidimensional deprivation: Contrasting social welfare and counting approaches', *Journal of Economic Inequality*, vol. 1(1), pp. 51–65.

Atkinson, A.B. (2015). *Inequality: What Can Be Done?* Cambridge, MA: Harvard University Press.

[50] However, for a range of k values near 0 (corresponding to union) and another range of k *values* near 1 (corresponding to intersection) M_0^γ will not rise in response to such a rearrangement. For every such k, the *M*-gamma measures (including the adjusted headcount ratio) would satisfy an appropriately expanded axiom that covers rearrangements that leave both parties poor. Outside of these ranges, all *M*-gamma measures (including the squared count measure) would violate this broader axiom. The table in Rippin (2017) does not distinguish between these ranges; other definitions and claims likewise need revision.

Atkinson, A.B. and Bourguignon, F. (1982). 'The comparison of multi-dimensional distribution of economic status', *Review of Economic Studies*, vol. 49, pp. 183–201.

Bérenger, V. (2017). 'Using ordinal variables to measure multidimensional poverty in Egypt and Jordan', *The Journal of Economic Inequality*, vol. 15(2), pp. 143–173.

Bossert, W., Chakravarty, S. and D'Ambrosio, C. (2013). 'Multidimensional poverty and material deprivation with discrete data', *Review of Income and Wealth*, vol. 59(1), pp. 29–43.

Bourguignon, F. and Chakravarty, S.R. (2003). 'The measurement of multidimensional poverty', *Journal of Economic Inequality*, vol. 1(1), pp. 25–49.

Burchi, F., Rippin, N., and Montenegro, C.E. (2018). 'From income poverty to multidimensional poverty—An international comparison', Working Paper number 174, International Policy Centre for Inclusive Growth.

Cameron, G., Dang, H., Dinc, M., Foster, J., and Lokshin M. (2019). 'Measuring the statistical capacity of nations', Policy Research Working Paper 8693, World Bank.

Chakravarty, S.R. and D'Ambrosio, C. (2006). 'The Measurement of Social Exclusion', *Review of Income and Wealth*, vol. 53(3), pp. 377–398.

Chakravarty, S.R., Mukherjee, D. and Renade, R.R. (1998). 'On the family of subgroup and factor decomposable measures of multidimensional poverty', *Research on Economic Inequality,* vol 8, pp. 175–194.

Chakravarty, S.R. and Silber, J. (2008). 'Measuring multidimensional poverty: The axiomatic approach', in (N. Kakwani and J. Silber, eds.), *Quantitative Approaches to Multidimensional Poverty Measurement*, pp. 192–209, Basingstoke: Palgrave Macmillan.

Chakravarty, S.R. (2009). *Inequality, Polarization and Poverty: Advances in Distributional Analysis*, Volume 6. New York: Springer.

Clark, D.A., Hemming, R. and Ulph, D. (1981). 'On indices for the measurement of poverty', *Economic Journal*, vol. 91(5), pp. 515–526.

CONEVAL (2009). *Methodology for Multidimensional Measurement of Poverty in Mexico*, Mexico City: CONEVAL.

CONPES (2012). 'Official methodology and institutional arrangements for the measurement of poverty in Colombia', *Documento Conpes Social,* 150. Bogota, 28 May, in Spanish [available at www.colaboracion.dnp.gov.co/CDT/Conpes/Social/150.pdf].

Datt, G. (2019). 'Distribution-sensitive multidimensional poverty measures?', *The World Bank Economic Review*, 33(3), pp. 551–572.

Dhongde S., Li Y., Pattanaik P. and Xu Y. (2016) 'Binary data, hierarchy of attributes, and multidimensional deprivation', *Journal of Economic Inequality*, vol.14(4), pp. 363–378

Foster, J., Greer, J. and Thorbecke, E. (1984). 'A class of decomposable poverty measures', *Econometrica*, vol. 52(3), pp. 761–766.

Foster, J. and Sen, A.K. (1997). 'On economic inequality after a quarter century', in (A.K. Sen, Expanded Edition), *On Economic Inequality*. Oxford: Clarendon Press, pp. 107–220.

Gajdos, T. and Weymark, J.A. (2005). 'Multidimensional generalized Gini indices', *Economic Theory,* vol. 26(3), pp. 471–496.

Government of Chile, Ministry of Social Development (2015). Nueva metodología de medición de pobreza por ingresos y multidimensional, *Documentos Metodológicos* 28, Santiago: Gobierno de Chile.

