Oxford Poverty & Human Development Initiative (OPHI)
Oxford Department of International Development
Queen Elizabeth House (QEH), University of Oxford

OPHI WORKING PAPER NO. 23

Inequality, Interactions, and Human Development

Suman Seth*

August 2009

Abstract

The Human Development Index, which is multidimensional by construction, is criticized on the ground that it is insensitive to any form of inequality across persons. Inequality in the multidimensional context can take two distinct forms. The first pertains to the spread of the distribution across persons, analogous to unidimensional inequality. The second, in contrast, deals with interactions among dimensions. The second form of inequality is important as dimensional interactions may alter individual-level evaluations as well as overall inequality. Recently proposed indices have incorporated only the first form of inequality, but not the second. It is an important omission. This paper proposes a two-parameter class of human development indices that reflects sensitivity to both forms of inequality. It is revealed how consideration of interactions among dimensions affects policy recommendations. Finally, the indices are applied to the year 2000 Mexican census data to contrast the present approach with the existing approaches.

Keywords: Human Development Index, multidimensional welfare, multidimensional inequality, association-sensitive inequality, generalized means, Mexican census data, measurement

JEL classification: D63, O15, I38

* Department of Economics, Vanderbilt University (suman.seth@vanderbilt.edu)

This study has been prepared within the OPHI theme on inequality.

OPHI gratefully acknowledges support for its research and activities from the Government of Canada through the International Development Research Centre (IDRC) and the Canadian International Development Agency (CIDA), the Australian Agency for International Development (AusAID), and the United Kingdom Department for International Development (DFID).

ISSN 2040-8188 ISBN 978-1-907194-08-5

Acknowledgements

This is a preprint of an article submitted for consideration in the *Journal of Human Development and Capabilities* [2009] copyright United Nations Development Programme.

I am thankful to Professor James Foster for his valuable comments. I thank to Luis Felipe López-Calva for providing access to the year 2000 Mexican census data. This paper is based on a background paper for the 2009 Latin America and the Caribbean Regional Human Development Report. I am grateful to the UNDP/RBLAC for financial support. All opinion and errors in this paper are mine.

Acronyms

CIS	*correlation increasing switch* concept
HDI	Human Development Index
SICS	axiom of *strictly increasing under common smoothing*
SIIA	axiom of *strictly increasing under increasing association*
WDIA	axioms that are *weakly decreasing under increasing association*
WIIA	axioms that are *weakly increasing under increasing association*

Oxford Poverty & Human Development Initiative (OPHI)
Oxford Department of International Development
Queen Elizabeth House (QEH), University of Oxford
3 Mansfield Road, Oxford OX1 3TB, UK
Tel. +44 (0)1865 271915 Fax +44 (0)1865 281801
ophi@qeh.ox.ac.uk http://ophi.qeh.ox.ac.uk/

1. Introduction

According to the United Nation Development Programme (UNDP): '*Human Development is a development paradigm that is about much more than the rise or fall of national incomes*'. That economic affluence cannot be an exclusive indicator of human development is now universally accepted, as it ignores the importance of other attributes, such as education and health. Any measure of human development, therefore, must consider various attributes rather than income alone. The concern for various attributes has inspired the measurement of human development to be multidimensional in nature. Measuring human development, however, has always been a difficult task for policymakers across the globe. In 1990, UNDP introduced the Human Development Index (HDI), which is currently the most widely known measure of human development.

The HDI rankings of countries have been published annually by the United Nations Development Programme (UNDP) in Human Development Reports. The HDI of a region[1] is a simple average of three different indicators of human development. The dimension-specific indicators are estimated by taking a simple average across persons. The method of aggregation makes the HDI readily comprehensible, attractive, and popular, but, at the same time, makes it completely insensitive to inequality across people. Ethically, increasing inequality is detrimental to the human development of a region. Therefore, the HDI certainly ignores an important aspect of the measurement of human development.

There are two different forms of inequality in the multidimensional context. The first form of multidimensional inequality is motivated by the concept of the single-dimensional inequality concerning the spread of the distribution (Kolm 1977). The second form of inequality, in contrast, is typically multidimensional in nature since it is based on the existing correlation[2] among different components of human development (Atkinson and Bourguignon 1982). The first form of inequality is called *distribution sensitive inequality*, whereas the second is called *association-sensitive inequality*. Both forms of inequality have already been incorporated in the construction of various multidimensional poverty indices (Tsui 2002, Bourguignon and Chakravarty 2003) and inequality indices (Tsui 1995, 1999, Bourguignon 1999, Decancq and Lugo 2008). Recently, several modified human development indices have also been proposed incorporating distribution-sensitive inequality, but they are silent about the second form. In this paper, a class of human development indices is proposed that is sensitive to both forms of multidimensional inequality.

Importance of the first form of multidimensional inequality can be traced back to the importance of the single-dimensional inequality concerning the dispersion of the distribution. The second form of multidimensional inequality is important for two reasons. First, the various components of human development are synergistically related to one another. When all dimensions are strongly correlated, then higher achievement in one dimension strongly enforces higher achievements in other dimensions and any one dimension is sufficient for measuring human development. Conversely, less correlation among dimensions makes multidimensional analysis more informative. Therefore, the degree of association among dimensions clearly has relevance for multidimensional evaluations of human development.

