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Human Capital and the Quality of Education in a Poverty Trap Model

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Abstract

This paper presents a model of a poverty trap that is caused by an unequal initial income and human capital distribution, and differences in the quality of education between children from the more and less advantaged social sectors. Under certain conditions, the economy converges to a situation with three stable and simultaneous equilibria, two of which constitute poverty traps, lowering the economy's current and steady-state aggregate output level as well as its growth rate. The model suggests that a policy oriented to equalizing the quality of education would, in the long run, have potential in reducing initial inequalities.

Keywords: poverty trap, income distribution, quality of education, educational policy, equality of opportunity

JEL classification: D31, O12, I21

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Acronyms

PISA	Programme for International Student Assessment
OECD	Organization for Economic Cooperation and Development
UNESCO	United Nations Educational, Scientific and Cultural Organization

The Oxford Poverty and Human Development Initiative (OPHI) is a research centre within the Oxford Department of International Development, Queen Elizabeth House, at the University of Oxford. Led by Sabina Alkire, OPHI aspires to build and advance a more systematic methodological and economic framework for reducing multidimensional poverty, grounded in people's experiences and values.

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1. Introduction

This paper presents a poverty trap model caused by an unequal income and human capital distribution and by the segmentation of social sectors, expressed by differences in the quality of education received by children from the more and less advantaged socioeconomic sectors. The model is based on Berti Ceroni (2001), with the important difference that this approach incorporates the quality of education.

The question driving this paper is whether the intergenerational transmission of poverty can actually be broken down through education. It is argued that even if poor children receive an education, if it is not of good quality, the cognitive skills they acquire will not suffice for obtaining an income that allows them to leave poverty.[1]

The policy implications derived from the model are related to Roemer's (1998) theory of equality of opportunity according to which unequal results cannot be justified if they are due to differences in circumstances rather than differences in efforts. This is also in line with the 2006 *World Development Report*: 'the equalizing promise of education can be realised only if children from different backgrounds have equal opportunities to benefit from quality education' (World Bank 2005: 140).

The case of Argentina motivates this paper. The country has experienced a significant increase in poverty and inequality since the 1970s (CEDLAS 2004, among others). At the same time, and contrary to what had occurred previously, the educational system presents some signs of segmentation. The results of the year 2000 Programme for International Student Assessment (PISA), developed by the Organization for Economic Cooperation and Development (OECD) and the United Nations Educational, Scientific and Cultural Organization (UNESCO) to test 15 year-old students, suggest that from all participating countries, Argentina is one with the highest test score gap between the lowest and the highest 20 per cent of the family wealth index. It is also among the countries in which the *between*-schools variation in students' performance accounts for a larger part of overall variation, and seems to be associated with differences in students' socioeconomic backgrounds (OECD 2003). Using other data sources, Llach and Schumacher (2004) and Cervini (2002, 2005) also argue that the educational system is segmented between the more and less advantaged social sectors. Such segmentation may have serious consequences in terms of the persistence of poverty in the long term.

Section 2 briefly reviews the literature on poverty traps. Section 3 describes the model. Section 4 derives the policy implications. Section 5 presents some basic empirical evidence supporting the assumption of segmentation and finally, Section 6 concludes.

2. Poverty Trap Models in the Literature

A poverty trap is any reinforcing mechanism that causes poverty to persist (Azariadis and Stachurski 2005: 33). From the beginning of the development theory, this concept has been useful in explaining the observed differences in per capita income between countries. Poverty trap models are associated with some type of departure from the neoclassical assumptions such as non-convexities (scale economies, positive externalities, and other increasing returns), the existence of imperfect competition, some market failures (especially capital markets), and acknowledgement of the importance of institutional frameworks. These

[1] It is considered that labour income is not exclusively determined by *sheepskin effects* (increases in income originated merely on the possession of an educational certificate) but by the real cognitive skills the individual has.

allow the emergence of multiple equilibria, at high and low income levels, so that if an economy starts below a certain threshold it remains trapped in a 'bad equilibrium'.

The concept of poverty trap was implicit in the early development theory of Rosenstein-Rodan (1943), Nurske (1952), Leibenstein (1957) and Myrdal (1957). These ideas were abandoned for some decades, possibly because of the lack of formalization (Ros 2001). However, given the difficulties of the neoclassical growth theory, in particular of Solow's (1956) model, in explaining the big differences in per capita income between countries, they re-emerged in a more formalized framework. This is the case of Murphy, Shleifer and Vishny (1989), and Matsuyama (1995) in which the complementarities in investment decisions lead to the existence of a *good* and a *bad* equilibrium. The models of Kremer (1993), Redding (1996) and Acemoglu (1997) share the feature that the existence of strategic complementarities in human capital investment leads to the emergence of multiple equilibria.[2]

The concept of poverty traps can also be applied to explain situations of *economic duality*, in which a fraction of the population reaches a *good equilibrium*, with a high income level, while another fraction remains trapped in a *bad equilibrium*, with a low-income level. This is labelled *fractal poverty traps* by Easterly (2001) and Barrett and Swallow (2003).

The papers of Galor and Zeira (1993), Galor and Tsiddon (1997), and Berti Ceroni (2001) correspond to this concept of poverty traps. In Galor and Zeira's (1993) model the poverty trap is driven by credit market imperfections, with a borrower's interest rate increasing with lenders' monitoring costs, which are in turn increasing in the amount lent. In this way, the initial distribution of wealth determines each dynasty's human capital accumulation path and steady state. The economy becomes segmented in two groups: the skilled and wealthy workers – for whom investing in education is an optimal decision – and the unskilled and unwealthy workers – who derive a higher utility from non-investing in education. Matsuyama (2000) has a similar model with the difference that the threshold that divides the rich and the poor is endogenously determined.

Galor and Tsiddon's (1997) model presents two types of positive externalities: a home environment externality, according to which an individual's level of human capital is an increasing function of the parental level of human capital, and a global technological externality, according to which technological progress is positively related to the average level of human capital in society. In the first phase of development, the home environment externality is the dominating factor, creating strong inequalities in human capital distribution. However, as investment in human capital of the highly educated segments of society increases, the global technological externality starts to dominate, leading to an income convergence. The model suggests that equality-enhancing policies implemented prematurely may lead to a low output trap.

Berti Ceroni (2001) criticizes Galor and Zeira's (1993) assumption of credit market restrictions to financing investment in education, arguing that credit market imperfections are more likely to take the extreme form of self-financing constraints, since expected human capital is not generally accepted as collateral. At the same time she criticizes Galor and Tsiddon (1997), remarking that empirical evidence on the existence of non-convexities in individual human capital accumulation is far from conclusive. The author presents a model in which a fraction of the population remains trapped in a low level of human capital and income, and another fraction is able to reach equilibrium at a high level of human capital and income. Education is privately financed: parents decide (motivated by altruism) to invest their disposable income in educating their children. The poverty trap is generated by non-homothetic preferences so that the poor require higher returns to education than the rich in order to invest in education.

