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Polarization and the Decline of the Middle Class: Canada and the US

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Abstract

Several recent studies have suggested that the distribution of income (earnings, jobs) is becoming more polarized. Much of the evidence presented in support of this view consists of demonstrating that the population share in an arbitrarily chosen middle income class has fallen. However, such evidence can be criticized as being range-specific – depending on the particular cutoffs selected. In this paper we propose a range-free approach to measuring the middle class and polarization, based on partial orderings. The approach yields two polarization curves which, like the Lorenz curve in inequality analysis, signal unambiguous increases in polarization. It also leads to an intuitive new index of polarization that is shown to be closely related to the Gini coefficient. We apply the new methodology to income and earnings data from the US and Canada, and find that polarization is on the rise in the US but is stable or declining in Canada. A cross-country comparison reveals the US to be unambiguously more polarized than Canada.

Keywords: Income distribution, Inequality, Lorenz curve, Gini coefficient, North America

JEL classification: D32, D63, I3

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Section A: Introduction

The presence of a sizable, well-off middle class is typically presumed to be an important factor in the growth and development of today's successful industrial economies. The middle class provides much of the labour force for the economy and is a key market for the national product. A large portion of a country's tax revenue is collected directly or indirectly from the middle class. It also has a special role in the relative political stability these nations have enjoyed. According to Lester Thurow (1984), 'A healthy middle class is necessary to have a healthy political democracy. A society made up of rich and poor has no mediating group either politically or economically.'

In the mid 1980s several researchers noticed a disturbing trend in the United States: the size and perhaps the relative affluence of the middle class appeared to be declining. Thurow was one of the first to point out the increased polarization of the income distribution. Defining the middle class as those with incomes between 75 and 125 per cent of the median income, he concluded that the percentage of middle class households fell from 28.2 per cent in 1967 to 23.7 per cent in 1983. Blackburn and Bloom (1985) broadened the middle income range to 60 to 225 per cent of the median and found a decrease from 62.4 per cent to 55.9 per cent over the same period. Other studies employing a variety of different definitions and data bases broadly confirm these findings (e.g. Bluestone and Harrison 1986, 1988; Bradbury 1986; Horrigan and Haugen 1988). A number of papers offer evidence to the contrary (e.g. Kosters and Ross 1988; Levy 1987; Rosenthal 1986; Levitan and Carlson 1984). In particular, Levy (1987) has noted that the share of income going to the middle 3/5 families in the US has stayed remarkably constant since 1945, at 52 to 54 per cent of the total. While the various studies on the US experience do not entirely agree on the extent or even the existence of increased polarization, the general topic is proving to be an important one for research.[1]

The condition of the Canadian middle class has only recently been investigated by researchers, primarily at the Analytical Studies Branch of Statistics Canada. One study by Myles et al. (1988) examined changes in the distribution of jobs between the years 1981 and 1986 using two unique surveys conducted by Statistics Canada. They found a slight decline in the middle class over this particular five year period. Leckie (1988) observed a similar modest decline in the middle class between 1971 and 1981, where the middle income range is defined as 85 to 115 per cent of the median wage. In a recent paper, Wolfson (1989) offered some preliminary findings on income polarization in Canada. He noted that the share of labour incomes in the range 75 to 150 per cent of the median declined steadily from 39.3 per cent in 1967 to 30.8 per cent in 1986. He also examined the share of income received by the middle fifth of the population and found a general decline amidst ups and downs. The results to date suggest that increased polarization may also be taking place in Canada.

In this work, little emphasis has been placed on the underlying methodology of measuring the middle class and the degree of polarization. The purpose of this paper is to provide a methodological base with which to analyze polarization. To do this, we begin with the question: are there certain stylized changes in the distribution that should be regarded as increasing polarization? We identify two such movements which we call 'increased spread' and 'increased bipolarity'. It is argued that any method of measuring polarization should be consistent with these basic elements of polarization.

We begin with the measurement of the middle class. The studies referred to above reveal a variety of incompatible definitions and measurement strategies and, naturally, there has been considerable potential for confusion and conflicting results. We provide a common framework within which the various approaches to defining and measuring the middle class may be compared. We note a potentially serious

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[1] See also Beach (1989), Harrison and Bluestone (1988), Levy and Murnane (1990), Maxwell (1990), and Winnick (1989).

problem in each of these studies: the range defining the middle class is essentially arbitrary. For example, why use a range of 75 to 125 per cent of the median income (as Thurow did) rather than 60 to 225 per cent? Alternatively, why focus on the middle three-fifths of the population (as Levy did) instead of the middle fifth?[2] If one range yields results that may be reversed at another reasonable range, then conclusions obtained at the original range are hardly trustworthy. When faced with a similar problem in poverty measurement, Foster and Shorrocks (1988) noted how allowing the cutoff point to vary leads to a useful graphical technique by which unambiguous rankings may be made. (This approach is quite similar to the Lorenz curve, used in ranking the distributions with respect to inequality.) The difference here is that there are two arbitrary cutoff points used in defining the middle class: the upper and lower levels of income. Allowing them to vary yields a curve which indicates when the cutoffs are irrelevant: when one curve is above another it means that no matter what cutoffs are chosen (as long as the upper cutoff is above the median and the lower is below) the first has a larger middle class than the second.

We apply a similar approach to obtain two polarization curves indicating when one distribution has unambiguously more polarization than another. The first curve is related to the 'increased spread' aspect of polarization. It ranks one distribution above the other in terms of polarization when, no matter what range of families is chosen around the median family, the range of incomes (or 'spread') necessary to capture all the families is larger. We show that this ranking method gives precisely the same answers as the above strategy for measuring the middle class: one distribution has unambiguously more polarization exactly when it has an unambiguously smaller middle class. Our second polarization curve incorporates the 'increased bipolarity' aspect of polarization as well. It is based on a notion of 'average distance from the median income', ranking one distribution above the other in terms of polarization when this average distance is higher for every range of families about the median.

In many analyses it is helpful to summarize all the data into a single numerical index, and so we next turn to the construction of an index of polarization. We opt for an intuitive measure based on the second polarization curve in a similar way that the Gini coefficient is based on the Lorenz curve. We demonstrate that the resulting formula has two nice interpretations. First, it can be depicted in terms of the Lorenz curve as (twice) the area beneath the curve above the tangent to the curve at the median family, renormalized to the median instead of the mean. Our polarization measure is thus seen to be a natural companion of the Gini measure of inequality. Second, we show that the polarization index may be expressed as a function of the 'between-group' inequality minus the 'within-group' inequality as measured by the Gini coefficient (where the two groups are families above and below the median). Inequality and polarization move together when the inequality between these groups rises; they move in opposite directions when there is more inequality within the groups.

To illustrate our methods, we provide a preliminary analysis of polarization in Canada and the US. Using data from the Luxembourg Income Study, the Survey of Consumer Finances in Canada, and the Current Population Survey in the US, we analyze how polarization and inequality changed during the 1980s and offer a cross-country comparison for the year 1988. Our results reveal that polarization and inequality have remained virtually unchanged in Canada, while at the same time both have been on the rise in the US Moreover, by 1988, there was an unambiguous ranking between the two countries: the US clearly has greater polarization and inequality than Canada.

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[2] This point was emphasized in the interesting paper of Horrigan and Haugen (1988).

Section B: Elements of Polarization

Our approach to measuring polarization will be based on techniques used in measuring the middle class. Before proceeding, though, we will identify certain stylized distributional changes leading to clear-cut increases in polarization. The first is an unambiguous movement away from the middle, as depicted in Figure B1, which may be termed increased spread. This is the classic example of the rich becoming richer and the poor becoming poorer, resulting in a distribution with a greater gap between the two groups.[3] Alternatively, the new distribution is obtained from the initial one by means of a mean-preserving regressive transfer (called a mean-preserving spread in the literature on risk) taking place across the middle. The restriction on the location of the transfer is crucial. Transfers across the middle, such as the one depicted in Figure B1, shift the population away from the middle and consequently lead to increased polarization via increased spread. Transfers on one side of the middle, from the rich to the very rich or from the very poor to the poor, move one group away from, and another towards, the middle. This possibility is not covered by our notion of increased spread and may have a very different effect on polarization.

Figure B1: Increased spread
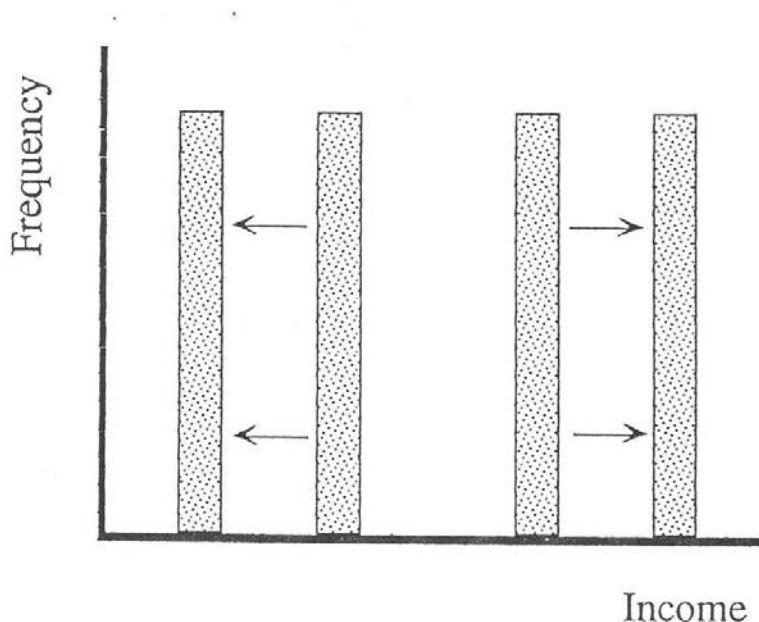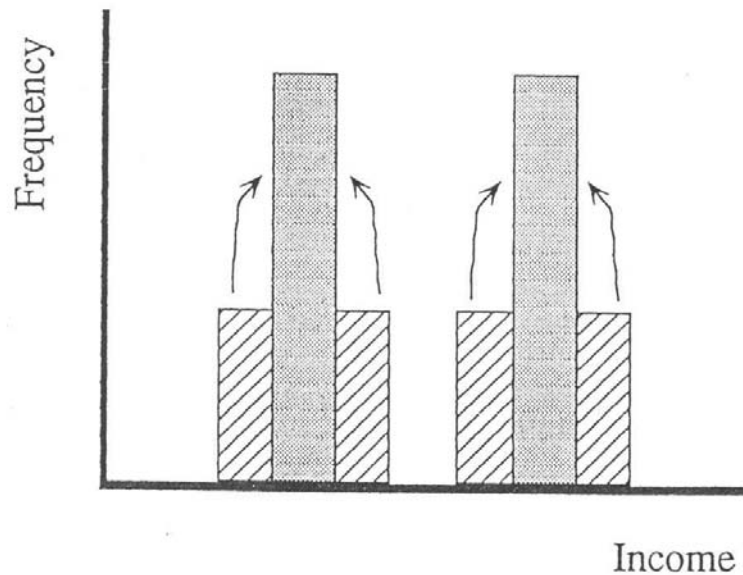