Second, association-sensitive inequality is important from the point of view of policy recommendation. Suppose policymakers of a region encompassing high inequality across people desire to improve the level of human development. They have a budget of one indivisible dollar, and their concern is: *Where the*

[1] A region can be a country, a state, or even a society.

[2] For the purpose of this paper, we assume the terms correlation and association as synonymous. The term association is broader, whereas in general, the term correlation implies Pearson's product moment correlation.

dollar should be spent to have the maximum improvement in human development of the entire region. If they measure human development in terms of the traditional HDI, they would have no incentive to undertake any kind of distributive policies. A distribution-sensitive HDI, on the other hand, persuades them to pay more attention towards the lower end of the distribution. Unlike in the single-dimensional context, it is highly probable in the multidimensional context that the poorest person in every dimension may not be the same person. This is a dilemmatic situation for the policymaker in deciding who should receive the dollar. An association-sensitive development index can become a saviour for the policymaker in this situation. However, proper knowledge on the relationship among dimensions is required since dimensions could be either substitutes or complements to each other (Bourguignon and Chakravarty 2003).

The traditional HDI satisfies neither forms of multidimensional inequality. This problem has not been overlooked and several classes of human development indices have been proposed that incorporate inequality across persons. Hicks (1997) proposes a class of indices based on the Sen welfare standard. The index, however, is not subgroup consistent, which means that it can show an improvement in human development of the entire region despite a worsening of the human development of a group while that of the rest is unaffected. This limitation has been pointed out by Foster, López-Calva and Székely (2005), who propose a class of indices based on generalized means. Indeed, this class of indices is distribution sensitive and subgroup consistent. None of these two indices is, however, association sensitive.

The two-parameter class of human development indices proposed in this paper is also based on generalized means. Apart from being both distribution sensitive and association sensitive, the class of indices is subgroup consistent. The sections of this paper are organized as follows. The second section introduces the basic framework and the basic axioms. The third section discusses both forms of multidimensional inequality and introduces the related axioms. The fourth section provides an outline and critical evaluation of the existing human development indices. The fifth section introduces the class of indices that reflects both forms of inequality and illustrates how the application of these indices affects policy recommendations. The sixth section is devoted to an application of the proposed indices to the year 2000 Mexican census data to contrast the results with previous studies and to show how interregional comparisons are reversed by incorporating association-sensitive inequality. The seventh section concludes the paper.

2 Basic Framework and Axioms

In this section, we introduce the basic framework of our analysis and discuss the basic axioms that the traditional HDI satisfies. There are numerous challenges behind measuring human development, such as the choice of proper dimensions, the collection of quality data, the selection of a proper aggregation method to estimate the appropriate level of human development, and many others. This paper focuses mostly on developing an improved methodology for aggregation.

2.1 The Traditional Human Development Index

In 1990, UNDP introduced the HDI to measure human development of countries. Despite a few early modifications since after its inception, the basic framework has remained the same. It is a composite index comprising normalized achievements in three different dimensions: economic prosperity, level of knowledge and skill, and quality of health. The standard of living is measured by the logarithm of per capita gross domestic product adjusted for purchasing power parity. The level of knowledge and skill is measured by a weighted average of two attributes: the adult literacy rate, and the combined gross enrolment ratio for primary, secondary and tertiary schools. The quality of life is measured by the life expectancy rate. Achievement scores for all three dimensions are normalized with respect to the corresponding minimum and maximum targeted scores.[3] These normalized scores are the indicator scores of the respective dimensions. The traditional HDI is a simple average of these three indicator scores.

2.2 Basic Framework

We follow the basic framework of the traditional HDI for developing all the results in this paper. We begin with the assumption that the data are available for all dimensions in normalized achievement form. We further assume that there are N persons and three dimensions. The normalized achievement of the nth person in the dth dimension is denoted by h_{nd}. The normalized achievement of all N persons in all three dimensions is summarized by an $N\times 3$ matrix H. As regions can differ in size, the achievement matrix with population N is denoted by H_N. The overall domain of the index is $H = \bigcup_N H_N$. The set of all possible achievement matrices is denoted by $\mathbf{H}$.

The nth row and the dth column of H is denoted by $h_{n\bullet}$ and $h_{\bullet d}$, respectively. Vector $h_{n\bullet}$ summarizes the normalized achievements of person n in three dimensions such that $h_{n\bullet} = (h_{n1}, h_{n2}, h_{n3})$. Similarly, the column vector $h_{\bullet d}$ summarizes the normalized achievements in dimension d for all N persons such that $h_{\bullet d} = (h_{1d}, \ldots, h_{Nd})$ for all $d = 1, 2, 3$. A human development index is a function W: $\mathbf{H} \to \mathbb{R}$, where $\mathbb{R}$ is the set of real numbers and $W(H)$ is the level of human development corresponding to matrix H in $\mathbf{H}$.