[2] For a recent thorough revision of the literature on poverty traps, see Azariadis and Stachurski (2005).

The model developed in this paper is based on Berti Ceroni (2001). The main difference is that here the quality of education is incorporated as a key element in addition to the segmentation of the economy in social sectors or networks given by education.[3] Instead of reaching a situation with two stable equilibria (one good and one bad), the economy converges to a situation with three stable equilibria, two bad and one good. Similarities and differences with Berti Ceroni (2001) are pointed out in the presentation of the model in Section 3.

3. The Model

This is based on Berti Ceroni (2001) in being a model of overlapping generations. Each family is composed of two individuals, father and son. Each individual is born with the same ability, lives two periods and is endowed with one unit of time in each period. Individuals can make decisions only in the second period of their lives. When young, individuals receive an education if their parents so decide, in which case they assign their unit of time to school. Children who do not go to school acquire a fixed level of human capital as a consequence of the passage of time. In the second period of their lives all individuals offer their time unit on the labour market, earn an income proportional to their level of human capital, and decide how to allocate it between consumption and spending on their children's education.[4]

Following Berti Ceroni (2001), the utility function of parent i in time t depends on consumption in period t and on the stock of human capital of the ith child in period $t+1$. It takes the form:

$$u^i(c_t^i, h_{t+1}^i) = \ln(c_t^i) + \delta h_{t+1}^i \tag{1}$$

where δ is a parameter that measures the altruistic motive, with $0 \le \delta \le 1$. The human capital production function presents the first departure from Berti Ceroni's model. It is assumed that there is segmentation in the economy between the families of the more educated parents and families of less educated parents. Such segmentation is usually observed, especially in developing countries. It can be seen in terms of social circles or networks, or even neighbourhoods. Superscript j denotes the social circle.

$$h_{t+1}^{ij} = \begin{cases} \mu^j & e_t^{ij} \le b^j \\ \ln[q^j(e_t^{ij} - b^j) + v^j] + a^j & e_t^{ij} > b^j \end{cases} \tag{2}$$

with $\mu^j = \ln(v^j) + a^j$.

h_{t+1}^{ij} is the level of human capital acquired by the son of parent i, from social circle j. μ^j is the level of human capital achieved by the child if he does not receive any formal education. This level varies with the social sector to which the parent belongs. It is assumed that, given two parents, one with a higher educational level than the other, $h_t^{i1} < h_t^{i2}$, if none of the parents decides to provide formal education to their children, the son of the better educated parent (h_t^{i2}) will enjoy a level of human capital equal or

3 The effect of social segmentation on inequality is also explored by Bowles, Loury and Sethi (2007).

4 Berti Ceroni (2001) also presents a stochastic generalization of the model in which children's ability is subject to exogenous shocks which become observable after education decisions have been made. The existence of poverty traps is not robust to the introduction of these shocks. This paper follows the deterministic model, leaving the consideration of idiosyncratic shocks for future research.

greater than the level of human capital of the son of the less educated parent (h_t^{i1}), that is: $\mu^1 \leq \mu^2$. Parameter μ^j depends on v^j and a^j; v^j represents the knowledge and basic abilities provided at home and it is assumed that for $h_t^{i1} < h_t^{i2}$, $v^1 \leq v^2$; a^j represents the socioeconomic environment in which the family lives. It is plausible to assume that when children grow up in better educated social networks, they enjoy positive externalities. The exchange with educated adults and children (*peer-effect*) whose parents have high education reinforces the knowledge and skills they learn at home and at school. Formally, this parameter moves upwards the whole human capital production function. As before, it is assumed that for $h_t^{i1} < h_t^{i2}$, $a^1 \leq a^2$.

Education here is considered to be public; this is the second difference with Berti Ceroni's model. However, there exists a private cost of education e_t^{ij}, given by the cost of complementary goods such as books and transportation to school, and by the opportunity cost of non-working. This cost is assumed to be independent of the parent's level of education. However, b^j is a parameter that depends on the social circle to which the child belongs. It is the education spending threshold level that is necessary so that the child's human capital starts to increase. In other words, it is the spending level at which spending in education starts to be *effective*. As before, for $h_t^{i1} < h_t^{i2}$, $b^1 \leq b^2$. A simple example might help to understand this assumption. It is very common that children whose parents are highly educated start first grade of primary school already knowing to read and write and performing some simple mathematical operations. This, on the other hand, is very rare among children whose parents have a low level of education. Therefore, the minimum level of education (and so the minimum educational spending) needed by the children of the better educated parents in order to exhibit an improvement in their skills is higher than the minimum required by children of the less educated parents.

Finally, q^j represents the quality of education received by the child belonging to social sector *j*, which is of particular interest in the model. The quality of education that the child receives is not a decision variable for the parents, which is determined by the allocation of public resources to each school. It is assumed – in principle – that schools with students coming from better educated families have a better quality of education than schools with students coming from less educated families, that is, for $h_t^{i1} < h_t^{i2}$, $q^1 \leq q^2$. This is a key assumption and its empirical validity will be explored in Section 5.

Figure 1 presents the human capital production function for a possible set of values of the parameters (v^j, a^j, b^j, q^j) and for two circles or social sectors $j = 1, 2$. Sector $j = 2$ parent's human capital level is higher than sector $j = 1$ parent's human capital level ($h_t^{i1} < h_t^{i2}$).[5] It can be noted that for both social sectors, when education spending is lower than the minimum required for it to be effective (b^1, b^2 correspondingly), children remain at the fixed human capital level μ^1 and μ^2 correspondingly, which is higher in the case of children of parents with higher education. As soon as education spending exceeds the threshold, children's human capital level starts to increase, and it does so at a decreasing rate. This diminishing returns behaviour is a natural way of thinking of human capital accumulation, reflecting the limits imposed by the capacity of any human being, which restricts the conversion of increasing education spending in ever-increasing cognitive skills.

5 The figure has been done with this particular set of values of the parameters: b^1=0.2, b^2=1.4, a^1=0, a^2=0.5, v^1=1.3, v^2=2.5, q^1=12, q^2=20.

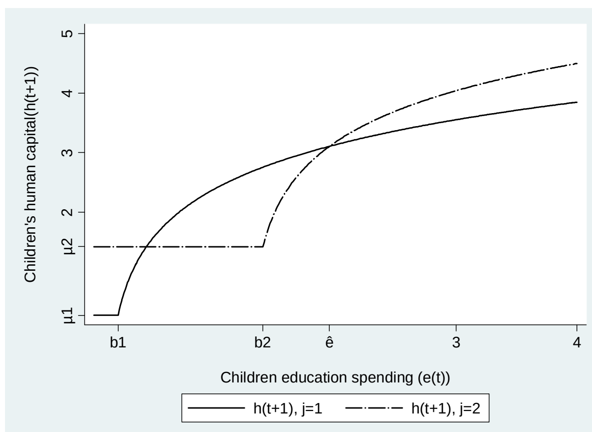
The figure clearly indicates the effect of segmentation and initial disadvantages. Children of the less educated parents have a lower education level at home, which results in a lower education spending threshold. At the same time, they benefit from lower external effects generated by the interaction with other people within their social circle, and they attend schools of lower quality education.