Government of Costa Rica INEC. (2015). *Methodology of the Costa Rican Multidimensional Poverty Index* (MPI). In Spanish. [Available at www.inec.go.cr/sites/default/files/documentos/pobreza_y_presupuesto_de_hogares/pobreza/metodologias/mepobrezaenaho2015-01.pdf].

IDB (2017). 'Better Jobs Index: An employment conditions index for Latin America', Technical Note IDB-TN-1326, Inter-American Development Bank.

Jayaraj, D. and Subramanian, S. (2010). 'A Chakravarty–D'Ambrosio view of multidimensional deprivation: Some estimates for India', *Economic and Political Weekly*, vol. 45(6), pp. 53–65.

Kolm, S. (1977). 'Multidimensional egalitarianisms', *Quarterly Journal of Economics*, vol. 91(1), pp. 1–13.

Maasoumi, E. and Lugo, M.A. (2008). 'The information basis of multivariate poverty assessments', in (N. Kakwani and J. Silber, eds.), *Quantitative Approaches to Multidimensional Poverty Measurement*, pp. 1–29, New York: Palgrave MacMillan.

Pattanaik, P., Reddy, S. and Xu, Y. (2012). 'On measuring deprivation and living standards of societies in a multi-attribute framework', *Oxford Economic Papers*, vol. 64, pp. 43–56.

Pattanaik, P.K. and Xu, Y. (2018). 'On measuring multidimensional deprivation', *Journal of Economic Literature*, vol. 56(2), pp. 657–672.

Pérez Pérez, J.E., Castelán, C.R., Trujillo, J.D. and Valderrama, D. (2015). 'Unpacking the MPI: A decomposition approach of changes in multidimensional poverty headcounts', Policy Research Working Paper 7514. The World Bank Group.

Piketty, T. (2014). *Capital in the Twenty-First Century.* Trans. Arthur Goldhammer. Cambridge, MA: Harvard University Press.

Ravallion, M. and Huppi, M. (1991). 'Measuring changes in poverty: A methodological case study of Indonesia during an adjustment period', *World Bank Economic Review*, vol. 5, pp. 57–82.

Rippin, N. (2017). 'Efficiency and distributive justice in multidimensional poverty issues', in (R. White, ed.), *Measuring Multidimensional Poverty and Deprivation: Incidence and Determinants in Developed Countries*, pp. 31–68. Basingstoke: Palgrave Macmillan.

RGOB-NSB. (2012). *Bhutan Multidimensional Poverty Index*, Royal Government of Bhutan National Statistics Bureau). Thimphu. www.nsb.gov.bt/publication/download.php?id=279 (last accessed: 9 Aug 2014).

Sen, A. (1973). *On Economic Inequality*. New York: Oxford University Press.

Sen, A. (1976). 'Poverty: An ordinal approach to measurement', *Econometrica*, vol. 44(2), pp. 219–231.

Sen, A. (1997). *On Economic Inequality. Expanded Edition with a Substantial Annex by James E. Foster and Amartya Sen*, Oxford: Clarendon Press.

Seth, S. (2013). 'A class of distribution and association sensitive multidimensional welfare indices', *Journal of Economic Inequality*, vol. 11(2), pp. 133–162.

Seth, S. and Alkire, S. (2017). 'Did poverty reduction reach the poorest of the poor? Complementary measures of poverty and inequality in the counting approach', in (S. Bandyopadhyay, ed.), *Research on Economic Inequality*, Volume 25, pp. 63–102. Bingley: Emerald Publishing.

Shorrocks, A. (2013). 'Decomposition procedures for distributional analysis: A unified framework based on the Shapley value', *Journal of Economic Inequality*, vol. 11, pp. 99–126.

Silber, J. and Yalonetzky, G. (2013) 'Measuring multidimensional deprivation with dichotomized and ordinal variables', in (G. Betti and A. Lemmi, eds.), *Poverty and Social Exclusion: New Methods of* Analysis, pp. 9-37, London and New York: Routledge.

Stevens, S.S. (1946). 'On the theory of scales of measurement', *Science*, New Series 103 (2684), pp. 677–680.

Tsui, K. (2002). 'Multidimensional poverty indices', *Social Choice and Welfare*, vol. 19(1), pp. 69–93.

UNDP and OPHI (2019a). *Global Multidimensional Poverty Index 2019: Illuminating Inequalities.* New York: United Nations Development Programme.