2.3 Generalized Mean

All results in this paper are presented in terms of generalized means. For a vector $x = (x_1, \ldots, x_M)$, where $x_m > 0$ for all $m = 1,\ldots, M$, the generalized mean of order γ is defined by:

$$\mu_\gamma(x) = \begin{cases} \left[\left(x_1^\gamma + \cdots + x_M^\gamma\right)/M\right]^{1/\gamma} & \text{if } \gamma \neq 0 \\ \left(x_1 \times \cdots \times x_M\right)^{1/M} & \text{if } \gamma = 0 \end{cases}.$$

Note that μ_γ is the arithmetic mean or the simple average for $\gamma = 1$. We will be denoting the simple average by μ from this point onward. For $\gamma = 0$, μ_γ is the geometric mean, whereas for $\gamma = -1$, it reduces to the harmonic mean. As the value of γ tends towards $-\infty$, $\mu_\gamma(x)$ converges towards the minimum value in vector x; whereas, $\mu_\gamma(x)$ converges towards the maximum value of x as the value of γ tends towards ∞. Application of generalized means requires all achievements to be strictly positive, i.e., $h_{nd} > 0$ for all $n = 1, 2, \ldots, N$ and for all $d = 1, 2, 3$.

2.4 Basic Axioms

Like the traditional HDI, any class of human development indices, W, should satisfy the following set of basic axioms. The first axiom requires that W should be *continuous* on H in $\mathbf{H}$. The axiom of *continuity* prevents the level of human development of any region from abruptly changing due to a change in any of the elements in the achievement matrix. The second axiom requires that W should remain unaffected if all normalized achievements are equal in H. For example, if the normalized achievement is equal to 0.6

3 For detail calculation please see the technical note of the *Human Development Report* (2006: 394).

for all persons in all dimensions, then $W = 0.6$. This axiom is called *normalization*. According to the third axiom, if all normalized achievements in H are changed in the same proportion, W should also change by the same proportion. Thus, if $H' = \delta H$, then $W(H') = W(H)$. This axiom is called *linear homogeneity*. The axiom of normalization and the axiom of linear homogeneity make the interpretation of any class of human development indices attractive and readily comprehensible.

In measuring the level of human development of a region, personal identity should ethically have no importance. The axiom of *symmetry in people* ensures that we treat all persons as being anonymous. Technically speaking, if achievement matrix H' is obtained by pre-multiplying H by an $N \times N$ dimensional permutation[4] matrix, then $W(H') = W(H)$. This axiom requires that each person receives equal weight during aggregation. Similarly, a human development index that treats each dimension with equal importance should satisfy the axiom of *symmetry in dimension*. According to this axiom, if H' is obtained by post-multiplying H by a 3×3 dimensional permutation matrix, then $W(H') = W(H)$. This axiom, however, is not too convincing in the sense that there is no appropriate justification as to why every dimension should receive equal importance. Therefore, this axiom can be relaxed if the dimensional priorities are of importance.

None of the previously introduced axioms allows us to compare two achievement matrices of varying population size. As we often perform cross-regional comparisons, we require an axiom that allows us to compare regions with varying populations. The *population replication invariance* axiom requires that if the population of a region is replicated several times, then the level of human development remains unchanged. Suppose H' is obtained by replicating the population of H by k times, where k is a positive integer. Then, according to this axiom, $W(H') = W(H)$.

The next axiom requires the overall human development of a region to increase if the achievement of any one person in any single dimension increases. Suppose H' is obtained from H by an increment in only one element of H, then the *monotonicity* axiom requires that $W(H') > W(H)$. This axiom, however, deals with only one person and one dimension. It does not state what happens to the overall level of human development if the human development of an entire group changes. The axiom of *subgroup consistency* requires the overall human development of a region to increase if the human development of a group increases, while that of the other group is unaltered. Suppose the entire region is divided into two subregions in terms of population. The achievement matrices of these two regions are H_{N_1} and H_{N_2} with the population size of N_1 and N_2, respectively, so that $N_1 + N_2 = N$. Let the level of human development of the first region, $W(H_{N_1})$, increase whereas that of the second region, $W(H_{N_2})$, remains unaltered. The axiom of *subgroup consistency*[5] requires the overall human development $W(H)$ to increase. This axiom can be easily extended to more than two subgroups.

Finally, for all classes of human development indices that we consider in this paper, the aggregation procedure involves two stages. The aggregation may occur either first across persons and then across dimensions, or first across dimensions and then across persons (Dutta, Pattanaik and Xu 2003). If both orders of aggregation yield the same level of human development, the human development index is

4 A permutation matrix is a square matrix where each row and column have exactly one element equal to one, while the other elements are equal to zero. An identity matrix is a special type of permutation matrix.

5 The subgroup consistency axiom that we discuss here is *population subgroup consistency* and not *dimension subgroup consistency*.

called path independent[6] (Foster, López-Calva and Székely 2005) and the corresponding axiom is called *path independence*.