Figure B2: Increased bipolarity

Figure B2 illustrates the second polarization-increasing change in distributions, which may be termed increased bipolarity. As in the first diagram, both distributions have two masses of individuals. But here it is not the relative positions of the masses that are changed – in fact the center of each mass is unaltered – it is the distribution around each center that is tightened up. When the spread is unchanged and the poles are better-defined, as with the second distribution in Figure B2, we shall say that polarization has increased via increased bipolarity. Intuitively, a change of this type involves movements away from the middle of the overall distribution by those nearer to the middle and a concurrent, equal-sized shift towards the middle by those who are further away. According to the increased-spread criterion, these two changes should work against one another, with the spread serving to increase polarization and the 'concentration' decreasing it. The bipolarity criterion decides the issue in favour of the (nearer-to-the-middle) spread, by positing that their combined effect is to raise overall polarization. This criterion implicitly places more weight on changes in the distribution occurring nearer the middle.

One immediate consequence of this approach is that the distinction between inequality and polarization, earlier noted by Love and Wolfson (1976), may be more clearly identified. Any regressive transfer (or mean preserving spread) leads to an unambiguous increase in inequality, irrespective of the location of the transfer. Thus, polarization and inequality move in the same direction when the transfer takes place across the middle: increased spread leads to greater inequality in Figure B1. However, increased bipolarity is associated with a pair of progressive transfers, one on each side of the middle, which necessarily diminish inequality. Polarization and inequality move in opposite directions when same-side transfers occur, as depicted in Figure B2. This distinction will be discussed at greater length below.[4]

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[4] Love and Wolfson (1976) were first to point out the distinction between inequality and polarization. Recent results by Amiel and Cowell (1992) suggest that the average individual's initiative notion of 'inequality' may be better represented by what we call polarization than by the economist's definition of inequality. Their international survey found that an overwhelming majority of college students polled considered a progressive transfer wholly on one side of the median to raise 'inequality' rather than lower it (as the Pigou-Dalton transfer axiom would require).

Section C: Measuring the Middle

As noted in the introduction, the most common proxy for increased polarization is a declining middle class. Various approaches have been used in measuring the middle-income group; in this section we will identify the common framework underlying them. The definitions of Thurow (1984), Blackburn and Bloom (1985), and Levy (1987) will be re-examined within this framework. One will be dismissed as a measure of the middle, while the fundamental arbitrariness in the remaining two will be noted. We then derive a useful partial ordering for measuring the middle that is related to the criterion of increased spread.

Framework

Most attempts at 'measuring the middle' may be broken down into four distinct steps: (i) choosing the 'space', (ii) defining the middle, (iii) fixing the range, and (iv) aggregating the data. The first, and most fundamental is the choice of the 'space' within which the middle class is to be identified. The most common choice is 'income space', where income is monthly salary, yearly expenditure, or some other single dimensional indicator of welfare. Sometimes the middle class is defined in 'people-space' as exemplified by the approach used by Levy.

Once this initial step is decided, the second step is to fix an appropriate definition of the middle. Most studies have opted for the median income or, in people space, the individual at the 50th percentile. Another possibility is the mean income, although the fact that less than half of the population typically achieves this standard lessens its intuitive appeal. The third step is to select a range around the middle, thus identifying the middle class. The final step is to aggregate the data on the middle class into an overall index reflecting its relative magnitude in some dimension of interest.

Examples

The approaches of Thurow (1984), Blackburn and Bloom (1985) and Levy (1987) all conform to this general framework. Let us begin with the first two, which work within income space. Both choose the median income m as the middle, select a range whose upper and lower cutoffs are percentages of the median, and use as their index M the proportion of the population falling within that range. Figures C1 and C2 depict their method of measuring the middle. In terms of the density function f for the income distribution, M is simply the area below f between the lower and upper limits, or the shaded region in Figure C1. Alternatively, M may be viewed as the vertical distance between the values of the cumulative distribution function (or cdf) F evaluated at the two cutoff points, as depicted in Figure C2. (Note that the median income m is located where F achieves the value 0.5.) The same basic diagrams apply to both papers, with the only difference being the particular ranges chosen.

Figure C1: Finding *M* for a density
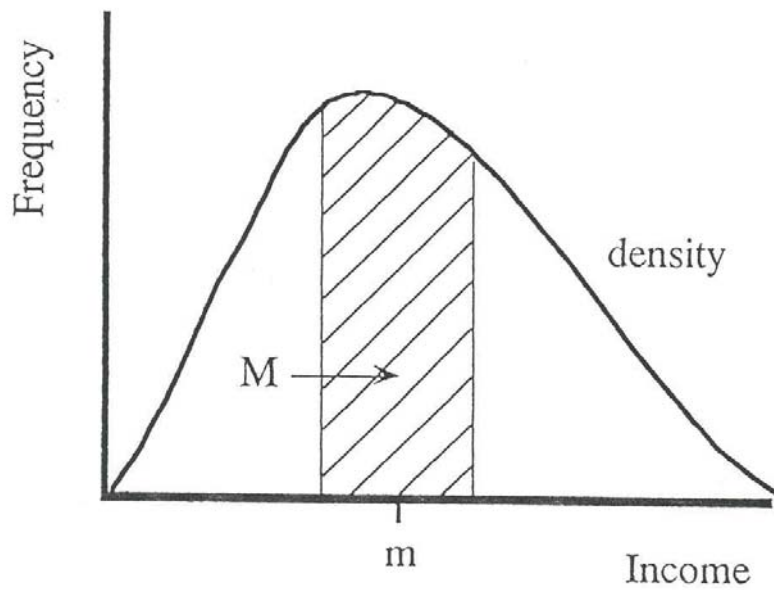
Figure C2: Finding *M* for a cumulative distribution function
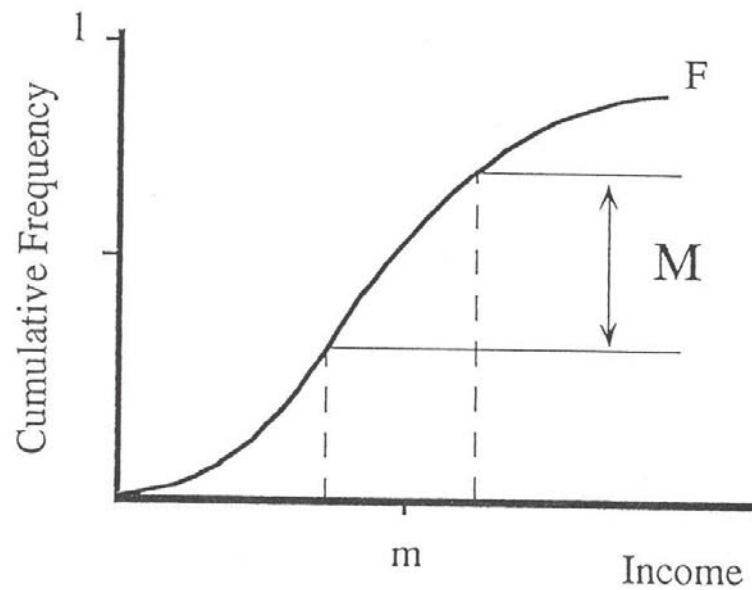
In contrast, Levy (1987) adopts a substantially different approach to measuring the middle. To begin with, the middle class is defined in people space rather than income space. The middle is taken to be the 50th percentile and a range from the 20th to the 80th percentile is identified as the middle class. Of course, it would be fruitless to use the measure *M* in this context. Consequently, an alternative indicator – the share of income received by the middle class – is used.

Figure C3: Levy's measure
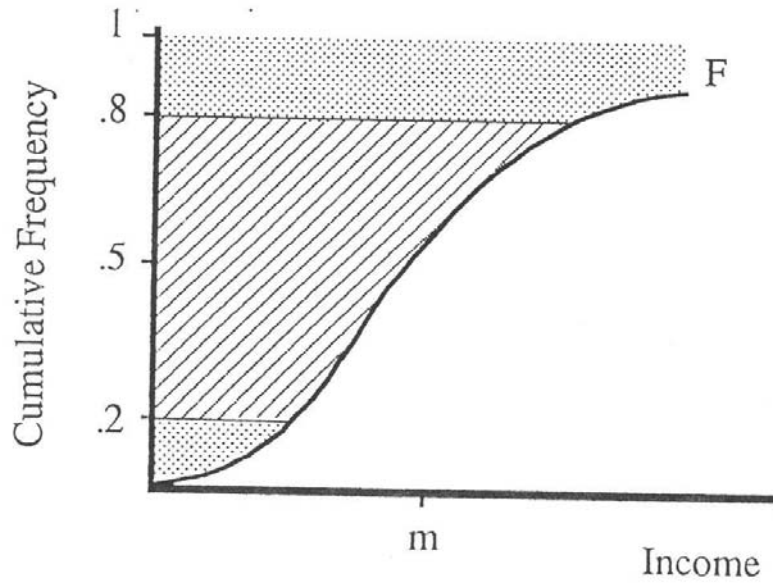
Figure C4: Lorenz curve and Levy's measure
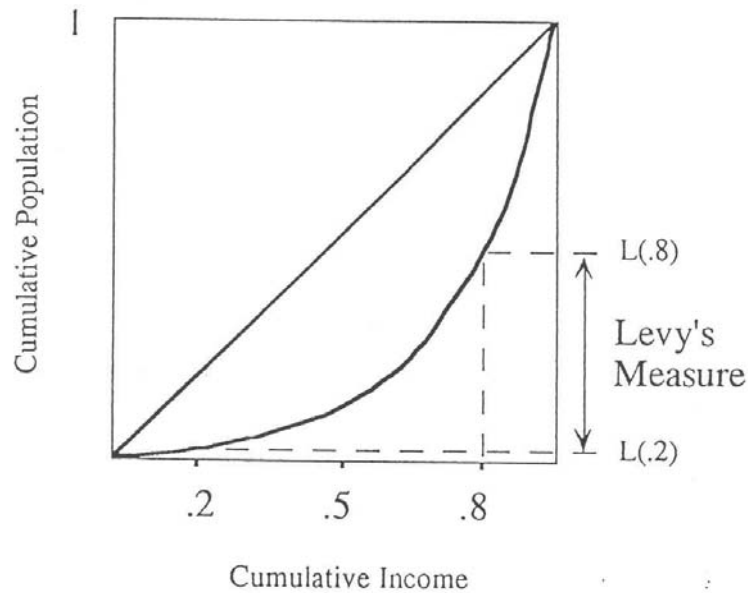
This method of measuring the middle also admits a simple graphical representation. Let F^{-1} denote the inverse of the cumulative distribution function F, so that $F^{-1}(p)$ is the income of the person at the pth percentile. The mean of F is $\mu = \int_0^1 F^{-1}(p)dp$, depicted as the shaded and crosshatched area in Figure C3. Levy's measure is $\int_{0.2}^{0.8} F^{-1}(p)dp/\mu$ or the ratio of the crosshatched area to the total shaded and crosshatched area in the diagram. An alternative representation in terms of the Lorenz curve L of the