UNDP and OPHI (2019b). *How to Build a National Multidimensional Poverty Index (MPI): Using the MPI to inform the SDGs.* New York: United Nations Development Programme.

UNGA (United Nations General Assembly). (2015). *Transforming our world: the 2030 agenda for Sustainable Development.* Resolution A/RES/70/L.1 (adopted 25 September 2015).

Ura, K., Alkire, S., Zangmo, T. and Wangdi, K. (2012). *An Extensive Analysis of the Gross National Happiness Index.* Thimphu: Centre of Bhutan Studies, Royal Government of Bhutan, www.grossnationalhappiness.com (last accessed: 23 March 2016).

World Bank (2017). *Monitoring Global Poverty: Report of the Commission on Global Poverty.* Washington, DC: World Bank.

Zuckerman, M., Faliszewski, P., Bachrach, Y. and Elkind, E. (2012). 'Manipulating the quota in weighted voting games', *Artificial Intelligence*, vol. 180−181, pp. 1−19.

Appendix

Proof of Th. 1: Verifications of the Basic Axioms are elementary and are left to the reader. To establish Ordinality for $\gamma \geq 0$, suppose that $(y'; z')$ is obtained from $(y; z)$ as an equivalent representation. Then $(y'; z')$ and $(y; z)$ have the same deprivation matrix and hence the same deprivation counts for all *i*, from which it follows that $\rho_k({y'}_i; z') = \rho_k(y_i; z)$, and hence the same persons are poor in both. This ensures that $M_0^0(y'; z') = M_0^0(y; z)$. It also implies that each person's censored deprivation score is the same in both, which means that $M_0^\gamma(y'; z') = M_0^\gamma(y; z)$ for every $\gamma > 0$. Thus, M_0^γ satisfies Ordinality for $\gamma \geq 0$.

To verify Dimensional Monotonicity for $\gamma > 0$, suppose that y' is obtained from y by a dimensional decrement among the poor, so that a poor person *i* has gained a deprivation and the rest of the population is unaffected. Then $c(k)' > c(k)$ and hence $c^\gamma(k)' > c^\gamma(k)$ for $\gamma > 0$, which implies that $M_0^\gamma(y'; z') > M_0^\gamma(y; z)$. Thus M_0^γ satisfies Dimensional Monotonicity for $\gamma > 0$.

To verify Dimensional Transfer for $\gamma > 1$, suppose that y' is obtained from y by a dimensional rearrangement among the poor, so that (a) – (d) hold for poor persons *u* and *i*. We want to show that $M_0^\gamma(y; z) > M_0^\gamma(y'; z)$ or, equivalently, that $|c^\gamma(k)| > |c^\gamma(k)'|$ for $\gamma > 1$. By definition $c(k)$ and $c(k)'$ differ only in coordinates *u* and *i*, and so this condition reduces to $c_u^\gamma(k) + c_i^\gamma(k) > c_u^\gamma(k)' + c_i^\gamma(k)'$ or

$$(c_u)^\gamma + (c_i)^\gamma > (c'_u)^\gamma + (c'_i)^\gamma \qquad \text{(A1)}$$

since *u* and *i* are poor. But notice that $c_u + c_i = c'_u + c'_i$ since a rearrangement does not alter the total deprivations in the population. Moreover, the vector dominance of initial deprivation vectors given in (d) ensures that $c_u > c_i$, while the subsequent absence of vector dominance ensures that both c'_u and c'_i lie strictly between c_u and c_i: the final pair of deprivation scores (c'_u, c'_i) can be obtained from the initial pair (c_u, c_i) by a mean preserving transfer of scores from *u* to *i*. Condition (A1) follows immediately from standard convexity results and the fact that $\gamma > 1$. Thus, M_0^γ satisfies Dimensional Transfer for $\gamma > 1$.