As in Berti Ceroni's model, the economy produces a final good only through a linear technology that uses human capital as the only production factor:

$$Y_t = H_t = \int_I h_t^i g_t(h_t^i) dh_t^i \qquad (3)$$

where H_t is the aggregate stock of human capital in period *t* and $g_t(h_t^i)$ is the density function that characterizes the distribution of human capital among parents in period *t*, so that $g_t(h_t^i) \geq 0$ and $\int_I g_t(h_t^i) dh_t^i = 1$. The distribution of human capital in the initial generation of parents is exogenously given: $g_0(h_0^i)$, with $h_0^i \in (\alpha, \beta)$ and $\mu_1 = \alpha < \beta$.

Figure 1: Human capital accumulation function



3.1 The Solution to the Microeconomic Problem

The individual maximization programme that parent *i* has to solve in time *t* is given by:

$$\max_{e_t^i} \quad u^i(c_t^{ij}, h_{t+1}^{ij}) = \ln(c_t^{ij}) + \delta h_{t+1}^{ij}$$

$$\text{s.t} \qquad (4)$$

$$c_t^{ij} = h_t^{ij} - e_t^{ij}$$

$$h_{t+1}^{ij} = \begin{cases} \ln(v^j) + a^j & e_t^{ij} \leq b^j \\ \ln[q^j(e_t^{ij} - b^j) + v^j] + a^j & e_t^{ij} > b^j \end{cases}$$

$$(e_t^{ij}, c_t^{ij}) \geq (0,0)$$

Following Behrman and Birdsall (1983), it is assumed that the quality of education is determined by public resource allocation to schooling out of general overall revenues so there is no direct relation between the quality in a particular area and the tax burden of a particular household located in that area. Therefore the budget constraint does not consider taxes.

As in Berti Ceroni's model, the utility function is non-homothetic. The marginal rate of substitution between the parent's consumption in period t and the child's human capital for a given ratio between the two is decreasing in the parent's human capital stock, so that the poor require relatively higher returns from education to start investing.

Replacing the budget constraint and the human capital production function in the utility function and maximizing with respect to e_t^{ij}, the expression of optimal spending in education is obtained:

$$e_t^{*i}(h_t^{ij}) = \begin{cases} b^j & h_t^{ij} \leq \bar{h}^j \\ \dfrac{\delta h_t^{ij} + b^j}{(1+\delta)} - \dfrac{v^j}{q^j(1+\delta)} & h_t^{ij} > \bar{h}^j \end{cases} \tag{5}$$

$$\text{where } \bar{h}^j = b^j + \frac{v^j}{\delta q^j} \tag{6}$$

Note that for human capital levels equal to or lower than the threshold $\bar{h}^j$, education spending is constant at the minimum required level of spending (b^j). For human capital levels above the threshold $\bar{h}_j$, the proportion of income assigned to education increases with the educational level of the parent. As expression (6) shows, the parent's human capital threshold level $\bar{h}_j$, at which education spending starts to increase, is increasing in b^j and in v^j, and decreasing in the quality of education q^j and parents' altruism δ. Replacing (5) in (2), the transition equation that describes the evolution of dynasty *i*'s human capital is obtained:

$$h_{t+1}^{ij} = \phi^j(h_t^{ij}) = \begin{cases} \mu^j & h_t^{ij} \leq \bar{h}^j \\ \ln\left[\dfrac{q^j\delta(h_t^{ij} - b^j) + \delta v^j}{(1+\delta)}\right] + a^j & h_t^{ij} > \bar{h}^j \end{cases} \tag{7}$$

Under the mentioned assumptions regarding the parameters, the dynamics of human capital accumulation of each dynasty is independent of the aggregate dynamic, but is dependent on the social circle *j* to which the dynasty belongs. This transition function $\phi^j(h_t^{ij})$ has a positive slope and is concave for $h_t^{ij} > \bar{h}_j$.

3.2 Conditions for the Emergence of Multiple Equilibria

It is assumed that the current income distribution determines the future one:

$$g_t(h_t^{ij}) = g_{t-1}[\phi^{-1}(h_t^{ij})] \qquad h_t^{ij} \in [\alpha, \beta] \tag{8}$$

Given the initial human capital distribution and the individual transition equation, it is possible to analyse the behaviour of income distribution over time.

The emergence of poverty traps requires the individual transition function to exhibit multiple steady states. In what follows, the conditions under which three stable equilibria emerge are analysed. One of these equilibria is such that dynasties that start with low levels of human capital and income remain uneducated; the second one is such that dynasties that also start with low levels of human capital and income (although higher than the levels of the previous ones), invest in education but as they receive low-quality education, they get trapped in an 'intermediate' level of human capital and income, and can never reach the *good* third equilibrium to which only the dynasties that start with a high level of human capital and income converge.[6]

It is assumed that there are only two sectors or social circles with initial human capital levels clearly differentiated: $j = 1, 2$, with $h_t^{i1} < h_t^{i2}$. The following conditions for the emergence of multiple equilibria are required:

C.1) Because of what each of the parameters represents, it is required that:

$$(v^j, b^j, a^j, q^j) \geq (0,0,0,0) \qquad \forall j = 1,2 .$$

C.2) Analogously, it is understood that the human capital level of a child that does not receive formal education is non-negative, so that:

$$\mu^j = \ln(v^j) + a^j \geq 0 \qquad \forall j = 1,2$$

C.3) At the same time, as it was explained in Section 3.1, it is assumed that:

$$v^1 \leq v^2; b^1 \leq b^2; a^1 \leq a^2; q^1 \leq q^2 \qquad \forall j = 1,2 .$$

C.4) It is also assumed that the human capital threshold at which the spending function starts to increase in the parent's human capital is higher for the dynasties with a higher initial education level. This is consistent with the assumption that the threshold at which education spending starts to be effective is higher for the children with more educated parents than for the children with less educated parents ($b^1 \leq b^2$). Formally, the condition is:

$$\bar{h}^1 < \bar{h}^2 .$$

Replacing (6) in both sides of the inequality and re-arranging, the condition is re-expressed as:

$$\left[\frac{v^1}{\delta q^1} - \frac{v^2}{\delta q^2}\right] < [b^2 - b^1].$$

C.5) It is required that for each *j*, the human capital threshold at which the education spending function starts to increase in the parent's human capital level is higher than the child's human capital level if he does not receive education:

$$\bar{h}^j > \ln(v^j) + a^j \quad \forall j = 1,2 .$$

6 In Berti Ceroni's model, dynasties converge either to the equilibrium of no-education and low income, or to the equilibrium of high education and income.