distribution is given in Figure C4. Recall that for every p between 0 and 1, the Lorenz value $L(p)$ is the share of income received by the poorest p of the population: $L(p) = \int_0^p F^{-1}(q)dq/\mu$. Levy's measure is $L(0.8) - L(0.2)$, the difference between the Lorenz values at the 80th and 20th percentiles.

Critique

At first glance, Levy's approach would appear to be a reasonable method of measuring the middle class. A closer examination, however, reveals certain major drawbacks. Consider a simple example of a uniform distribution between $10,000 and $30,000 having median m = $20,000. The middle three quintiles extend from $14,000 to $26,000, and since there are just as many persons with income $m - \varepsilon$ as $m + \varepsilon$, the middle 60 per cent receives 60 per cent of the overall income. Now suppose that the distribution spreads out to become uniform between $0 and $40,000, as would result if those having an income of $m - \varepsilon$ gave a transfer of $\varepsilon > 0$ to those with $m + \varepsilon$. This is a clear migration away from the middle that reduces the size of the middle class according to Thurow or Blackburn and Bloom. However, Levy's measure indicates <u>no</u> <u>change</u> in the middle class: the middle 60 per cent still receives 60 per cent, by the same argument as before. Indeed, <u>any</u> symmetric distribution will have the same 'size' of the middle class using Levy's approach, irrespective of whether the incomes range widely or fall within one dollar of the median income. The fact that the income range necessary to capture 60 per cent of the population may have to vary extensively (from $12,000 in the initial uniform distribution to $24,000 in the second) is totally ignored. Clearly this approach misses out on an important aspect of the distribution: its <u>spread</u>.

Now what does Levy's index actually measure? We will show that it is more a measure of the <u>skewness</u> of the distribution than a measure of the middle class. For any $p < 0.5$, let $D(p)$ denote the difference between the slope of the 45° line of equality and the slope of the chord running from $L(p)$ to $L(1 - p)$, as depicted in Figure C5. Notice that as p tends to 0.5, the slope of the chord tends to the slope of the tangent to the Lorenz curve at 0.5. This slope, in turn, is simply m/μ, and so $\lim_{p \to 0.5} D(p) = (\mu - m)/\mu$, a natural measure of skewness. As an approximation of this limiting formula, $D(p)$ is also a measure of skewness in its own right: Clearly, $D(p) = 1 - [L(1 - p) - L(p)]/(1 - 2p)$ or in terms of $V(q) = q - L(q)$, the vertical shortfall of the Lorenz curve from the diagonal, $D(p) = [V(1 - p) - V(p)]/(1 - 2p)$. For any symmetric distribution, the shortfalls are symmetric about $p = 0.5$, or equivalently, $V(q) = V(1 - q)$. Consequently, in the symmetric case, $D(p) = 0$ and the slope of the chord is 1. However, if the distribution is positively skewed, as is typically the case with empirical income distributions, the upper shortfall will exceed the lower shortfall (as in Figure C5), and $D(p)$ will then be positive. The degree of skewness, as measured by $D(p)$, increases as the shortfall discrepancy $V(1 - p) - V(p)$ increases and as the Lorenz ordinates $L(p)$ and $L(1 - p)$ move closer together.

Figure C5: A measure of skewness
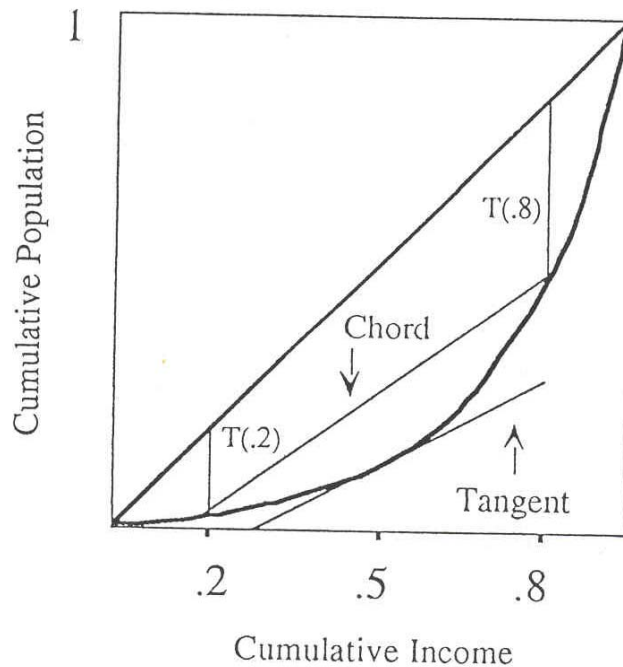
These observations offer a clearer understanding of Levy's measure and his empirical results. For symmetric distributions, Levy's measure $L(0.2) - L(0.8)$ takes a value of 0.6. The observed value of 0.53 suggests that the distribution is positively skewed. The associated chord has a slope of 53/60, which is less than the slope of the diagonal, and the Lorenz curve is closer to the diagonal at the 20th percentile than it is at the 80th. Moreover, the relative constancy of Levy's index indicates that skewness as measured by $D(0.2) = 7/60$ was basically unchanged over the reported time period. These results are interesting, but have little to say about the middle class.[5]

A Partial Ordering

The approach of Thurow and Blackburn-Bloom is certainly more compelling. Even so, there remains a certain arbitrariness in the method they employ. Thurow chooses a middle-class range of 75 per cent to 125 per cent of the median; Blackburn and Bloom select cut-off points at 60 per cent to 225 per cent of the median. As the choice of a range is arbitrary, any number of alternatives would be equally justifiable. If one of these alternative ranges leads to contradictory results, this would surely weaken their range-specific evidence. On the other hand, if every range about the median yields the same direction of change, the robustness of their conclusions could not be challenged along these lines. In this section we determine whether and when such an unambiguous determination can be made.

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[5] In private communications, Levy has indicated that he no longer uses this approach to 'measuring the middle', and instead is pursuing an alternative method based on the generalized Lorenz curve. See Levy and Murnane (1990) for related discussions.

We begin with some definitions. Let $R=(\underline{z},\bar{z})$ be an interval whose endpoints satisfy $0 \leq \underline{z} \leq 1 \leq \bar{z}$. Given an income distribution F with median m_F, we may use R to represent the middle-income range whose lower cutoff is $\underline{z}m_F \leq m_F$ and whose upper cutoff is $\bar{z}m_F \geq m_F$. The index $M(F;R)=F(\bar{z}m_F)-F(\underline{z}m_F)$ gives the share of the population in the range defined by R. To see whether one distribution, F, has a larger middle class than another, G, for the range R, we simply compare $M(F;R)$ and $M(G;R)$. Note that since m_F and m_G may be different, the absolute cutoff points will in general be different; it is the cutoffs as a proportion of the median that are fixed in the comparison.

Our motivating question is whether it is possible for F to have a larger middle class than G, irrespective of which range R is actually chosen. If this happens we shall say that F has an <u>unambiguously larger</u> middle class than G, written $F\,M\,G$. More precisely, $F\,M\,G$ if and only if $M(F;R) \geq M(G;R)$ for all R, with $M[F;R] > M[G;R]$ for some R. Figure C6 depicts an example where $F\,M\,G$ holds. It is clear that the requirements for such an unambiguous comparison are quite strong, so one would expect the resulting ranking M over distribution to be highly incomplete: for many pairs of distributions, neither $F\,M\,G$ nor $G\,M\,F$ is true. Because of this, M is said to be a <u>partial ordering</u>.
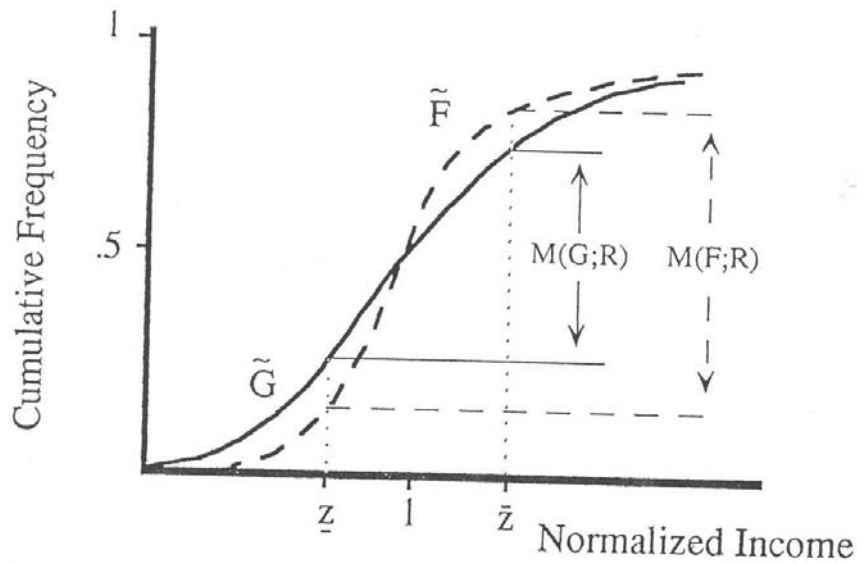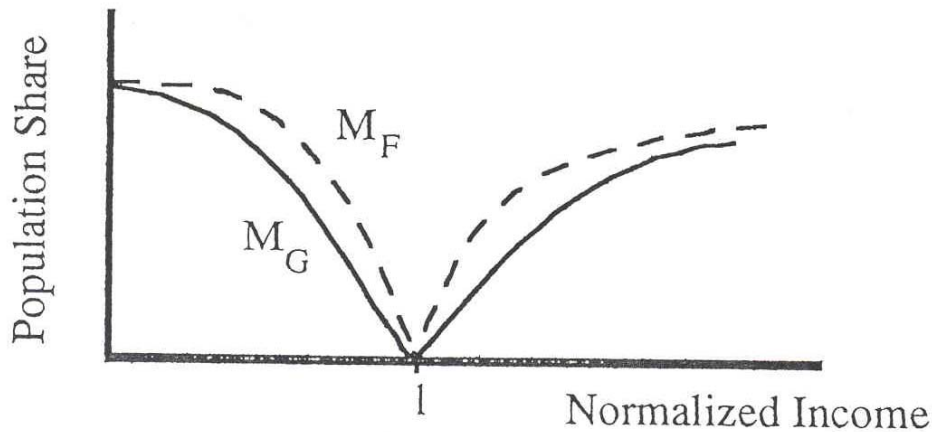
Figure C6: An example where F M G