Finally, turning to Weak Rearrangement for $\gamma \geq 1$, suppose that y' is obtained from y by an association-decreasing rearrangement among the poor, so that (a) – (c) hold for persons *u* and *i*. We want to show that $M_0^\gamma(y;z) \geq M_0^\gamma(y';z)$ or equivalently that

$$(c_u)^\gamma + (c_i)^\gamma \geq (c'_u)^\gamma + (c'_i)^\gamma. \qquad \text{(A2)}$$

If (d) also happens to hold, then condition (A2) follows from standard convexity results and the fact that $\gamma \geq 1$. Alternatively, one or more of the inequalities in (d) could fail to hold, in which case it can be shown that (A2) must be true with equality. Indeed, if inequality $g_u^0 > g_i^0$ fails to hold, then by (c) the only other possibility is $g_u^0 = g_i^0$, which implies that $g_u^{0\prime} = g_u^0$ and $g_i^{0\prime} = g_i^0$ and so (A2) is true with equality. If $g_u^0 > g_i^0$ holds but after the rearrangement $g_i^{0\prime} > g_u^{0\prime}$ also holds, then we must have $g_u^{0\prime} = g_i^0$ and $g_i^{0\prime} = g_u^0$, which implies that (A2) is true with equality. If $g_u^0 > g_i^0$ holds but after the rearrangement $g_u^{0\prime} > g_i^{0\prime}$ holds, then $g_u^{0\prime} = g_u^0$ and $g_i^{0\prime} = g_i^0$ which also implies that (A2) is true with equality. Thus, M_0^γ satisfies Weak Rearrangement for $\gamma \geq 1$. ■

Proof of Theorem 2: Suppose that $\mathcal{M}$ satisfies Symmetry and Dimensional Breakdown and let y' be obtained from y by a dimensional rearrangement among the poor. By Symmetry, without loss of generality, we can assume that 1 and 2 are the poor persons involved in the rearrangement and that $y_1 > y_2$. Now let J be the set of all dimensions *j* that are unchanged in the rearrangement, so that both $y'_{1j} = y_{1j}$ and $y'_{2j} = y_{2j}$ hold. Let x be the achievement matrix obtained from y by lowering person 1's achievement level in each $j \in J$ to person 2's achievement level, leaving all remaining entries unchanged. Similarly construct x' from y' by lowering person 1's achievement level in each $j \in J$ to that of person 2. Person 1 remains poor in both x and x', and so all four achievement matrices have the same poverty status vector r. By Dimensional Breakdown, then, there exist $v_j > 0$ summing to one and normalized component functions $m_j: Y_{rj} \times R_{++}^d \to R_+$ for $j = 1, \dots, d$, such that expression (3) holds for all matrices in Y_r. Applying (3) to y and y' yields

$$M(y';\ z) - M(y;\ z) \ = \sum_{j \notin J} v_j [m_j\left(y'_{.j}; z_j\right) - m_j\left(y_{.j}; z_j\right)]$$

since y' and y are the same in all dimensions of *J*. Applying (3) to x' and x yields

$$M(x';\ z) - M(x;\ z) \ = \sum_{j \notin J} v_j [m_j(x'_{.j}; z_j) - m_j(x_{.j}; z_j)]$$

for the analogous reason. By construction, it follows that $y'_{.j} = x'_{.j}$ and $y_{.j} = x_{.j}$ for all $j \in J$ and hence that $M(y'; z) - M(y;\ z) = M(x'; z) - M(x; z)$. However, x' is simply a permutation of x (between persons 1 and 2) and so by Symmetry we have $M(x'; z) - M(x; z) = 0$. This implies that $M(y'; z) = M(y; z)$ and thus Dimensional Transfer is violated. ■

Proof of Lemma 1: In a one-person society y the squared count measure $M_0^2(y; z)$ is simply $c_1^2(k)$, or the square of the person's score $c_1(k)$, where

$$c_1(k) = w_1 g_{11}^0(k) + \cdots + w_d g_{1d}^0(k) = w_1 H_1 + \cdots + w_d H_d = |b|$$

since $g_{1j}^0(k) = H_j$ in the case $n = 1$. It follows that $M_0^2(y; z)$ can also be written as $|b|^2$ or the square of the sum of the entries in b, the dimensional breakdown vector for $M_0^1(y; z)$.