This condition guarantees that the human capital accumulation curve for each *j* intersects the 45° line, so that corner solutions appear at low income levels, constituting a stable equilibrium: once the dynasty reaches the human capital level μ^j, it remains there forever. Replacing (6) at the left of the inequality and re-arranging, the condition can be re-expressed as:

$$\frac{v^j}{\delta q^j} > [\ln(v^j) + a^j - b^j] \qquad \forall j = 1, 2 \,.$$

For the existence of other equilibria at higher income levels and for the emergence of poverty traps, it is required that the derivative of each transition function j in its concave part (when $h_t^{ij} > \bar{h}^j$), has a slope greater than 1 when it is evaluated at the point where $h_t^{ij} = h_{t+1}^{ij}$. This point will be called h_u^j. Formally, it is required that:

$$\frac{\partial \phi^j (h_t^{ij})}{\partial h_t^{ij}}\Big|_{h_{t+1}^{ij}=h_t^{ij}} = \frac{\partial \phi^j (h_u^j)}{\partial h_u^j} > 1 \,.$$

To find an expression of this condition in terms of the parameters, it is necessary to find first an expression for h_u^j, for which the transition equation (7) needs to be solved, evaluated at the point in which $h_{t+1}^{ij} = h_t^{ij} = h_u^j$:

$$\ln\left[\frac{q^j \delta (h_u^j - b^j) + \delta v^j}{1+\delta}\right] + a^j = h_u^j \tag{9}$$

For simplicity, the following notation will be used: $m^j = \frac{\delta}{1+\delta} q^j$, $p^j = \frac{\delta}{1+\delta}(v^j - q^j b^j)$, and $d^j = e^{a^j}$. Applying the exponential to both sides of equation (9), it can be re-written as:

$$(m^j h_u^j + p^j) d^j = e^{h_u^j} \tag{10}$$

Multiplying both sides of equation (10) by $-e^{(-p^j/m^j)}$ and re-arranging the terms, equation (11) is obtained:

$$-\frac{e^{(-p^j/m^j)}}{m^j d^j} = -\frac{(m^j h_u^j + p^j)}{m^j} e^{\left\{-\frac{(m^j h_u^j + p^j)}{m^j}\right\}} \tag{11}$$

Using the W Lambert Function (cited in Euler 1783), the solution to equation (11) is given by:[7]

$$h_u^j = \frac{-\left\{W\left(-\frac{e^{(-p^j/m^j)}}{m^j d^j}\right) m^j + p^j\right\}}{m^j} \tag{12}$$

[7] In general terms, given an expression $Y = Xe^X$, the W function (also called Product Log) provides a solution to it given by: $X = W(Y)$. The W Lambert function can be expanded in series: $W(Y) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^{n-2}}{(n-1)!} Y^n$.

Replacing expression (12) in the condition for the existence of multiple equilibria, which requires that in h_u^j the transition function has a slope higher than 1, the condition can be stated as:

$$\frac{\partial \phi^j(h_u^j)}{\partial h_u^j} = -\frac{1}{W\left(-\frac{e^{(-p^j/m^j)}}{m^j d^j}\right)} > 1 \tag{13}$$

For condition (13) to be satisfied, it must hold that:

$$-1 < W\left(-\frac{e^{(-p^j/m^j)}}{m^j d^j}\right) < 0 \tag{14}$$

Given that $m^j, d^j, e^{(-p^j/m^j)} > 0$, the argument of the *W* function is negative. At the same time, for real values between $[-1/e, 0]$, function *W* takes values between [–1,0], with W(–1/e)=–1, and W(0)=0. Then for (14) to be satisfied, it is required that:

$$-\frac{1}{e} < -\frac{e^{(-p^j/m^j)}}{m^j d^j}$$

C.6) Replacing m^j, d^j, p^j by their original expressions and re-arranging the inequality, it is obtained that the condition for the existence of multiple equilibria is:

$$\frac{\delta}{1+\delta} > \frac{e^{\left[1-\frac{v^j}{q^j}-a^j+b^j\right]}}{q^j}$$

If condition (C.6) is satisfied, the transition function $\phi^j(h_t^{ij})$ has a slope higher than 1 in h_u^j, and so h_u^j is an unstable point. Given that the function is concave, the slope decreases tending to zero and it intersects the 45° line one more time, at a point that will be called h_L^* for *j*=1 and in h_H^* for *j*=2. h_L^* and h_H^* are stable equilibria.

Figure 2: Transition functions

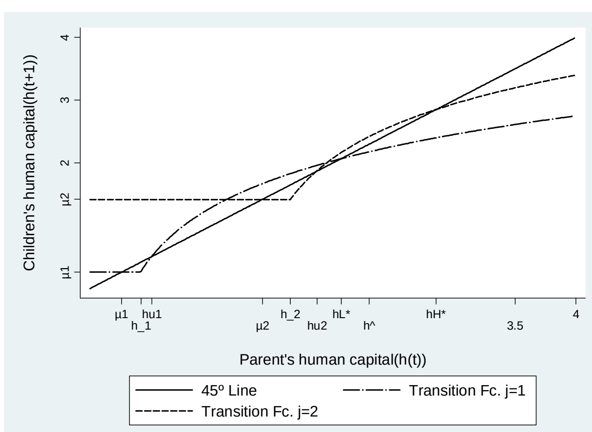


Figure 2 presents one possible set of values of the parameters satisfying the mentioned conditions and producing multiple equilibria for each *j*-transition function.[8] Each individual transition function $\phi^1(h_t^{i1})$ (labelled in the figure as Transition Fc. *j*=1) and $\phi^2(h_t^{i2})$ (labelled in the figure as Transition Fc. *j*=2), presents three steady states, at the human capital levels μ_1, h_u^1, h_L^*, μ_2, h_u^2, h_H^* correspondingly.

3.3 Equilibria at the Aggregate Level: The Poverty Traps

While each of the two *j*-transition functions presents three equilibria, the equilibria that prevail at the aggregate level depend on the interval of human capital levels in which each human capital accumulation function $h_{(t+1)}^{i1}$ and $h_{(t+1)}^{i2}$ operates. In other words, the number and type of equilibria that are determined in the economy depend on the human capital level that distinguishes the two social circles: the more and the less educated. This threshold will be called $\hat{h}$. As an example, $\hat{h}$ could correspond to tertiary (university or other post-secondary) education.

To define this level, two additional conditions will be defined, which allow the configuration of equilibria of interest in this paper. In the first place, it is assumed that:

C.7) $\mu^2 < h_L^*$

This condition requires the human capital level achieved by the children of the more educated parents when they do not receive formal education ($e_t^{i2} < b_2$) to be lower than the human capital level achieved by the children of the less educated parents when they receive formal education ($e_t^{i1} \geq b_1$). If this condition is not satisfied, it would mean that the children of the rich, educated parents who receive no formal education end up having a steady-state human capital (μ^2) higher than the human capital of the children of the uneducated and poor parents who do receive formal education (h_L^*), which would be counter-intuitive. Formally, this condition guarantees the intersection of the curves of human capital accumulation of the two social sectors, $\phi^1(h_t^{i1})$ and $\phi^2(h_t^{i2})$.