3 Sensitivity to Inequality across Persons

The basic axioms that we have already discussed in the previous section do not incorporate existing inter-personal inequality. Lower inequality should, ethically, increase overall human development of a region (Foster and Sen 1997). A policymaker, who truly cares about improving the level of overall human development but uses an index that is insensitive to inequality, would have no incentive to implement any redistributive policy. Therefore, it is crucial for any human development index to be sensitive to inequality across persons. There are two distinct forms of inequality in the multidimensional context. The first is the *distribution-sensitive inequality* (Kolm 1977) and the second is the *association sensitivity inequality* (Atkinson and Bourguignon 1982).

3.1 Multidimensional Distribution Sensitivity

To introduce the first form of inequality, we need to understand the notion of inequality in the single dimensional context. Suppose the only dimension that we consider is income. Let income distribution y be obtained from distribution x by a *Pigou-Dalton transfer*,[7] so that for any two persons, m_1 and m_2, $y_{m_1} = x_{m_1} + \delta$, $y_{m_2} = x_{m_2} - \delta$, $x_{m_2} - x_{m_1} > \delta$, where $\delta > 0$ and $x_m = y_m$ for all $m \neq m_1, m_2$. Intuitively, it requires that income be transferred from a richer person to a poorer person such that the existing income difference is reduced. A Pigou-Dalton transfer, and so a series of Pigou-Dalton transfers, is inequality reducing and we say that distribution y is more equal than distribution x with the same mean income and the same population size. According to another definition,[8] distribution y is obtained from x by a series of Pigou-Dalton transfers if y is obtained from x by pre-multiplying x by a bistochastic[9] matrix B, which is not a permutation matrix, so that $y = Bx$. Any single dimensional HDI that is sensitive to inequality across persons must yield $W(y) > W(x)$. Note that our primary objective is to rank different distributions in terms of the level of human development incorporating inter-personal inequality.[10]

We now introduce distribution sensitivity in the multidimensional framework. Following Kolm (1977), we state that achievement matrix J is more equal than achievement matrix H, if J is obtained from H by pre-multiplying H by a bistochastic matrix, such that $J = BH$. Following Foster, López-Calva and Székely 2005, we call this process as *common smoothing* since we reduce the spread of all dimensions by the same bistochastic matrix. The axiom of *strictly increasing under common smoothing* (SICS) requires that $W(J) > W(H)$ if J is obtained from H by common smoothing. In other words, the human development for J is higher than the human development for H. Any human development index that satisfies this property is

6 Note that the concept of path independence in the single dimensional context is different from this concept of multidimensional path independence.

7 This type of transfer is also called *progressive transfer*.

8 To see the relation between these two definitions in the single-dimensional and the multidimensional context, see Weymark (2006).

9 A bistochastic matrix is a non-negative square matrix whose row sum and column sum are both equal to one. Thus, a permutation matrix is always a bistochastic matrix, by definition, but the reverse is not true.

10 If there are two distributions with the same mean and the same population size, and one is less dispersed than the other, then the former is more equal than the latter, yielding higher human development. These two distributions need not necessarily be obtained from each other resulting from direct transfers.

strictly distribution sensitive. In an analogous manner, the axiom of *weakly increasing under common smoothing* (WICS) requires that $W(J) \geq W(H)$ if J is obtained from H by common smoothing. Any index that satisfies this axiom is weakly distribution sensitive as there is always a possibility that it would yield the same level of human development due to common smoothing. The applicability of the transfer principle is often misunderstood when some of the concerned dimensions are non-transferable such as education and health. As in the single dimensional context with income, the objective of the transfer principle is to rank various existing distributions having the same mean but less dispersion as if one distribution is obtained from another by common smoothing. The latter distribution may not necessarily be obtained from the former by direct transfers.

3.2 Policy Question

Suppose policymakers of a region with high inequality have one indivisible dollar that they can spend only on one dimension of a person. In other words, they can neither split the dollar to spend on multiple dimensions of a person, nor distribute the dollar between people along a dimension. Suppose further that they are concerned about improving the level of human development of the region. The question is: *Where should the dollar be spent to have the maximum improvement in human development of the entire region*? If human development is measured by an index that is insensitive to any form of inequality, the policymaker could spend the dollar on any dimension of any person. The policymaker would have no incentive to undertake redistributive policy that would require the human development index to be distribution sensitive.

Consider for a moment, a situation where the measurement of human development involves a single dimension and a distribution-sensitive index. The policymakers would be motivated to undertake proper redistributive policies besides improving the level of human development. They would most likely spend the dollar on the poorest person in the distribution. In the multidimensional context, the decisionmaking becomes more complicated. If there were a person who was the poorest in all dimensions, any distribution-sensitive index would suggest assisting that person with the dollar. However, if there are three different persons who are most deprived in three different dimensions, decisionmaking becomes dilemmatic. The poorest person in one dimension is not necessarily the poorest one in other dimensions. *Who is the most appropriate person to receive assistance from the policymaker in this situation*? A distribution-sensitive index is silent on this question. Let us consider the following three-person three-dimension example to illustrate the above phenomenon.