Figure C7: M-Curves

But just how 'partial' is the middle class ordering M; and in what cases is it able to make a judgement? To answer these questions, we will characterize M with the help of some straightforward definitions and graphs. Let $\tilde{F}$ be the (median-) normalized distribution derived from F, defined by $\tilde{F}(z)=F(zm_F)$. Clearly $\tilde{F}(1)=0.5$, implying that all normalized distributions intersect at $z=1$. Now let $M_F(z)$ denote the proportion of the population in F lying between m_F and zm_F, so that $M_F(z) = \left|\tilde{F}(z) - \tilde{F}(1)\right| = \left|F(zm_f) - F(m_f)\right|$. Noting that $M(F;R) = M(\tilde{F};R) = M_F(\underline{z}) + M_F(\bar{z})$, it is apparent that the middle class population can be broken down into a 'lower middle class', measured by $M_F(\underline{z})$, and an 'upper middle class', measured by $M_F(\bar{z})$. Consequently, whenever two distributions F and G satisfy $M_F(z) \geq M_G(z)$ for all z, and $>$ for some, it follows that $F\,M\,G$. In words, if F has larger lower and upper middle classes, irrespective of the cutoffs, then it must have an unambiguously larger middle class as well. This case is illustrated in Figures C6 and C7.

It is perhaps less obvious that the <u>converse</u> must hold true as well. Suppose that $F\,M\,G$, and recall that no limitations are placed on $\underline{z}$ and $\bar{z}$ beyond $0\leq \underline{z} \leq 1\leq \bar{z}$. By choosing $\underline{z}=z$ and $\bar{z}=1$, and then $\underline{z}=1$ and $\bar{z}=z$, the condition $M_F(z) \geq M_G(z)$ for all z, and $M_F(z) > M_G(z)$ for some z, follows immediately from the definition of $F\,M\,G$, which establishes our first result.

<u>Proposition 1</u>. $F\,M\,G$ is and only if $M_F(z) \geq M_G(z)$ for all z, with $M_F(z) > M_G(z)$ for some z.

The graph of the function $M_F(z)$, which we call the 'M-curve' of the distribution F, is depicted in Figure C7, along with the analogous curve for G. Proposition 1 shows that the 'M-curve' represents the partial ordering M in much the same way as the Lorenz curve represents the Lorenz ranking: A higher M-curve indicates an unambiguously larger middle class. Even when M is unable to rank two distributions, its curve can be a useful tool of analysis, identifying the precise locations where crossings occur and thus the cause of the ambiguous ranking. In particular, the M-curve may be used to extend the benchmark partial ordering M in practice. For example, if the curves reveal that the source of the ambiguity is a crossing far from the median, the researcher may choose to conclude that a sufficiently robust ranking can be made.

Many measures of the income distribution are closely related to stochastic dominance orderings,[6] and the present approach is no exception. Figure C6 reveals that M is analogous to first degree stochastic dominance of G by F to the left of 1 (since $\widetilde{F}(z) \leq \widetilde{G}(z)$ for all $z \leq 1$), while it is first degree stochastic dominance of F by G to the right (since $\widetilde{F}(z) \geq \widetilde{G}(z)$ for all $z \geq 1$). Intuitively, the combined effect of these two requirements is to leave F with more of its population near its median than G. In particular, $F\,M\,G$ requires the density function f of F to be higher at its median m_F than the density function g of G is at m_G.

Figures C6 and C7 also illustrate the link between the partial ordering M and the abstract notion of 'increased spread'. Any mean preserving spread across the median lowers the M-curve on both sides of 1, and consequently leads to a distribution having an unambiguously smaller middle class. Indeed, M goes beyond the stylized spreads of this type. Any <u>decrement</u> in income taking place below the median, or any <u>increment</u> in income above the median, will unambiguously diminish the middle class.

However, the ordering can render no judgement when a transfer occurs to one side of the median. Any progressive transfer of this type will lower the M-curve somewhere near the median and raise it somewhere further away, resulting in M-curves that cross.

Viewing a decreasing middle class as increased polarization, we now have available a polarization partial ordering that reflects the 'increased spread' component, but not the 'increased bipolarity' component of polarization. The next section will analyze the converse of this partial ordering, and then will show how it can be extended to accommodate the 'increased bipolarity' aspect of polarization.

Section D: Polarization Curves

In this section we explore partial orderings of polarization, and the curves that describe them, focusing on people space (the vertical axis) rather than income space (the horizontal axis) to define the middle class. The first partial ordering is motivated by the question: Given any range around $p=0.5$ in people space, how wide would its range in income space have to be to contain all its members? This leads to a ranking and a curve representing it that captures the notion of increased spread, and is directly linked to the middle class ordering M. The second partial ordering will place greater weight on incomes closer to the middle, being motivated by the question: what is the average income shortfall of those in a given range around $p=0.5$? The resulting curve and ranking will fully reflect the spread and bipolarity aspects of polarization.

Polarization as Increased Spread

Whether people space or income space is chosen as the space for defining the middle class, the general methodology – as outlined above – is the same. This choice may initially seem unimportant since, for any given distribution F, every middle income range $R=[\underline{z},\overline{z}]$ satisfying $\underline{z} \leq 1 \leq \overline{z}$ has a corresponding middle class population range $Q = [\underline{q},\overline{q}]$ satisfying $\underline{q} \leq 0.5 \leq \overline{q}$ and vice versa. However, when the distribution changes, the original income range may no longer correspond to the original population range. For example, if we fix an income range in Figure C6, the associated population range for F is

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[6] See Fishburn and Vickson (1978) for a brief introduction to the three degrees of stochastic dominance. In particular, F stochastically dominates G in the first degree if $F(y) \leq G(y)$ for all y.

larger than the one for *G*. Alternatively, for any given range in people space, *F* has a smaller income range than *G*. It was the fact that a given Q can have widely varying income spreads for different distributions which tripped up Levy's method of measuring the middle class: increasing the spread can leave the income share received by Q unchanged when the income range of Q varies. In contrast, the middle class index *M* is based on the very fact that different distributions have different Qs for a given *R*. *M* is in fact the length of the associated Q.

Perhaps the people-space approach can be resurrected by selecting an index which is sensitive to this spread. To this end, define $S(F;Q)$ to be the normalized income distance $\tilde{y}(\overline{q})-\tilde{y}(\underline{q})$ associated with Q, where $\tilde{y}(q)=\tilde{F}^{-1}(q)$ is the median normalized income of the person at the *qth* percentile. Just as *M* measures the length of the associated range in people space, *S* measures the length of the range in median normalized income space associated with a given Q. A greater level of *S* indicates that there are fewer incomes near the middle, as illustrated in Figure D1, so *S* is seen to be an index of polarization rather than a measure of the middle class. It is the dual index to *M*, and as such is subject to the same critique that it depends on a range that is entirely arbitrary. This consideration leads to a range-independent partial ordering of polarization, defined as follows:

$F\ S\ G$ if and only if $S(F;Q)\geq S(G;Q)$ for all Q, with $S(F,Q)>S(G,Q)$ for some Q.

In words, $F\ S\ G$ indicates that no matter which middle-class range about $p=0.5$ is chosen, the normalized income spread required to capture it is larger (or no smaller) for *F* than for *G*. This is clearly the case in Figure D1.
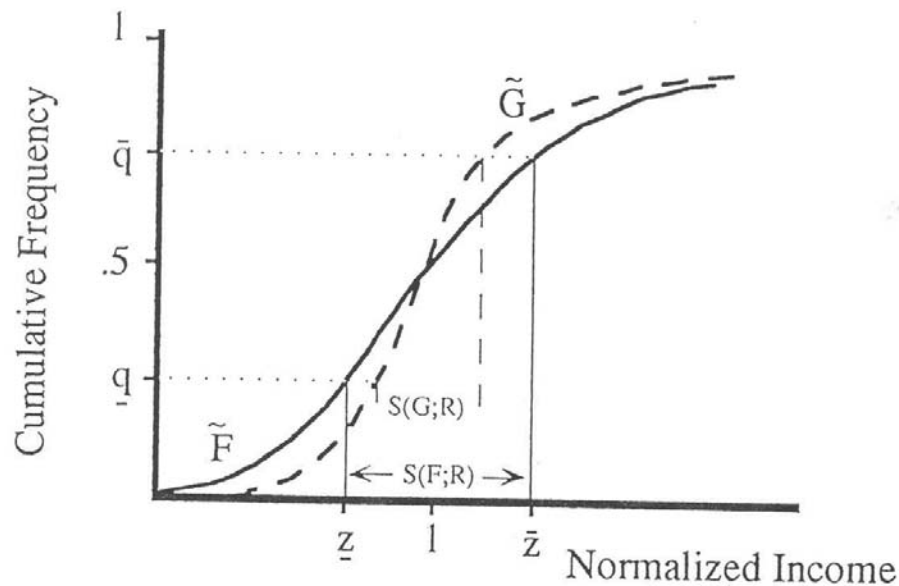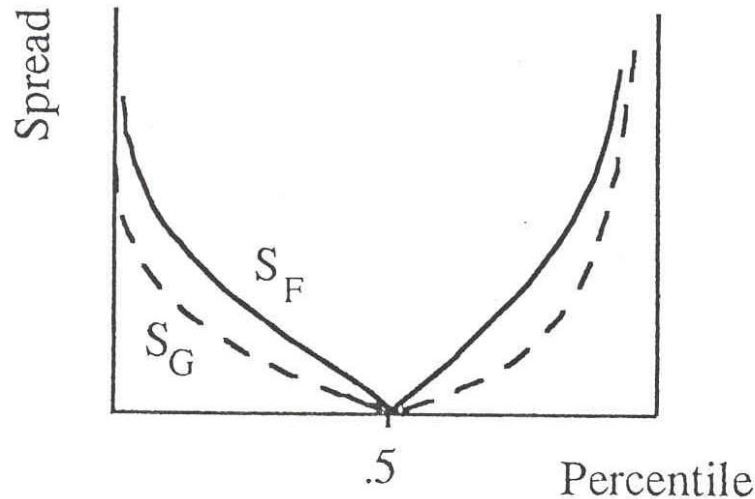
Figure D1: An example where F S G