Second, it is required that:

C.8) $h_u^2 < h_L^* < \hat{h}$

The first part of this condition requires the human capital level that works as a threshold in sector $j = 2$ below which dynasties end up being non-educated, to be lower than the maximum human capital level that dynasties from sector $j = 1$ can achieve. Although this sounds arbitrary, it may be argued that the opposite case would be extreme in the assumptions regarding social segmentation. The second part of this condition requires the human capital level that distinguishes the two sectors ($\hat{h}$) to be higher than the human capital level to which the initially less educated dynasties that invest in education converge (h_L^*). Again, although it seems an arbitrary condition, together with the previous ones it guarantees an intuitive result: that at least some of the poor decide to invest in education.

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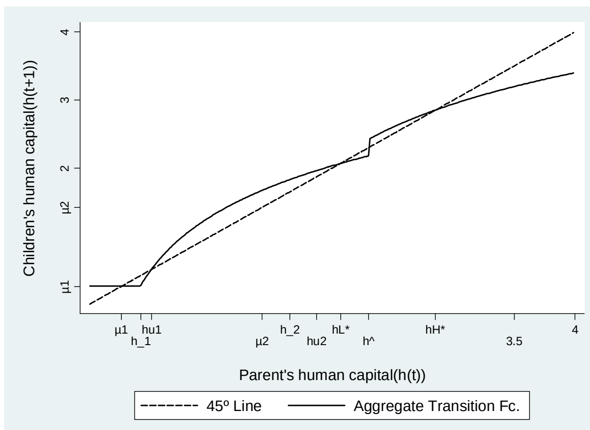
8 Figure 2 has been done with the same values of the parameters as in Figure 1 and for a value of δ=0.5.

Considering all the mentioned conditions, the aggregate transition function is obtained. For parents' human capital levels lower than the threshold ($h_t < \hat{h}$), the human capital accumulation function that prevails is the one corresponding to $j = 1$; for parents' human capital levels above the threshold ($h_t \geq \hat{h}$), the prevailing human capital accumulation is the one that corresponds to $j = 2$. The expression for this aggregate function is given by:

$$\phi(h_t^i) = \begin{cases} \ln(v^1) + a^1 & h_t^i \leq \bar{h}^1 \\ \ln\left[\dfrac{q^1\delta(h_t^i - b^1) + \delta v^1}{1+\delta}\right] + a^1 & \bar{h}^1 < h_t^i < \hat{h} \\ \ln\left[\dfrac{q^2\delta(h_t^i - b^2) + \delta v^2}{1+\delta}\right] + a^2 & h_t^i \geq \hat{h} \end{cases} \tag{15}$$

Figure 3 presents the aggregate transition function under the stated conditions:

Figure 3: Aggregate transition function



The figure shows that the aggregate transition function has a discontinuity at the threshold level $\hat{h}$, and that three stable equilibria are defined at levels μ^1, h_L^*, h_H^* (and an unstable one at h_u^1).[9] Then, in the long run, dynasties are concentrated in three groups. The *very poor* are the dynasties with an initial human capital level below h_u^1. The parents of these dynasties may initially invest in educating their children (with a spending higher than the required threshold $e_t^{i1} > b^1$), but eventually they will stop doing so because the human capital stock decreases from one generation to the next, and they will converge to the steady-state level of human capital given by μ^1, remaining forever below the level h_u^1, that is, poor and uneducated. This result is analogous to the first equilibrium in Berti Ceroni's model.

9 If all conditions of Section 3.2 are satisfied (so that the transition equation of each sector has three stable equilibria) but conditions C.7 or C.8 are not satisfied, other equilibria configurations can occur at the aggregate level. Although these may not be interesting for the purpose of this paper, they are detailed in the Appendix.

The second equilibrium h_L^* corresponds to the *poor*. Dynasties that converge to this equilibrium are those whose initial human capital is above h_u^1 but below $\hat{h}$.[10] The parents of these dynasties invest enough in their children's education ($e_t^{i1} > b^1$). However, given that they move in a low-educated social circle and receive low-quality education, the human capital level to which they eventually converge is considerably lower than the one corresponding to the third possible equilibrium h_H^*. This one corresponds to the *non-poor*, and is reached only by those dynasties with an initial human capital above $\hat{h}$, not only because of their favourable initial conditions, but also because they receive high-quality education.

The two poverty traps constituted by equilibriums μ^1 and h_L^* can be seen not only in relative terms but also in absolute terms if one thinks of two poverty lines, one for extreme poverty – z_1 – and one for poverty – z_2 – such that $\mu^1 < z_1 < h_L^*$, and $h_L^* < z_2 < h_H^*$.

3.4 Income Distribution and Macroeconomic Equilibrium

Once the three long-run equilibria to which different fractions of the society converge are identified, it is possible to obtain the aggregate level of education spending, each period *t* output and the long-run output.

As in Berti Ceroni (2001), at any point in time, income distribution determines current aggregate investment in education and aggregate human capital and income of the next period. Given the human capital (or income) distribution function $g_t(h_t^i)$, define $G_t(\bar{h}^1) = \int_{\mu^1}^{\bar{h}^1} g(h_t^i)dh_t$, and $G_t(\hat{h}) = \int_{\mu^1}^{\hat{h}} g(h_t^i)dh_t$.

Considering expression (5), the threshold localization given by conditions (C.7) and (C.8) ($\mu^2 < h_L^* < \hat{h}$), and that $\int_I g_t(h_t^i)dh_t^i = 1$, the aggregate education spending E_t is given by:

$$E_t = b^1 G(\bar{h}^1) + \frac{\delta}{1+\delta}\int_{\bar{h}^1}^{\beta} h_t g_t(h_t^i)dh_t + \left(\frac{q^1 b^1 - v^1}{q^1(1+\delta)}\right)[G(\hat{h}) - G(\bar{h}^1)] + \left(\frac{q^2 b^2 - v^2}{q^2(1+\delta)}\right)[1 - G(\hat{h})] \quad (16)$$

Considering expressions (3) and (15), the aggregate output is given by:

$$\begin{aligned} Y_t = {} & \mu^1 G_{t-1}(\bar{h}^1) + \int_{\bar{h}^1}^{\hat{h}} \ln[q^1(h_{t-1} - b^1) + v^1] g_{t-1}(h_{t-1})dh_{t-1} \\ & + \int_{\hat{h}}^{\beta} \ln[q^2(h_{t-1} - b^2) + v^2] g_{t-1}(h_{t-1})dh_{t-1} \\ & + a^1[G_{t-1}(\hat{h}) - G_{t-1}(\bar{h}^1)] + a^2[1 - G_{t-1}(\hat{h})] + \ln\left(\frac{\delta}{1+\delta}\right)[1 - G_{t-1}(\bar{h}^1)] \end{aligned} \quad (17)$$

[10] It is worth noting that there are dynasties that despite the fact that they start with a human capital level higher than h_L^* (but lower than $\hat{h}$), they converge to the steady state human capital level h_L^*, lower than the initial one.