It is evident from the achievement matrix presented in Table 1 that Person 1 has a very high level of achievement in all three dimensions. Person 2 has higher achievement in education and health compared to Person 3, but enjoys the same level of achievement in health. For sake of simplicity of our discussion, let us assume that marginal return to the dollar for any dimension at any level is the same. If the measurement of human development is based on any inequality insensitive index, the policymaker is indifferent between spending the budget on education of the first person and on the education of the third person. The policymaker would have no incentive to reduce the prevailing inequality across

Table 1: The first achievement matrix

	Health	Education	Income
Person 1	90	90	90
Person 2	30	80	80
Person 3	30	40	40

persons. On the contrary, if a distribution-sensitive index is used, the government assists either the second or the third person to improve health.

Let the budget of one-dollar increase the score on any dimension of any person by ε units, and the overall achievement of the nth person is obtained by the aggregation function $(h_{n1} + h_{n2} + h_{n3})/3$. The distribution of overall achievement of these three persons is $z = (90, 63.3, 36.7)$. If the second person receives the assistance, the post-spending distribution of overall achievements is $z' = (90, 63.3 + \varepsilon/3, 36.7)$; whereas if the third person is assisted, the post-spending distribution of overall achievements is $z'' = (90, 63.3, 36.7 + \varepsilon/3)$. Indeed, both z' and z'' yield higher human development compared to z by the axiom of monotonicity, but distribution z'' is more equal than distribution z'. Thus, the policymaker should assist the third person, in this situation to obtain a less unequal distribution with improved human development. Note that the first stage aggregation function is a simple mean, where the dimensions are assumed perfect substitutes. It can be also shown that the policymaker should target the second person when dimensions are perfect complements. For the same achievement matrix, the government should assist different persons, depending on how different dimensions are related to each other. Therefore, it is evident from the above example that any human development index should incorporate interdimensional *interaction* at the individual or disaggregated level. An index that is sensitive to interdimensional interaction enables the policymaker to implement proper policy in these circumstances.

3.3 Multidimensional Association Sensitivity

Now that we know that interdimensional interaction is capable of ensuring improved policy recommendation, it is important to note that the concept is closely related to prevailing correlation or association among dimensions (Bourguignon and Chakravarty 2003). High correlation among dimensions is detrimental to the human development of a region whenever dimensions are substitutes, since high correlation reflects high inequality across persons. On the other hand, high correlation is beneficial to the human development of the region whenever dimensions are complements, because dimensions are enjoyed more effectively by people if consumed together. Suppose G and H are two achievement matrices with the same population size and the same dimensional means, but G is obtained from H by increasing the correlation among dimensions. If the dimensions are substitutes, then $W(G) < W(H)$, whereas if the dimensions are complements, then $W(G) > W(H)$.

There are several ways in which an achievement matrix can be obtained from another by increasing correlation between dimensions. Bourguignon and Chakravarty (2003) introduce a concept called *correlation increasing switch* (CIS), where an achievement matrix G' could be obtained from H by increasing the correlation between any two dimensions. Consider the following set of matrices, where we denote persons in rows and dimensions in columns. Matrix G' is obtained from matrix H by a CIS that increases the correlation between the second and the third dimension. However, correlations between the second dimension and other two dimensions have fallen at the same time. Thus, the definition of CIS is not completely transparent in terms of increasing correlation between dimensions. There is another approach introduced by Tsui (1999) following Boland and Proschan[11] (1988) that seems to be more appealing. Suppose there are two persons, n_1 and n_2, with person n_1 having strictly higher achievement in some dimensions and strictly lower achievement in some others.

11 This approach is applied by Bourguignon (1999), Tsui (1999, 2002), and Alkire and Foster (2007). Bourguignon and Chakravarty (2003) use an almost similar concept called *correlation increasing switch* in the two dimensional context.

$$H=\begin{bmatrix}3&3&3&3\\2&2&3&2\\3&3&2&3\end{bmatrix},\ G'=\begin{bmatrix}3&3&3&3\\2&\mathbf{3}&3&2\\3&\mathbf{2}&2&3\end{bmatrix},\ G=\begin{bmatrix}3&3&3&3\\\mathbf{3}&\mathbf{3}&3&\mathbf{3}\\\mathbf{2}&\mathbf{2}&2&\mathbf{2}\end{bmatrix}$$

Achievement matrix G is obtained from H by an *association increasing transfer* if $G \neq H$, $g_{n_1,\bullet} = (\min[h_{n_1,1}, h_{n_2,1}], \min[h_{n_1,2}, h_{n_2,2}], \min[h_{n_1,3}, h_{n_2,3}])$, $g_{n_2,\bullet} = (\max[h_{n_1,1}, h_{n_2,1}], \max[h_{n_1,2}, h_{n_2,2}], \max[h_{n_1,3}, h_{n_2,3}])$, and $g_n = h_n$ for all $n \neq n_1, n_2$. Intuitively, association among dimensions increases if one person has strictly higher achievement in some dimensions but strictly lower in others before the transfer, and obtains higher achievement in all dimensions than the other does after the transfer. In the example above, G is obtained from H by an association increasing transfer, because Person 2 has higher achievements in all dimensions compared to Person 3. Note that the definition of CIS and association increasing transfer is identical if there are only two dimensions, but they differ when there are more dimensions.