Figure D2: First degree polarization curves

Figures C6 and D1 suggest that when *F* has an unambiguously greater spread, *G* has an unambiguously larger middle class, and vice versa. It is not difficult to confirm this in general. Perhaps the simplest way to see it is by defining a curve which represents *S* in the same way the *M*-curve represents the ranking *M*. The (first degree) polarization curve $S_F(q)$ of a distribution *F* is defined by $S_F(q)=|\tilde{y}(q)-\tilde{y}(0.5)|$ $=\left|F^{-1}(q)-F^{-1}(0.5)\right|/m_F$. For each *q*, the quantity $S_F(q)$ represents the distance between the median and the income of the person at the *qth* percentile, expressed in medians. The polarization curves of the distributions in D1 are depicted in D2. An argument entirely analogous to the verification of Proposition 1 demonstrates that the polarization curve represents the partial ordering *S*.

Proposition 2. *F S G* if and only if $S_F(q) \geq S_G(q)$ for all *q*, with $S_F(q) > S_G(q)$ for some *q*.

Moreover, the polarization curve of one distribution is higher exactly when its *M*-curve is lower than the other distribution's *M*-curve, as can be seen by comparing diagrams C6 and D1-2. In the lower half of the distribution, a higher polarization curve means that the normalized cdf is to the left and above the second normalized cdf, and hence the *M*-curve is lower. In the upper half, it means that the normalized cdf is to the right and below the other normalized cdf which translates once again to a lower *M*-curve. By Propositions 1 and 2, then, we have the following result:

Proposition 3. *F S G* if and only if *G M F*.

In sum, the polarization ranking *S* is the converse of the middle-class ranking *M*. It is represented by the first degree polarization curve, which acts like a pair of Lorenz curves signaling greater polarization. It has an intuitive interpretation in terms of increased spread. In addition, our previous comments about extending *M*, and its interpretation in terms of stochastic dominance, also apply to *S* (with the order reversed, of course). One final observation may also be made: the polarization ranking is preserved under arbitrary increasing transformations of the income variable when distributions have the same median. In other words, the ranking *S* (as well as *M*) can be meaningfully applied to ordinal data, such as IQ or educational attainment, without the kind of criticism that has been leveled against the use of the Lorenz criterion in these cases. Since the Lorenz curve 'adds up' incomes and relies on the mean, Lorenz

comparisons can be altered by transformations of the variable. The polarization curve uses ranges, not sums, and is based on the median, which is invariant to increasing transformations. This is a practical advantage of S in cases where the variable is not cardinally meaningful.

When the income variable <u>is</u> cardinally meaningful, though, it may be argued that S does not go far enough in identifying polarization, since it ignores increased bipolarity. This may be seen by the fact that any progressive transfer wholly on one side of the median <u>lowers</u> the polarization curve for one range of incomes, but <u>raises</u> it over another range closer to the median, resulting in crossed curves (as in Figure D3). The normalized distribution function of F has greater spread near its median than does G, but further away the relationship reverses. To reflect the increased bipolarity, the change closer to the median would have to be emphasized. Just as the Lorenz curve builds on low incomes by cumulating from the left to the right, we build on the middle incomes by cumulating from the median outwards. The next section constructs a second polarization curve based on this idea.

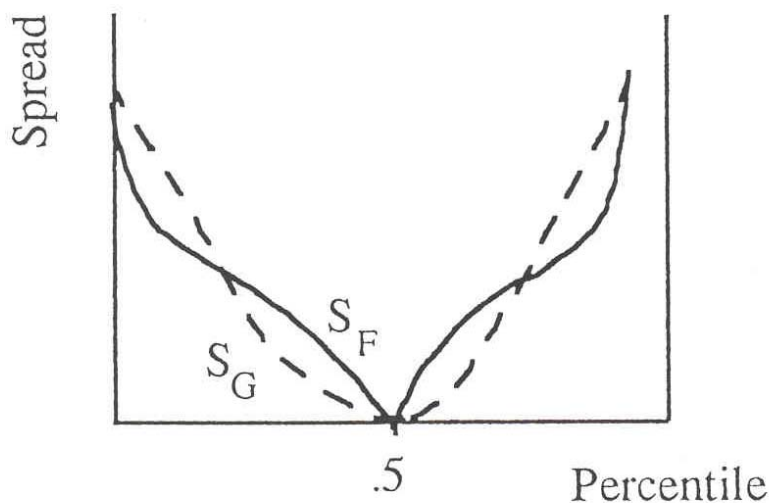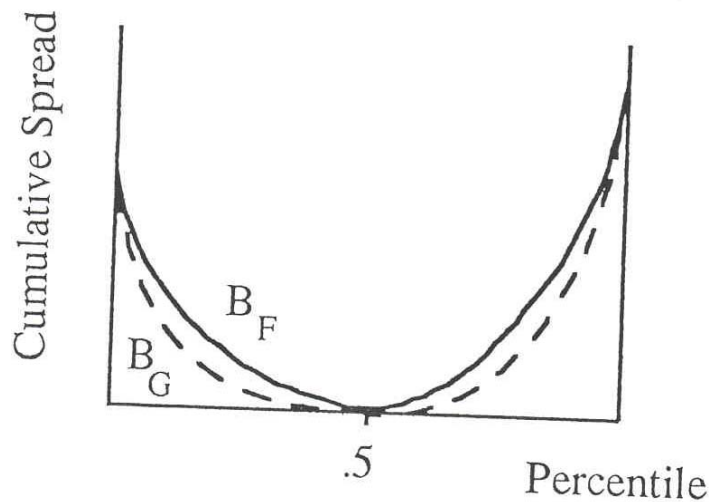Figure D3: Increased bipolarity and crossing curves

Figure D4: Second degree polarization curves

Polarization as Increased Bipolarity

For a given distribution *F*, define the <u>(second-degree) polarization curve</u> B_F by $B_F(q) = \left| \int_q^{0.5} S_F(p)\,dp \right|$ for $0 \le q \le 1$, which is simply the area under the first polarization curve *S* between 0.5 and *q*. Since a higher first-degree curve integrates to become a higher second degree curve, both curves reflect increased spread. The second degree polarization curve, however, is also sensitive to increased bipolarity. Figure D4 depicts B_F and B_G for the distributions in Figure D3. Recall that the distribution *F* has increased bipolarity as compared to *G*: the progressive transfers shift income away from the middle, resulting in higher first- and second-degree curves for *F* near the median. As we move further from the middle, the spread S_F falls below S_G, but the cumulative spread B_F is still larger than B_G. Thus the new polarization curve reflects increased bipolarity as well as spread.

Let *B* denote the partial ordering generated by the new curves, so that *F B G* if and only if $B_F(q) \ge B_G(q)$ for all *q*, with $B_F(q) > B_G(q)$ for some *q*. It turns out that *B* has the following interesting interpretation.

<u>Proposition 4.</u> *F B G* is equivalent to the requirement that for any middle class population *Q*, the average distance of its members' incomes from the median (in terms of medians) is no lower in *F* than in *G*, and for some *Q* it is higher.

The verification of this result follows from the observation that $B_F(\overline{q}) + B_F(\underline{q})$ represents the aggregate distance from the median (in median units) for the members of *Q*, while $(\overline{q} - \underline{q})$ represents *Q*'s population size, and hence $(B_F(\overline{q}) + B_F(\underline{q}))/(\overline{q} - \underline{q})$ is the average distance for *Q*'s members. Therefore, if B_F never falls below B_G, the sum $B_F(\overline{q}) + B_F(\underline{q})$ must be greater than or equal to $B_G(\overline{q}) + B_G(\underline{q})$ and so the average distance must follow suit. The converse follows from an argument analogous to the one for Proposition 1. Thus, a higher second degree polarization curve provides information about the average distance to the median for every middle class and vice versa.

It is easily seen that the second polarization curve is always convex, and resembles a pair of Lorenz curves facing one another. It achieves a minimum of 0 at $p=0.5$. The left side of the curve reaches a maximum at a value of $B_F(0) = \int_0^{0.5} [\tilde{y}(0.5) - \tilde{y}(q)]dq = \frac{1}{2}(m - \mu^L)/m$, where μ^L is the mean income among the lower half of the distribution; the right side has a highest value of $B_F(1) = \int_{0.5}^1 [\tilde{y}(q) - \tilde{y}(0.5)]dq = \frac{1}{2}(\mu^U - m)/m$, where μ^U is the mean of the upper half. Since the overall mean μ is halfway between μ^L and μ^U, this implies that for the typical positively skewed distribution with $m < \mu$, the right side reaches a higher value than the left. For two distributions with the same median, a necessary condition for $F\ B\ G$ is for $\mu_F^L \le \mu_G^L$ and $\mu_G^U \le \mu_F^U$, so that both the lower and upper means are as far from the median in F as in G. Additionally, it must be the case that the density of F at the median is no higher than the density of G at the median. For if the opposite were true, then S_F would be below S_G near the median, implying the same for B_F and B_G.