As in Berti Ceroni's model, the current – and therefore the initial – income distribution affects the aggregate accumulation of human capital and the output growth along the transition path to the steady state. The more unequal the initial income distribution is, the slower the human capital accumulation. Dynasties with an initial human capital below h_u^1 have (zero or) negative growth rates, as do dynasties with an initial human capital level $h_L^* < h < \hat{h}$. Only dynasties with an initial human capital level $h_u^1 < h < h_L^*$ or $h > \hat{h}$, have positive growth rates.

The negative effect of inequality in the initial human capital distribution on aggregate output persists in the long run because of the existence of poverty traps. The steady-state aggregate output is given by:

$$\begin{aligned} Y_\infty &= \mu^1 G_0(h_u^1) + h_L^*[G_0(\hat{h}) - G_0(h_u^1)] + h_H^*[1 - G_0(\hat{h})] \\ &= h_H^* - (h_H^* - h_L^*)G_0(\hat{h}) - (h_L^* - \mu_1)G_0(h_u^1) \end{aligned} \tag{18}$$

where one can see that the highest potential output level h_H^* is reduced because fraction $[G_0(\hat{h}) - G_0(h_u^1)]$ of the population can reach only output level h_L^*, and fraction $G_0(h_u^1)$ can reach only output level μ^1.[11]

4. Economic Policy Implications

Among the set of parameters $v^j, b^j, a^j, q^j, \delta$ that affect the human capital transition equation, only parameter q^j, the quality of education, is a potential policy instrument. Parameters v^j, b^j, a^j refer to intrinsic characteristics of the two social sectors, the more and the less educated, while parameter δ measures the degree of altruism of parents to children and is common to both social sectors. None of these can be influenced by public policy.

A natural question that arises in this setting is what would be the effect of a policy that guaranteed the same quality of education to all children, independent of the social sector from which they come. Figure 3 depicts the effect of equating the quality of education in the two sectors ($q^1 = q^2$) and holding everything else constant.[12] The slope of the transition equation for $j = 1$ dynasties increases substantially, so that now all dynasties with an initial human capital level above h_u^1 converge to the steady-state human capital level h_H^*, independent of whether their human capital was below or above $\hat{h}$, that is, independent of whether they belong to the more or the less educated sector.[13]

A fraction of the population still remains uneducated, with the human capital level μ^1; they are the dynasties that start with a human capital level lower than h_u^1. Note, however, that the increase in the quality of education for all $j = 1$, makes the threshold level $\bar{h}^1$ decrease (recall expression 6). This

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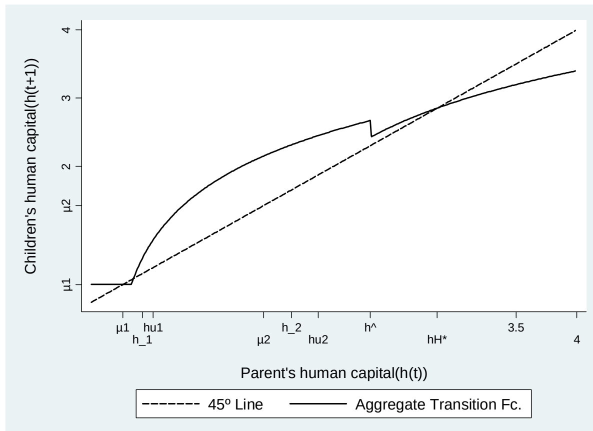
[11] Note that although the steady state aggregate output level depends on the initial income distribution, it does not depend on the initial output level. This is the same as in Berti Ceroni's model.

[12] All the values of Figure 4 are the same as those in Figure 3, except that now $q_1 = q_2 = 20$.

[13] A part of condition (C.8) no longer holds, because now $\hat{h} < h_L^*$.

decrease makes the value h_u^1 also lower (the whole curve $\phi^1(h_t^{i1})$ is now to the left of the original one). Therefore, the fraction of the population that remains uneducated ($G(h_u^1)$) is lower. It can be verified that both the aggregate education spending as well as the current output level and the steady-state output level are higher.[14]

Figure 4: Aggregate transition function with equal quality of education



This policy exercise exemplifies the crucial role that the quality of education can play in the economy. If – despite being public – the education received by a child from an advantaged social circle is higher than that received by a child from a disadvantaged one, social inequalities will be perpetuated causing *poverty traps*. Even when the years of schooling are the same for these two children, the acquired cognitive skills (i.e., acquired human capital) will not be, affecting their future employability in the labour market, and therefore, their future incomes.

The model shows in a very schematic way, that even with marked initial disadvantages represented by lower values of parameters v^j, a^j and b^j, if children from the disadvantaged sectors receive the same quality of education as the children from the advantaged sectors, this can eliminate the initial inequalities in the long run (at least in part).

5. Evidence of Segmentation for an Argentinean Sample

This section provides some empirical intuition regarding one of the key assumptions of the model: segmentation in the quality of education received by children from more and less educated parents. Data from PISA 2000 are used. PISA is an internationally standardized assessment of 15-year-olds, developed by the OECD and UNESCO. The main focus of the assessment in 2000 was in literacy (reading).

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14 If the quality of education was set equal for the two sectors at a higher level than the one used for Figure 4, the *j*=1 transition function would not intersect the 45° line at low human capital levels, eliminating the two first equilibria (μ^1, h_u^1), and all dynasties would converge to the high human capital and income equilibrium h_H^*. In that case, condition (C.5) would not be satisfied. Although this is a possibility in the model, it can be argued that it might be an extremely optimistic point of view on the potential that such a policy could have.

PISA dataset was used to analyze whether the assumption of differences in the quality of education received by children from different social sectors is plausible in the case of Argentina. A thorough testing of this hypothesis should estimate multilevel education production functions, analysing the impact of socioeconomic characteristics on the between-schools variance in test scores. This has been done by Cervini (2002, 2005) using national data, finding empirical support for the idea of social segmentation of the Argentinean educational system.[15] Such an estimation exercise falls beyond the scope of this paper. However, although a basic tool, the hypothesis tests of the difference in means performed here provide some intuition for the assumption.

In the model, h_t^{ij} and h_{t+1}^{ij} represent, correspondingly, the human capital level of the parent and the child *i* from social sector *j*. The social sector is given by the parent's human capital level and income. The case of two social circles, $j=1$ and $j=2$, distinguished by being below and above the human capital level $\hat{h}$, is analysed. It is hypothesized that $\hat{h}$ could correspond to tertiary education. The same threshold is assumed for the hypothesis tests done in this empirical exercise.