Now let Π be the set of all permutations of d dimensions. A typical permutation $\sigma \epsilon \Pi$ indicates the order in which dimensions will have their deprivations suppressed when computing the Shapley average marginal impact on the squared count measure; in particular, $\sigma_1 \epsilon \{1, \ldots, d\}$ denotes the first dimension to have its deprivations suppressed, $\sigma_2 \epsilon \{1, \ldots, d\}$ is the second, and so on. Let $c_1^2(T) = \left(\sum_{j \epsilon T} b_j\right)^2$ denote the squared count when all dimensions outside of *T* have had their deprivations suppressed. For any dimension j', define $T(j', \sigma)$ to be the set of all dimensions that follow or appear to the right of j' in σ and hence remain intact after dimension j' has had its deprivations suppressed in permutation σ. Then the dimension j' term in the Shapley breakdown of $M_0^2(y; z)$ is defined by

$$\phi_{j'} = \frac{1}{d!} \sum_{\sigma \epsilon \Pi} \ [c_1^2(T(j', \sigma) \cup \{j'\}) - c_1^2(T(j', \sigma))]$$

Clearly,

$$c_1^2(T(j',\sigma)\cup\{j'\}) - c_1^2(T(j',\sigma)) = \left(\sum_{j\epsilon T(j',\sigma)\cup\{j'\}} b_j\right)^2 - \left(\sum_{j\epsilon T(j',\sigma)} b_j\right)^2$$

$$= b_{j'}^2 + 2b_{j'}\sum_{j\epsilon T(j',\sigma)} b_j$$

Therefore,

$$\phi_{j'} = b_{j'}^2 + \frac{2}{d!} b_{j'} \sum_{\sigma\epsilon\Pi} \sum_{j\epsilon T(j',\sigma)} b_j = b_{j'}^2 + \frac{2}{d!} b_{j'} \sum_{j\neq j'} a_j b_j$$

where a_j is an integer indicating the number of times b_j appears in the sum $\sum_{\sigma\epsilon\Pi}\sum_{j\epsilon T(j',\sigma)} b_j$ or, equivalently, the number of permutations σ for which $j\epsilon T(j',\sigma)$. But the number of permutations where j follows j' equals the number where j precedes j', and hence $a_j = \frac{d!}{2}$ for every $j \neq j'$. It follows that

$$\phi_{j'} = b_{j'}^2 + b_{j'}\sum_{j\neq j'} b_j = b_{j'}|b|$$

and so $\phi = b|b|$, as claimed. ■

Proof of Theorem 3: Since M_0^2 satisfies Subgroup Decomposability, we know that $M_0^2(y;z) = \frac{1}{n}\sum_{i=1}^n M_0^2(y_i;z)$. Let ϕ denote the Shapley breakdown vector for $M_0^2(y;z)$ and let ϕ^i be the Shapley breakdown vector for $M_0^2(y_i;z)$ where $i = 1,\dots,n$. By the linearity property of the Shapley value, $\phi = \frac{1}{n}\sum_{i=1}^n \phi^i$. By Lemma 1, the *j*th component of ϕ^i is given by $\phi_j^i = w_j g_{ij}^0(k) c_i(k)$ and hence

$$\phi_j = w_j \frac{q_j}{n} \frac{\sum_{i=1}^n g_{ij}^0(k) c_i(k)}{q_j} = w_j H_j A_j = w_j M_{0j}$$

as asserted. ■

Table 1. MPI and Censored Headcount Ratios for Cameroon and its Regions

		Education		*Health*		*Living Standards*					
	MPI (M_0)	Schooling	School Attendance	Child Mortality	Nutrition	Electricity	Sanitation	Water	Housing	Cooking Fuel	Assets
Cameroon	**24.3**	**23.4**	**17.6**	**9.8**	**24.2**	**36.9**	**40.2**	**28.8**	**39.0**	**44.6**	**22.8**
Yaoundé	01.5	1.4	0.4	1.2	2.3	0.4	3.4	1.5	1.2	2.3	1.6
Douala	02.2	2.1	2.0	1.5	3.4	0.5	4.7	0.9	1.6	4.0	1.1
Littoral (sans Douala)	07.8	4.3	0.8	3.7	9.1	12.3	15.3	14.8	15.7	18.6	10.2
Sud	12.8	3.2	4.5	8.4	19.0	15.0	27.8	19.8	21.7	30.0	10.4
Sud-Ouest	13.7	5.1	4.1	6.8	14.4	24.7	28.2	25.9	28.7	31.3	16.8
Ouest	13.9	12.3	2.0	3.2	16.0	18.3	30.5	24.9	24.6	33.0	18.7
Centre (sans Yaoundé)	14.4	6.0	4.2	7.4	18.5	24.4	28.2	25.0	25.7	33.1	14.5
Nord-Ouest	17.4	7.7	3.9	3.7	23.4	32.9	36.1	24.0	35.7	39.6	29.0
Est	30.7	28.0	19.6	14.3	32.2	43.3	49.3	42.1	45.5	56.9	33.2
Adamaoua	31.5	41.3	27.0	11.8	30.0	44.9	40.7	33.3	35.5	57.5	25.8
Nord	42.3	43.5	37.2	21.3	37.7	63.1	60.6	48.7	65.8	72.2	32.8
Extrême-Nord	48.0	53.7	43.0	17.0	43.6	74.5	76.6	45.6	76.4	79.4	40.1