Section E: An Index of Polarization

Partial orderings and their associated curves have proved to be important tools in analyzing the income distribution. With their help, the analyst can graphically depict the aspect of the distribution of particular interest and then identify pairs of distributions whose ranking is unambiguous. For example, inequality analyses often begin by checking whether one distribution Lorenz dominates another. Numerical indices, however, are the most common tools in analyzing the income distribution. A numerical index summarizes the aspect of interest in a single number and thus induces a complete ordering over distributions. It can be easier to work with than a partial ordering, particularly when comparing large numbers of distributions or searching for factors that 'explain' changes in the distribution. We now turn to the construction of an index of polarization.

There are a number of qualities that a 'good' index should possess. First, the index should conform to the basic, underlying notions of the concept being measured; e.g., inequality measures should be Lorenz-consistent. In the present context, this means that the index agrees with the second degree (and hence first degree) polarization curve when it applies. Second, the index should be well motivated and understandable. For example, the Gini coefficient can be interpreted as twice the area between the Lorenz curve and the diagonal of equality, while the meaning of Theil's inequality index is somewhat more obscure. Third, the index may be required to satisfy certain useful properties or axioms, such as decomposability by population subgroup. Since this is a first attempt at constructing a polarization index, we will focus on the first two criteria and will leave the third for future work.

Using the Gini coefficient as a model, we propose to measure polarization as twice the area beneath the second degree polarization curve. In symbols, our polarization index P is defined by $P = \int_0^1 2B_F(q)dq$. Any distribution with a higher polarization curve will obviously have a higher index value, so P is consistent with the unambiguous partial ordering B. When polarization curves cross, it makes determinations in much the same way as the Gini coefficient does with intersecting Lorenz curves: it decides on the basis of the areas contained by the curves. This is, of course, an entirely arbitrary solution but has considerable merit in its simplicity. Moreover, as we shall see below, the resulting index offers important new perspectives on the relationship between polarization and inequality.

The Relative Median Deviation, Inequality and Polarization

Generally speaking, inequality and polarization increase when the distance between those above the median and those below the median rises. We can measure the extent of the average distance by $T = (\mu^U - \mu^L)/\mu$, where, as before, μ^U is the mean of those above the median, μ^L is the mean of those below, and μ is the overall mean. By analogy to a similar mean-based index[7] we shall call T the <u>relative median deviation</u>. It turns out that T has an interesting interpretation in terms of the Lorenz curve.[8] Recall that μ is the average of μ^L and μ^U, so that $(\mu^U - \mu)/\mu = (\mu - \mu^L)/\mu$ is the amount of income, as a share of the mean, which would be necessary to raise the income of everyone initially below the median to the overall mean. This, in turn, is simply the vertical distance of the Lorenz curve from the line of equality at $p = 0.5$, denoted as $V(0.5) = 0.5 - L(0.5)$ above. It follows from its definition, then, that T is twice the magnitude of the Lorenz shortfall, or $T = 2V(0.5) = 1 - 2L(0.5)$.

Figure E1: Polarization, relative median deviation, and the Gini
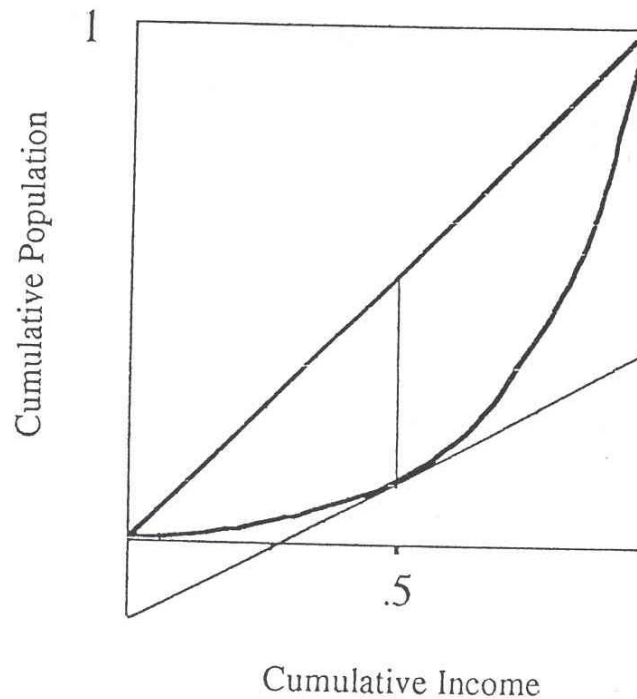
Figure E1 depicts T, and also points out an interesting fact concerning T and the Gini coefficient G, namely that T is inevitably greater than G when the latter is nonzero. To see this, we have drawn the tangent line to the Lorenz curve at $P = 0.5$. In combination with the line of equality, this line defines a

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[7] The relative <u>mean</u> deviation uses the mean income rather than the median as the demarcation line. It can be defined in terms of population weighted group means or more simply as twice the shortfall between the diagonal and the Lorenz curve evaluated at the mean income.

[8] T also is related to the first degree polarization curve. The area below this curve is

$$\int_0^1 S(q)dq = B(1) + B(0) = \tfrac{1}{2}(\mu^U - m)/m + \tfrac{1}{2}(m - \mu^L)/m = \tfrac{1}{2}T(\mu/m),$$ so T is twice the area below the first degree polarization curve, normalized by the mean rather than the median.

quadrilateral whose area is easily seen[9] to be $0.5-L(0.5)$. Thus T is twice the area of this quadrilateral, and since G is twice the area of the region between the Lorenz curve and the line of equality, T is greater than G. Consequently, the remaining area of the quadrilateral is never zero when G is nonzero.

The next result shows that our polarization index is related to the residual area of the quadrilateral below the Lorenz curve. In fact, P is the median normalized difference between T and G.

<u>Proposition 5</u>. $P=(T-G)\dfrac{\mu}{m}$

The proof is given in the appendix. From Proposition 5, P is the difference between T and G, scaled up by the measure of positive skewness μ/m. When the distribution is symmetric, the relative median deviation T is the sum of inequality G and polarization P. Positive skewness in the distribution leads to a value of μ/m greater than unity, and augments the measure of polarization. It is easy to see how inequality and polarization can move in the same direction: An increase in T is likely to bow out the Lorenz curve at the same time that it expands the quadrilateral, which may well lead to a greater residual area. This is consistent with the effect of a regressive transfer across the median; an increased spread is reflected in greater inequality as well as polarization. However, when T is fixed, inequality and polarization can move in opposite directions, as in the case of progressive transfers wholly on either side of the median. Increased bipolarity is therefore associated with increased polarization, but must lead to a decrease in the level of inequality.

Proposition 5 also shows that the polarization index is easily constructed from four readily available statistics: the mean, the median, the Gini coefficient and the relative median deviation (or $1-2L(0.5)$). Thus, it should be straightforward to derive polarization values for previous studies from reported results.

Polarization and Decomposing the Gini

Inequality indices are often decomposed by population subgroups into a term representing between-group inequality and a term representing (average) within-group inequality. The Gini coefficient does not generally admit such a simple decomposition, but it does in the special case where the groups are defined by nonoverlapping income ranges. Let the overall population be divided into two subgroups: those above the median and those below. The between-group term G^B is found by applying the Gini to the 'smoothed' distribution in which those above m receive the group mean μ^U, and those below receive μ^L. The within-group term G^W is a population weighted average of the inequality levels within the two groups. The overall inequality G is thus broken down as $G=G^B+G^W$.

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[9] The width of the quadrilateral is 1 and the average height is given by the height at its middle, which is $0.5 - L(0.5)$.

Figure E2: Polarization and decomposition of the Gini
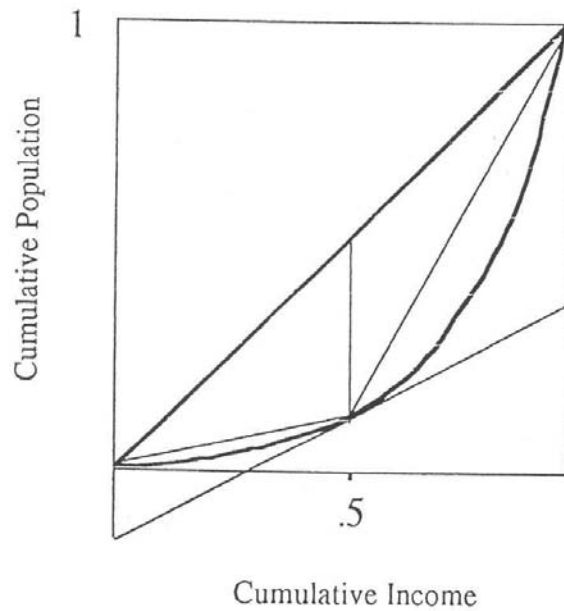
This decomposition of the Gini is illustrated in Figure E2. The Lorenz curve of the smoothed distribution is formed by connecting the three points along the original Lorenz curve where $p=0$, 0.5, and 1. The slope of the lower segment is μ^L/μ while the slope of the upper segment is μ^U/μ. The between-group term is twice the area captured by this piecewise linear between-group Lorenz curve. The degree to which the original Lorenz curve bows away from the between-group Lorenz curve reflects the extent of within-group inequality. In fact, by the decomposition formula, G^W is twice the area between the original and the between-group Lorenz curve.

The triangle formed by the between-group Lorenz curve and the diagonal has an area of $(0.5-L(0.5))/2$. Therefore, $G^B=0.5-L(0.5)$ and since T is $2(0.5-L(0.5))$ we know that $T=2G^B$. The relative median deviation is twice the between-group term. Substituting this identity along with the decomposition formula into Proposition 5 yields the following result:[10]

<u>Proposition 6</u>. $P=(G^B-G^W)\dfrac{\mu}{m}$

Thus the polarization index is just the inequality between the upper and lower halves of the distribution <u>minus</u> the inequality within the two subgroups, all measured by the Gini coefficient and renormalized by the median. Since the overall Gini is the sum of G^B and G^W, this provides another clear indication of how inequality differs from polarization. More inequality between the upper and lower halves of the distribution will tend to raise both inequality and polarization; a greater level of within-group inequality raises overall inequality, but <u>lowers</u> polarization.

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[10] The authors would like to thank Professor Chinhui Juhn for pointing out this nice interpretation of *P*.