Table 1 presents the results of the hypothesis test for the difference in the mean value of an indicator of children's human capital and different indicators of the quality of education they receive (q^j in the model) for two groups of students: group $j=1$, where parents lack tertiary education (either university or other post-secondary education), and group $j=2$, where parents have tertiary education. The mean values for the whole sample are also included in the table. Given that the model assumes education as being public, the reported results correspond to tests conducted only with students who attend public schools. The total sample is 1,222 students, corresponding to 54 schools, of which 860 students are in group $j=1$, and 362 students in group $j=2$.[16]

As expected, the mean value of the International Socioeconomic Index of Occupational Status is significantly higher for the group of children whose parents have tertiary education than for the group whose parents lack similar education (the simplified assumption that income equals human capital is used in the model).[17] Also, measured by PISA's reading score, the average human capital of the children of parents with tertiary education is also significantly higher than the average human capital of those children whose parents lack tertiary education, suggesting the intuitive positive relation between children and parents' human capital assumed in the model. These results are standard and predictable. However, it is interesting to note that among the students attending public schools, there seems to be significant differences in the average quality of education received by children from the more and less advantaged social sectors.

Seven different indicators of the quality of education are used: an index of the quality of the school's educational resources, index of the quality of schools' physical infrastructure, the number of computers per student, index of teachers' behaviour, the teachers-students ratio and the proportion of language teachers who have a third-level qualification. These indicators are constructed by PISA from

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15 Cervini (2002) uses data from the 1997 Operativo Nacional de Evaluación de la Calidad. Cervini (2005) uses data from the 1998 Censo Nacional de Finalización del Nivel Secundario. The collection of both data sources was done by the Ministerio de Cultura y Educación of Argentina.

16 The total sample size of PISA 2000 for Argentina is 3,983 students in 156 schools. However, only 2,094 students, representing 89 schools, have complete data for the selected variables. Among them, 286 students attend 13 private schools, and 586 attend 22 government-sponsored private schools. This is why the sample size is reduced to 1,222 students attending 54 schools that are totally public.

17 The index follows the methodology of Ganzeboom, De Graaf and Treiman (1992). It varies from a minimum value of 16 to a maximum of 90. For more details on its construction, see OECD (2002).

questionnaires to school principals.[18] PISA contains other indicators of school quality but this exercise does not pretend to be exhaustive. Note, however, that the seven indicators cover aspects of educational, physical and human resources of the schools.[19] Results indicate that for the seven indicators on the quality of education, the mean value of the group of children with more educated parents is significantly higher than that of the group with less educated parents. The index of the teachers' behaviour is the only indicator for which the difference is less significant (only at the 10 per cent level).

The same exercise was performed with the sample split into three groups instead of two: students with primary-educated parents or less, those with at least one parent with some secondary education (at a level of either completed lower or upper secondary school), and students with at least one parent who completed tertiary education. The results of the hypothesis tests of differences in the means between the three groups are similar to those reported in Table 1. The difference in the mean values of the education quality indicators between the three possible pairs of groups is statistically significant in most cases, suggesting that children coming from better educated households attend better quality schools.[20] Finally, the same exercise was repeated to include government-dependent private schools as well as private schools and in most cases, the difference in the mean values is significant. In summary, in line with previous evidence, PISA data seem to support the assumption of segmentation in the quality of education between social sectors in Argentina, at least among secondary school students.

One possible explanation for the observed differences may be that many public schools in Argentina form private cooperatives to which parents contribute voluntarily to complement funds received from the government. Although this fund usually represents a small percentage of the school budget, schools with children from the more advantaged social sectors will be in a better position to buy additional educational material or improve the infrastructure. Moreover, schools with students coming from the very disadvantaged social sectors cannot count on such extra funds (or they are very meagre). These schools also need to use a large fraction of public funding for purposes other than strictly educational, i.e., satisfying the students' most urgent basic needs such as providing them with daily meals. Clearly, government's allocation of funds should recognize that some schools have very limited possibilities of supplementing the public budget with voluntary funding from parents, and also that these schools usually have to meet other needs of the students, apart from education.

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18 The indices of the school's educational resources, the quality of its infrastructure and teacher behaviour are based on the perceptions of the school principals. They are asked about the extent (scale ranging from: not at all, a little, some, a lot) to which 15-year-olds are hindered in their learning by different educational aspects. The issues considered are as follows: for the index of the quality of school's educational resources: lack of instructional material, lack of computers, poor library, poor multi-media resources, poor science equipment and poor facilities for the fine arts; for the index of the quality of school's physical infrastructure: poor condition of buildings, poor heating and cooling and/or lighting systems and lack of instructional space; for the index of teachers' behaviour: low expectations of teachers; poor student-teacher relations; teachers not meeting individual students' needs, teacher absenteeism, staff resisting change, teachers being too strict with students, and students not being encouraged to achieve their full potential. In all cases they range from negative to positive values. For detailed information on how the indices are constructed, see OECD (2002).

19 Other available indicators include the hours of schooling per year, the proportion of computers with internet access, the proportion of teachers who are certified by the appropriate authority, an index of teacher shortage, an index of teacher morale and commitment, and indices of school and teacher autonomy. The PISA dataset for Argentina has a high number of missing observations. Therefore, the greater the number of indicators used, the smaller the sample with complete data.

20 With regard to the number of computers per student and the percentage of teachers with third-level qualifications, the difference in means is not significant between the group of children with primary-educated parents and those children whose parents have, at a maximum, secondary education. With respect to teacher-behaviour index, the difference in means is not significant between the group of children whose parents have, at a maximum, secondary education, and the group whose parents have tertiary education.

Table 1: Hypothesis test for difference in mean human capital level and quality of education between children with the more and less educated parents, PISA 2000 data

	h_t^j Parent's human capital measured by the maximum educational level achieved			Mean value Total observations (*j*=1 and *j*=2)
	Parents without tertiary education (*j*=1)		Parents with tertiary education (*j*=2)	
h_t^j Mean value of the international socioeconomic index of occupational status	36.3 (0.40) (35.5 ; 37.1)	t = -22.21 (0.75) (P-Val<1%)	58.5 (0.87) (56.8 ; 60.3)	42.7 (0.79) (41.2 ; 44.3)
h_{t+1}^j Children's human capital measured by PISA 2000 average reading score	421.1 (3.34) (414.4 ; 427.7)	t = -8.77 (4.18) (P-Val<1%)	457.7 (6.19) (445.4 ; 470.0)	431.6 (4.26) (423.2 ; 440.1)
q^j Mean value of the index of the quality of school's educational resources	-0.99 (0.07) (-1.12 ; -0.85)	t = -4.60 (0.07) (P-Val<1%)	-0.66 (0.07) (-0.81 ; -0.52)	-0.89 (0.06) (-1.02 ; -0.77)
q^j Mean value of the index of the quality of schools' physical infrastructure	-0.55 (0.09) (-0.74 ; -0.36)	t = -3.85 (0.001) (P-Val<1%)	-0.23 (0.08) (-0.40 ; -0.07)	-0.46 (0.08) (-0.63 ; -0.29)
q^j Mean value of the number of computers per student	0.027 (0.001) (0.025 ; 0.030)	t = -3.11 (0.001) (P-Val<1%)	0.032 (0.002) (0.028 ; 0.035)	0.029 (0.001) (0.026 ; 0.031)
q^j Mean value of the index of teachers' behaviour	-0.32 (0.06) (-0.43 ; -0.20)	t = -1.86 (0.06) (P-Val<10%)	-0.21 (0.08) (-0.37 ; -0.04)	-0.29 (0.06) (-0.41 ; -0.16)
q^j Teacher-student ratio	0.14 (0.004) (0.13 ; 0.15)	t = -2.77 (0.005) (P-Val<1%)	0.15 (0.006) (0.14 ; 0.16)	0.14 (0.004) (0.14 ; 0.15)
q^j Mean value of the percentage of language teachers with third-level qualifications	0.31 (0.02) (0.28 ; 0.35)	t = -4.06 (0.02) (P-Val<1%)	0.41 (0.03) (0.35 ; 0.47)	0.34 (0.02) (0.30 ; 0.38)
v	860		362	1222