Association-sensitive Axioms

The axiom of *strictly decreasing under increasing association* (SDIA) requires that if G is obtained from H by a series of association increasing transfers, then $W(G) < W(H)$. On the contrary, the axiom of *strictly increasing under increasing association* (SIIA) requires that if G is obtained from H by a series of association increasing transfers, then $W(G) > W(H)$. The corresponding weak versions of the axioms are *weakly decreasing under increasing association* (WDIA) and *weakly increasing under increasing association* (WIIA), respectively.

3.4 Sequence of Aggregation versus Association Sensitivity

We assume in this paper that all classes of human development indices can be aggregated in two stages. Either the first stage aggregation is across persons and the second stage aggregation is across dimensions, or the first stage aggregation is across dimensions and the second stage aggregation is across persons. Association sensitivity, however, restricts these sequences of aggregation.

Theorem 1. *A human development index that aggregates first across persons and then across dimensions does not satisfy SDIA and SIIA.*

Proof. Let us assume that W is a human development index that first aggregates across persons and then across dimensions. If G is obtained from H by a series of association increasing transfers, it leaves the dimension-specific distributions unchanged and increases the association among dimensions only. The first stage aggregation yields the same distribution of overall achievements for both G and H. Therefore, W yields the same level human development for both G and H.

However, W satisfies SDIA (SIIA) if $W(G) < (>)\ W(H)$. This is a contradiction. Hence, W does not satisfy SDIA (SIIA).[12] ▪

Corollary 1. A *path independent human development index does not satisfy SDIA and SIIA.*

Proof. For a path independent human development index, the sequence of aggregation does not matter and it yields the same result. Therefore, a path independent measure also aggregates across persons first

12 The idea behind the theorem is analogous to Proposition 1 and Proposition 2 in Pattanaik, Reddy and Xu (2007).

and then aggregates across dimensions. Hence, according to the theorem above, a path independent measure does not satisfy SDIA and SIIA. ▪

Therefore, among the class of human development indices that we consider in this paper, a strictly association-sensitive index must aggregate first across dimensions and then across persons, and it must not be path independent.

4. Existing Human Development Indices

The first obvious index is the traditional HDI. Given achievement matrix H, the traditional HDI first aggregates across persons and then across dimensions, both by simple mean. We denote the traditional index by W_A and formulate as:

$$W_A(H) = \mu(\mu(h_{\bullet 1}), \mu(h_{\bullet 2}), \mu(h_{\bullet 3})).$$

The traditional HDI satisfies all basic axioms including path independence (Dutta, Pattanaik and Xu 2003) and subgroup consistency. Aggregation by the simple mean, however, prevents the index from being strictly distribution sensitive; and the axiom of path independence prevents it from being strictly association sensitive. Thus, the traditional HDI does not satisfy SICS, SDIA, and SIIA, but satisfies WICS, WDIA, and WIIA.

Hicks (1997) comes up with the first modified human development index that is sensitive to any kind of inequality across persons. Hicks suggests aggregating across persons in the first stage, applying Sen's welfare standard to obtain dimension-specific standards, and then applying simple mean across dimensions in the second stage to obtain the distribution-sensitive HDI. The Sen welfare standard is defined by $S(h_{\bullet d}) = \mu(h_{\bullet d})[1 - \text{Gini}(h_{\bullet d})]$ for all $d = 1, 2, 3$, where $\text{Gini}(h_{\bullet d})$ is the Gini coefficient of distribution $h_{\bullet d}$. The formal definition of the Hicks index is:

$$W_{Hi}(H) = \mu(S(h_{\bullet 1}), S(h_{\bullet 2}), S(h_{\bullet 3})).$$

This index satisfies all basic axioms except path independence and subgroup consistency. The index is not path independent because the change in the sequence of aggregation would yield different levels of human development. The index is not subgroup consistent because the Gini coefficient is not subgroup consistence. Hicks' index is distribution sensitive due to the fact that the Gini coefficient is Lorenz consistent. However, according to Theorem 1, the sequence of aggregation prevents it from being strictly association sensitive. Therefore, Hicks' index satisfies SICS, WDIA, and WIIA, but does not satisfy SDIA and SIIA.