Section F: Empirical Analysis

In the previous sections, we have developed new tools for evaluating the polarization of an income distribution. We now apply the methodology to income data from Canada and the US.

The Data

For each country, we have obtained data on family disposable income over three years. The Canadian data for 1981 and 1987 are drawn from the Luxembourg Income Study (LIS), an international collaborative effort to assemble consistent income distribution and related microdata sets.[11] This source has the benefit of having been gathered over a number of countries using a consistent methodology. It is available, though, in a somewhat aggregated form, so in addition to this source we have made use of the original microdata from the Survey of Consumer Finance (SCF) for this year 1988. The LIS was also the source of the first two US years we examine, 1979 and 1986. The 1988 US data are derived from the public use file of the March Current Population Survey (CPS). The variable analyzed is family size adjusted disposable family income constructed using a 40-30 equivalence scale (an extra adult is 40 per cent of the first adult, extra children are 30 per cent). Commentary on data of this type is available elsewhere (e.g. Wolfson, 1986 and Wolfson and Murphy, 1992), so we will move directly to the results.

Results

Table F1 summarizes our numerical results. The first two rows give the mean and median incomes in current dollars in each country's own currency. To get an idea of how theses figures compare across the countries, we could use the OECD's Purchasing Power Parity conversion factor of $1.25 Canadian per American in 1988 to see that the US mean income in Canadian dollars was $20,932, while the median similarly expressed was $18,273. The remaining statistics are based on normalizations of the data, so we do not need to worry about exchange rates to compare across countries. The third row is the median income over the mean, or the slope of the Lorenz curve at $p=0.5$. Recall that 1 minus this ratio is a measure of skewness of the distribution. As expected, the distributions for both countries in all years are positively skewed, with a more definite increasing trend exhibited in the UsS.

Table F1: Canada and US income distributions compared

	Family Disposable Income per Equivalent Adult						Labor Income	
	Canada			US			Canada	US
	LIS	LIS	SCF	LIS	LIS	CPS	LIS	LIS
	1981	1987	1988	1979	1986	1988	1987	1986
Mean	13,624	19,171	19,768	9,313	15,251	20,932	24,098	20,184
Median	12,372	17,237	17,825	8,483	13,378	18,273	21,786	17,957
Median/mean	0.908	0.899	0.902	0.911	0.877	0.873	0.904	0.890

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[11] O'Higgins et al. (1990).

	Family Disposable Income per Equivalent Adult						Labor Income	
	Canada			US			Canada	US
Quantiles								
Q1	6.9	7.2	7.3	5.9	5.3	4.9	3.6	3.8
Q2	13.0	13.0	13.0	12.6	11.8	11.5	11.2	10.9
Q3	18.2	18.0	18.0	18.2	17.6	17.5	18.2	17.6
Q4	24.3	23.9	23.8	24.7	24.6	24.8	25.7	25.2
Q5	37.6	37.9	37.8	38.7	40.7	41.4	41.4	42.5
D9	15.6	15.4	15.2	16.0	16.3	16.5	16.7	17.0
D10	22.1	22.5	22.6	22.7	24.4	24.9	24.6	25.5
V19	9.3	9.3	9.2	9.7	10.0	10.2	10.0	10.6
V20	12.8	13.1	13.4	13.0	14.3	14.7	14.7	14.9
Inequality								
Gini *G*	0.300	0.298	0.297	0.320	0.345	0.357	0.372	0.379
%Pop with incomes:								
< 40% of median	9	9	8	13	14	15	19	18
< 50%	15	14	14	19	20	21	25	23
< 60%	22	21	21	24	26	27	29	29
60% to 75%	10	11	11	9	9	9	7	9
75% to 100%	18	18	18	17	15	14	14	12
100% to 125%	16	16	17	15	14	13	12	14
125% to 150%	12	13	11	12	10	11	12	9
150% to 200%	14	13	14	14	14	14	15	14
> 200%	8	8	8	9	12	12	11	13
% in *M* given income range:								
75% to 150% of median	46	47	46	44	39	38	38	35
75% to 120%	34	34	35	32	29	27	26	26
50% to 150%	63	65	64	58	54	53	49	50

	Family Disposable Income per Equivalent Adult						Labor Income	
	Canada			US			Canada	US
S given pop range:								
40% to 60%	0.291	0.294	0.289	0.301	0.345	0.365	0.365	0.374
35% to 65%	0.445	0.439	0.436	0.475	0.524	0.557	0.579	0.585
30% to 70%	0.611	0.606	0.594	0.655	0.718	0.750	0.781	0.783
25% to 75%	0.792	0.766	0.758	0.843	0.932	0.968	1.025	1.011
20% to 80%	0.971	0.938	0.936	1.042	1.147	1.185	1.245	1.232
Avg distance given pop range:								
40% to 60%	1.024	1.025	1.018	1.024	1.028	1.031	1.032	1.018
35% to 65%	1.027	1.026	1.020	1.027	1.032	1.035	1.037	1.020
30% to 70%	1.031	1.029	1.023	1.031	1.037	1.040	1.039	1.023
25% to 75%	1.037	1.034	1.029	1.035	1.045	1.047	1.042	1.029
20% to 80%	1.043	1.042	1.037	1.042	1.055	1.057	1.046	1.039
Polarization								
Median share $L(0.5)$	0.283	0.286	0.287	0.269	0.252	0.243	0.230	0.227
Relative median deviation T	0.434	0.428	0.426	0.462	0.496	0.514	0.540	0.546
Polarization index P	0.148	0.145	0.143	0.156	0.172	0.180	0.186	0.188

The next series in the table gives quantile shares, or the shares of income received by various quantile groups. These figures start with the quintiles, or the shares received by groups comprising 20 per cent of the families (Q1-Q5), then the ninth and tenth deciles – 10 per cent group shares – are given (D9 and D10), and finally the 19th and 20th vingtiles – 5 per cent group shares – are presented (V19 and V20). The Canadian data reveal a fairly stable picture from 1981 to 1987, with the lowest and highest quintiles gaining slightly at the expense of the third and fourth. Summing over the lowest k quantiles gives the Lorenz coordinate for $p=0.2k$. The resulting Lorenz curves cross somewhere between 0.6 and 0.8, with the 1987 curve initially higher. In the US, each of the four lowest quantiles lost share to the highest quintile between 1979 and 1986. This suggests an unambiguous increase in inequality according to the Lorenz criterion. The final four rows show that the very highest have gained substantially in the US over time, and modestly in Canada. For example, the highest 5 per cent in the US received 13 per cent of the income in 1979 and 14.3 per cent in 1986. The comparable figures for Canada are 12.8 per cent and 13.1 per cent. When all the shifts in the distribution are summarized in the Gini coefficient, we find a modest decrease in inequality in Canada and a reasonably large jump in inequality in the US.

In the next three rows we turn to indicators of the prevalence of low income. A relative definition of low income is used, where the cutoff is a percentage of the median income. The results show that the proportion of low income recipients has risen somewhat in the US and fallen somewhat in Canada, irrespective of whether a 40 per cent, a 50 per cent or a 60 per cent cutoff is used.

The rest of the table presents data on the degree of polarization in the two countries. Data pertaining to the middle class are given in the first series. The first five rows show how various median ranges fared, while the bottom three lines give the proportion of families in the middle class when the middle income range is from 75 per cent to 150 per cent of the median, 75 per cent to 125 per cent, and 50 per cent to 150 per cent, respectively. We see a rather mild increase in the size of this middle class between 1981 and 1987 in Canada, owing to an increase at the upper part of the range. In the US there is a reasonably large erosion (5 percentage points) of the middle class for each range. The exodus was both upwards to the range beyond twice the median and downwards to the low income ranges.

This phenomenon is likewise reflected in the figures on the spreads of the distribution. For Canada, in all but the smallest population range, the income spread (in medians) necessary to capture ranges is smaller in 1987 than in 1981. For instance, the spread necessary to encompass the middle 60 per cent of the population in 1981 was about 97 per cent of the median income; by 1987 this was about 94 per cent. The direction of change was notably different in the US The middle 60 per cent had a spread of about 104 per cent of the median income in 1979 and approximately 115 per cent in 1986. A similar picture is obtained for each of the symmetric ranges given.

The final rows report our index of polarization *P* and the relative median deviation *T*. We see that the level of polarization in Canada fell a modest amount from 1981 to 1987. Over the same period the relative median deviation tracked similarly, which indicates that difference in mean incomes of the group above the median income, and the group below, fell somewhat over the period in question. Reviewing our results on the Gini coefficient, we conclude that both inequality and polarization have declined in Canada over the period examined. In the US data, a rather different picture is obtained. The polarization index began at a higher level and rose even further during the period 1979 to 1986. The relative median deviation increased a significant amount, reflecting a widening in the mean incomes of the upper and lower groups. These figures suggest that both inequality and polarization increased in the US over this time period.

A graphical cross-section comparison of the US and Canada is presented in Figures F1 and F2 for the year 1988. Each country's cumulative distribution function (in own dollars) is given in the top graph of F1. The US distribution is above the Canadian distribution due to the difference in the value of the currencies. The figure below the cdfs at the left presents the same data, mean normalized, rotated and reflected, resulting in what is often called 'Pen's parade' after Pen (1971). For each percentile on the horizontal axis, the curves show the associated levels of income, measured in means, of the family at that percentile. The Pen's parade for Canada is initially higher than the US curve and then falls below at the higher incomes: the family at the first decile receives a higher relative income in Canada than in the US, while the order is reversed for the family at the ninth decile. Consequently, the Lorenz curve for the US is below that for Canada, reflecting an unambiguously higher level of inequality in the US than in Canada in 1988.

At the right of Figure F1, the relative levels of polarization are depicted. The first graph is the first degree polarization curve which indicates the spread of the distribution. It is obtained from the cdf by normalizing by the median then reflecting the graph so that the axes are reversed, cutting the graph at 1, and folding the left hand side of the graph up. The US curve dominates the Canadian curve, which means that no matter which middle class range on the horizontal axis is selected, the spread needed to capture these families is higher in the US than in Canada. By our results relating the polarization ranking to the middle class ranking, then, it is apparent that the middle class is unambiguously smaller in the US

than in Canada. The final graph is the second degree polarization curve which is obtained from the previous curve by integrating outwards from the middle. It shows that the average departure from the median associated with a given range on the horizontal axis is larger for the US than for Canada. It also indicates a rather noticeable difference in the two country's polarization index values. Since the area under the US curve is greater than the area under the Canadian curve, the US has the higher level of overall polarization.