Notes: Tertiary education includes university and post-secondary non-university education. Each cell presents the variable's average value for each group, the standard deviation, and the 95% confidence interval. Below, the t-statistic value of the hypothesis test of equal means for *j*=1 and *j*=2 groups is presented, its standard deviation and the P-Value. For the hypothesis test it was assumed that the two groups *j*=1 and *j*=2 have unknown but equal variances. Hypothesis tests were performed considering the weights provided in the database, which correspond to the Fay's Balance Repeated Replication Weights Methodology. Stata command *svy* was used for this. Results do not vary if these weights are not used. Source: Own elaboration using PISA 2000 data.

6. Conclusions

This paper presents a poverty trap model based on Berti Ceroni (2001), with the important motivation that it incorporates the quality of education. Starting with initial inequality in the distribution of human capital and income, the human capital accumulation dynamics (which respond to optimizing behaviour) lead, under certain conditions, to a situation with three simultaneous steady-state equilibria. For those individuals with an initial, low human capital endowment, it is not profitable to invest in education, and these remain in that situation for ever. Those exceeding a certain human capital threshold will invest in education, but as they receive low-quality education, they remain trapped at low levels of human capital and income. Only the dynasties which initially have high levels of human capital reach the steady state at a high human capital and income level. The existence of poverty traps lowers the economy's current and steady-state aggregate output level as well as the growth rate.

However, the model suggests the possibility to break out of the vicious circle: by equating the quality of education for all social sectors. Such a policy would eliminate the intermediate equilibrium, making it possible for a greater part of the population to converge to the highest levels of human capital and income, producing a higher aggregate steady-state output level and increasing in this way the aggregate social welfare.

The model constitutes only the first step in formalizing the concept that differences in the quality of education may lead to poverty traps. It has some restrictive assumptions, such as the linear production function and non-consideration of physical capital. At the same time, the equilibria configuration analysed here is not the only one possible and depends on the position of the threshold level $\hat{h}$ which distinguishes the two social sectors. The Appendix details the other possible equilibria configurations. Therefore, it cannot be stated that differences in the quality of education received by children from different social sectors necessarily lead to poverty traps, but they do constitute a possibility. Some basic statistical tools applied to PISA 2000 suggest that the model may be appropriate for the Argentinean case. Moreover, and in line with the *World Development Report 2006*, the model gives theoretical support not only to policies targeted to guarantee equal access to education but also to equity in the quality of education.

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Appendix

This appendix analyses the other possible equilibria that arise when the two additional conditions C.7 and C.8 are not satisfied. The case of the three stable equilibria ($\mu^1 < h_L^* < h_H^*$) described in Section 3.5 will be called *Scenario 1*.

1. In the case condition C.7 is not satisfied, that is, if $\mu^2 \geq h_L^*$, it may occur: that the curves $\phi^1(h_t^{i1})$ and $\phi^2(h_t^{i2})$ do not intersect (curve $\phi^2(h_t^{i2})$ would be above curve $\phi^1(h_t^{i1})$), that they intersect for the first time at $\mu^2 = h_L^*$, or that they intersect at some point to the right of μ^2. In any of these cases, because condition C.7 is not satisfied, the first part of condition C.8 will not be satisfied either, so that $h_u^2 > h_L^*$. Under these conditions, the following possibilities can occur regarding condition C.8:

1.a) One possibility is that $\hat{h} < h_L^* < h_u^2$. In this case, there will be three equilibria at $\mu^1 < \mu^2 < h_H^*$. This will be called *Scenario 2*. In it, all the poor (who start with a human capital level lower than $\hat{h}$), end with a steady-state human capital level equal to μ^1, that is, they are trapped in a very low poverty trap forever. On the other hand, the rich (those with a human capital level higher than $\hat{h}$), may converge to two possible steady states, depending on their initial human capital level. If $h_t < h_u^2$, they will converge to the steady-state level μ^2, whereas if $h_t > h_u^2$, they will converge to the steady-state level h_H^*.

1.b) Another possibility is that $h_L^* < \hat{h} < h_u^2$, in which case, to avoid indeterminacies, it is required that $\hat{h} < \mu^2$. Under these conditions, four equilibria are produced at $\mu^1 < h_L^* < \mu^2 < h_h^*$. This will be called *Scenario 3*. In it, the poor with an initial human capital level $h_t < h_u^1$ will end up being not educated, converging to equilibrium μ^1; the poor with an initial human capital level $h_u^1 < h_t < \hat{h}$ invest in education and converge to equilibrium h_L^*. On the other hand, the rich with an initial human capital level $\hat{h} < h_t < h_u^2$ do not invest in education and converge to equilibrium μ^2. Only the rich with an initial human capital level $h_u^2 < h_t$ converge to the highest equilibrium h_H^*. In this case there are three poverty traps at different human capital and income levels: μ^1, h_L^*, μ^2.

1.c) Finally, it is also possible that $h_L^* < h_u^2 < \hat{h}$, in which case the resulting equilibria are the same as with *Scenario 1*.

2. It is possible that condition C.7 is satisfied ($\mu^2 < h_L^*$, so that the two curves $\phi^1(h_t^{i1})$ and $\phi^2(h_t^{i2})$ intersect), but that condition C.8 is not satisfied. In that case, the following options are possible:

2.a) It can happen that $h_u^2 < \hat{h} < h_L^*$, in which event two equilibrium are produced at human capital levels $\mu^1 < h_H^*$. This will be called *Scenario 4*. In this setting all the poor ($h_t < \hat{h}$) remain poor and with low human capital, since they converge to μ^1. All the rich $h_t > \hat{h}$ stay rich and with high human capital, since they converge to h_H^*.

2.b) It may also happen that $h_L^* < h_u^2 < \hat{h}$, in which case the equilibria of *Scenario 1* arise.

2.c) If $h_L^* < \hat{h} < h_u^2$, there is an indeterminacy for dynasties that start with a human capital level $\hat{h} < h_t < h_u^2$, since μ^2 is not an equilibrium in this case.

2.d) Another possibility is that $\hat{h} < h_L^* < h_u^2$, in which case the equilibria of *Scenario 2* arise.

2.e) Finally, it is also possible that $\hat{h} < h_u^2 < h_L^*$, in which case it is required that $\hat{h} < \mu^2$ to avoid indeterminacies. Under these conditions the equilibria of *Scenario 2* arise.