That the Hicks index does not satisfy subgroup consistency has been recently pointed out by Foster, López-Calva and Székely (2005), who propose a class of modified indices based on the generalized means. In the first stage, the aggregation takes place across persons using generalized mean of order α so that $\alpha \leq 1$. In the second stage, the aggregation takes place across dimensions using the same order of generalized mean. Parameter α can be interpreted as both an inequality aversion parameter and a parameter measuring the degree of substitution among dimensions. The class of indices can be formally expressed as:

$$W_{FLS}(H) = \mu_\alpha(\mu_\alpha(h_{\bullet 1}), \mu_\alpha(h_{\bullet 2}), \mu_\alpha(h_{\bullet 3})).$$

This class can also be reformulated as:

$$W_{FLS}(H) = \mu_\alpha(\mu_\alpha(h_{1\bullet}), \ldots, \mu_\alpha(h_{n\bullet})) = \mu_\alpha(H).$$

This class of indices satisfies all basic axioms including path independence. The major contribution of the paper is that the class satisfies subgroup consistency in addition to distribution sensitivity. For $\alpha = 1$, $W_{FLS}(H) = W_A(H)$. The secret behind satisfying the path independence axiom is the use of the same parameter for both the degree of substitution and inequality aversion. The generalized mean places more emphasis on the lower end of the distribution for $\alpha < 1$. Therefore, if a distribution is obtained from another by common smoothing, the generalized mean of the former is higher than the latter. The generalized mean is Lorenz consistent and makes the Foster-López-Calva-Székely class of indices strictly distribution sensitive for $\alpha < 1$. Therefore, it satisfies SICS for $\alpha < 1$ and WICS for $\alpha \leq 1$. However, this class of indices is not strictly association sensitive by Lemma 1. Hence, the Foster, López-Calva-Székely class does not satisfy SDIA and SSIA, but does satisfy WDIA and WSIA.

5. Human Development Index: Incorporating Interactions

In this section, we propose a two-parameter class of indices that is sensitive to both forms of multidimensional inequality. The class is based on generalized means and is motivated primarily by the Foster et al. single-parameter class of indices. Note that the first stage aggregation must take place first across dimensions and then across persons in order to ensure association sensitivity in strict sense. Therefore, instead of aggregating both stages by the generalized means of the same order, the proposed class aggregates first across dimensions and then across persons using different orders of generalized mean. The first stage aggregation applying generalized means is analogous to the *constant elasticity of substitution* aggregation function. Several studies have already proposed the CES functional form for the first stage aggregation, while constructing various classes of association-sensitive inequality (Bourguignon 1999) and poverty indices (Bourguignon and Chakravarty 2003). The proposed class of human development index is defined as:

$$\Omega(H) = \mu_\alpha(\mu_\beta(h_{1\bullet}), \ldots, \mu_\beta(h_{n\bullet}));$$

where $\alpha, \beta \leq 1$.[13] To establish the link with the previously proposed class of indices, we use the same symbols for both parameters as has been previously proposed by Bourguignon (1999). Parameter β can be interpreted as the parameter measuring the degree of substitution among dimensions, whereas parameter α can be interpreted as the inequality aversion parameter. For a characterization of the proposed class of indices, see Seth (2009).

Let us provide a brief description of the characterization of the class. First, for the sake of simplicity of representation and transparency while weighting various dimensions, it is assumed that the first stage aggregation functions are additively separable and are identical across persons. Technically speaking, the first stage aggregation function $\Psi(\bullet)$ for any person n can be formulated as:

$$\Psi(h_{n\bullet}) = \Phi(\upsilon_1(h_{n1}) + \upsilon_2(h_{n2}) + \upsilon_3(h_{n3})),$$

where Φ is continuous and υ_d is continuous for all $d = 1, \ldots, D$. Then it can be shown that the proposed class is the natural class of indices given the following set of axioms: *continuity*, *normalization*, *linear homogeneity*, *symmetry in people*, *symmetry in dimension*, *population replication invariance*, *monotonicity*, *subgroup consistency*, and *SICS*. It can be further shown that the class of indices satisfies *SDIA* if, and only if,

[13] For $\alpha = 0$ and $\beta = 0$, the proposed indices take the corresponding geometric mean forms.

$\alpha < \beta \leq 1$, and satisfies *SILA* if, and only if, $\beta < \alpha \leq 1$. We have already discussed in the previous section that *SDLA* corresponds to the situation where dimensions are substitutes since higher correlation increases inequality across persons, whereas *SILA* corresponds to the situation where the dimensions are complements.

In the first stage, achievements of each person are aggregated by the generalized mean of order β. The first stage aggregation function for the *n*th person is:

$$\Psi(h_{n\bullet}) = \mu_\beta(h_{n\bullet}) = [(h_{n1}^\beta + h_{n2}^\beta + h_{n3}^\beta)/3]^{1/\beta} \text{ for all } n,$$

with $\Phi(\bullet) = [\bullet]^{1/\beta}$ and $\upsilon_1(h_{nd}) = (h_{nd})^\beta/3$ for all $d = 1,2,3$. In this formulation, every dimension is treated with equal importance, as in the traditional HDI.[14]

The distribution of overall achievement from the first stage aggregation yields $(\mu_\beta(h_{1\bullet}), \mu_\beta(h_{2\bullet}), \ldots, \mu_\beta(h_{n\bullet}))$. In the second stage, these overall achievements are aggregated across persons again by using generalized means, but of a different order α, which is the inequality aversion parameter. Lower α entails more emphasis at the lower end of the distribution, and the distribution with more inequality is punished. The proposed class of indices satisfies all basic axioms including subgroup consistency for all values of α and β but does not always satisfy the axiom of path independence. Note that the proposed class of indices is identical to the traditional HDI for $\alpha = \beta = 1$ and is identical to the Foster-López-Calva-Székely class of indices for $\alpha = \beta \leq 1$. However, Theorem 1 and Corollary 1 rule out all indices for which $\alpha = \beta$.