Table 2. *M*-gamma Measures, Intensities, and Inequalities for Cameroon and Its Regions

	M_0^0	M_0^1	M_0^2	A	A'	V	V_p
Cameroon	**45.3**	**24.3**	**14.2**	**53.6**	**31.3**	**0.083**	**0.026**
Yaoundé	3.9	1.5	0.6	37.7	14.5	0.005	0.003
Douala	5.5	2.2	0.9	40.6	17.0	0.009	0.005
Littoral (sans Douala)	19.1	7.8	3.3	40.9	17.4	0.027	0.006
Sud	30.1	12.8	5.7	42.4	19.0	0.041	0.010
Sud-Ouest	32.0	13.7	**6.2**	**42.8**	**19.3**	**0.043**	**0.010**
Ouest	33.2	13.9	**6.1**	**42.0**	**18.5**	**0.042**	**0.009**
Centre (sans Yaoundé)	33.4	14.4	6.5	43.1	19.5	0.045	0.010
Nord-Ouest	39.6	17.4	8.1	43.9	20.4	0.050	0.011
Est	57.7	30.7	17.7	53.2	30.6	0.083	0.023
Adamaoua	58.7	31.5	18.2	53.7	31.0	0.083	0.022
Nord	73.0	42.3	26.4	58.0	36.1	0.084	0.024
Extrême-Nord	80.0	48.0	30.9	60.0	38.6	0.079	0.026

Table 3. Distribution of Deprivation Scores of the Poor by Intensity Bands

	33–39%	40–49%	50–59%	60–69%	70–79%	80–89%	90–100%
Cameroon	26%	17%	20%	20%	9%	6%	2%
Yaoundé	69%	12%	19%	0%	0%	0%	0%
Douala	82%	18%	0%	0%	0%	0%	0%
Littoral	44%	18%	22%	11%	0%	4%	0%
Sud-Ouest	49%	21%	18%	12%	0%	0%	0%
Sud	57%	20%	17%	4%	3%	0%	0%
Ouest	62%	14%	17%	7%	0%	0%	0%
Nord-Ouest	47%	23%	23%	7%	0%	0%	0%
Centre	55%	20%	16%	7%	0%	2%	0%
Est	39%	9%	28%	16%	4%	2%	1%
Adamaoua	26%	22%	21%	16%	6%	7%	1%
Nord	16%	15%	20%	25%	11%	9%	3%
Extrême-Nord	14%	12%	15%	24%	19%	9%	5%

Table 4. Breakdowns for M_0^1 and M_0^2

Indicator	Censored Headcount Ratio H_j	Dimensional Breakdown $w_j H_J$	Censored Intensity A_j	Censored Adjusted Headcount M_{0j}^1	Shapley Breakdown $w_j M_{0j}^1$	Relative Contribution $w_j H_j / M_0^1$	Relative Contribution $w_j M_{0j}^1 / M_0^2$	Percentage Point Diff. Δ
Years of Schooling	23.4	3.9	61.5	14.4	2.4	16.1%	16.9%	-0.8
School Attendance	17.6	2.9	65.1	11.4	1.9	12.1%	13.4%	-1.4
Child Mortality	9.8	1.6	63.9	6.3	1.0	6.7%	7.4%	-0.6
Nutrition	24.2	4.0	58.9	14.3	2.4	16.6%	16.7%	-0.1
Electricity	36.9	2.1	56.0	20.7	1.1	8.4%	8.1%	0.3
Sanitation	40.2	2.2	54.4	21.9	1.2	9.2%	8.6%	0.6
Water	28.8	1.6	54.5	15.7	0.9	6.6%	6.1%	0.4
Flooring	39.0	2.2	55.1	21.5	1.2	8.9%	8.4%	0.5
Cooking Fuel	44.6	2.5	53.8	24.0	1.3	10.2%	9.4%	0.8
Assets	22.8	1.3	55.5	12.6	0.7	5.2%	4.9%	0.3
		M_0^1 =24.3			**M_0^2 =14.2**			