Figure F2 combines the bottom two diagrams, representing both inequality and polarization in a single diagram. The relative median deviation is twice the vertical departure of the Lorenz curve at the midpoint $p=0.5$. The Gini coefficient is twice the area between the Lorenz curve and the line of equality. The slope of the tangent line is the median over the mean, so the area beneath the Lorenz curve above the line, divided by the slope of the line, is our index of polarization. It is clear from the graph that the lower US Lorenz curve has higher values for the relative median deviation and for the Gini coefficient. Moreover, the area beneath the Lorenz is greater, while the slope of the tangent is less, resulting in a higher level of polarization in the US than in Canada.

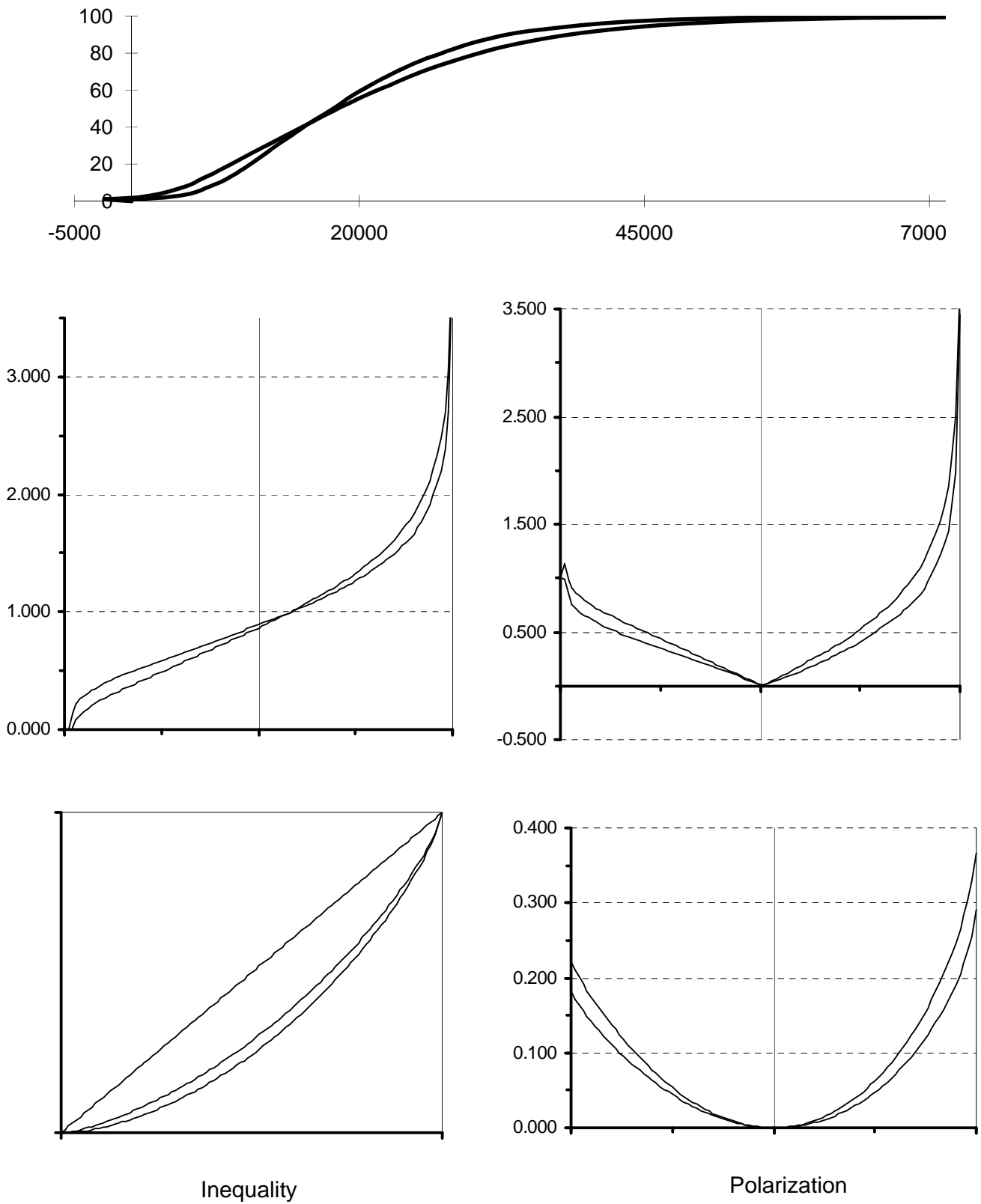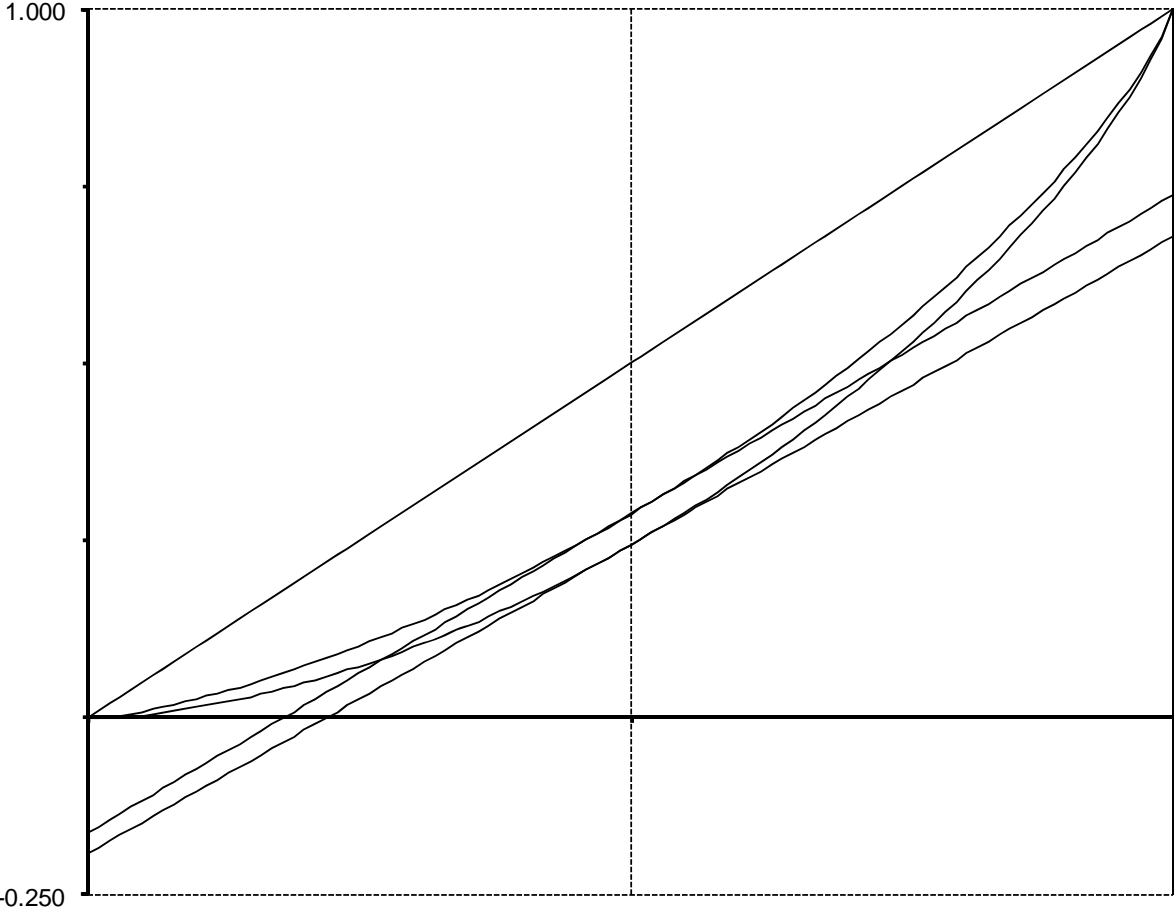Figure F1: Canada and US compared: two strands of measures

Inequality

Polarization

Figure F2: Canada and US compared: a grand synthesis

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Appendix

The proof of Proposition 5 relies on the following lemma.

<u>Lemma</u> $B_F(q) = 0.5 - \frac{\mu}{m} L_F(0.5) - (q - \frac{\mu}{m} L_F(q))$ for all q.

<u>Proof of Lemma</u>

Case 1: $q \leq 0.5$

$$
\begin{aligned}
B_F(q) &= \int_q^{0.5} S_F(p)dp = \frac{1}{m}\int_q^{0.5}(F^{-1}(0.5) - F^{-1}(p))dp \\
&= (0.5 - q) - \frac{1}{m}[\int_0^{0.5} F^{-1}(p)dp - \int_0^q F^{-1}(p)dp] \\
&= (0.5 - q) - \frac{\mu}{m}(L_F(0.5) - L_F(q)).
\end{aligned}
$$

Case 2: $q \geq 0.5$

$$
\begin{aligned}
B_F(q) &= \int_{0.5}^q S_F(p)dp = \frac{1}{m}\int_{0.5}^q(F^{-1}(p) - F^{-1}(0.5))dp \\
&= \frac{1}{m}[\int_0^q F^{-1}(p)dp - \int_0^{0.5} F^{-1}(p)dp] - (q - 0.5) \\
&= \frac{\mu}{m}(L_F(q) - L_F(0.5)) - (q - 0.5)
\end{aligned}
$$

<u>Proof of Proposition 5</u>

By the Lemma,
$$
\begin{aligned}
P &= 2\int_0^1 [0.5 - \frac{\mu}{m} L_F(0.5) - (q - \frac{\mu}{m} L_F(q))]dq \\
&= \int_0^1 \frac{m-\mu}{m}dq + \frac{\mu}{m}\int_0^1 2(.5 - L_F(.5))dq - \int_0^1 \frac{m-\mu}{m}2qdq - \frac{\mu}{m}\int_0^1 2(q - L_F(q))dq \\
&= \frac{m-\mu}{m} + \frac{\mu}{m}T - \frac{m-\mu}{m} - \frac{\mu}{m}G
\end{aligned}
$$
$$
= (T - G)\frac{\mu}{m}
$$