That the proposed class of indices satisfies SICS for the above restrictions follows directly from Theorem 6 of Kolm (1977). According to the theorem, if the first stage aggregation function is concave and the second stage aggregation function is quasi-concave, the class of indices is sensitive to common smoothing. For parameter restrictions α, $\beta \leq 1$ and $\alpha \neq \beta$, the first stage aggregation function is concave and the second stage aggregation function is quasi-concave.[15]

Next, we provide an intuitive interpretation why the proposed class of indices satisfies *SDLA* and *SILA* under appropriate restrictions. Consider a two-person two-dimension example, where the achievement vectors of both persons are $h_1 = (h_{11}, h_{12})$ and $h_2 = (h_{21}, h_{22})$ so that $h_{11} > h_{21}$ and $h_{12} < h_{22}$. The proposed class of indices would yield:

$$\Omega(h_1, h_2) = \left[\frac{1}{2}\left(\frac{h_{11}^\beta + h_{12}^\beta}{2}\right)^{\frac{\alpha}{\beta}} + \frac{1}{2}\left(\frac{h_{21}^\beta + h_{22}^\beta}{2}\right)^{\frac{\alpha}{\beta}}\right]^{1/\alpha}.$$

Now, suppose that an association-increasing transfer takes place and the new achievement vectors are $g_1 = (h_{11}, h_{22})$ and $g_2 = (h_{21}, h_{12})$; the first person has more achievements in both dimensions. The proposed class of indices yields:

14 The above formulation can be generalized to consider unequal weights.

15 Generalized mean $\mu_\gamma(x)$ is both concave and quasi-concave for $\gamma \leq 1$. Similarly, $\mu_\gamma(x)$ is both strictly concave and strictly quasi-concave for $\gamma \leq 1$.

$$\Omega(g_1, g_2) = \left[\frac{1}{2}\left(\frac{h_{11}^{\beta} + h_{22}^{\beta}}{2} \right)^{\frac{\alpha}{\beta}} + \frac{1}{2}\left(\frac{h_{21}^{\beta} + h_{12}^{\beta}}{2} \right)^{\frac{\alpha}{\beta}} \right]^{1/\alpha}.$$

Due to the transfer, the overall achievement of the first person increases but that of the second person falls. In the second stage, the direction of change in the level of human development is determined by the interaction between parameters α and β. If $\alpha < \beta$, then the human development index is 'punished' by the increase in association. On the contrary, for $\alpha > \beta$, increase in association is rewarded.[16]

The proposed class of indices seems to be analogous to the formulation of the welfare function proposed by Bourguignon (1999), while commenting on the class of inequality indices proposed by Massoumi (1999). Bourguignon proposes the following class of welfare indices:[17]

$$W_B(H) = \mu([\mu_\beta(h_{1\bullet})]^{\alpha}, \ldots, [\mu_\beta(h_{n\bullet})]^{\alpha}) = [\Omega(H)]^{\alpha},$$

where α is the inequality aversion parameter and β is the parameter measuring the degree of substitution among dimensions so that $\beta < 1$ and $0 < \alpha < 1$. This class is certainly strictly sensitive to correlation among dimensions. Besides, it satisfies the basic axioms and SICS but does not satisfy path independence.

Indeed, these two classes differ from each other in many aspects. First, Bourguignon states the result of association sensitivity for the class in terms of only two dimensions, in the same line as Bourguignon and Chakravarty 2003) do for the class of poverty indices. No clear definition of the multi-dimensional (more than two dimensions) version of the correlation-increasing transfer principle is provided. If the multidimensional version is equivalent to the CIS, then this is not an appropriate form of the correlation-increasing transfer discussed earlier. Second, the role of the inequality-aversion parameter α is not clear for this class. The Dalton-type inequality index that is constructed from this welfare index does not have a monotonic relationship with α (an example is provided in the next section). It is difficult for any policymaker to decide what value of α enables the Bourguignon class of indices to be more sensitive to inter-personal inequality. The proposed class of indices is free from this problem. Finally, unlike the proposed class, no characterization result is provided for choosing the particular form of welfare function for the Bourguignon index.

The Dalton type inequality index that Bourguignon proposes can be formulated as:

$$I_{W_B} = 1 - W_B(H)/W_B(\hat{H}),$$

where $\hat{H}$ is a matrix with all persons having the same average achievement vectors. Likewise, it is possible to construct a Dalton type inequality index from the proposed class of indices as:

$$I_\Omega = 1 - \Omega(H)/\Omega(\hat{H}).$$

16 For detail proof, see Seth (2009).

17 The original welfare index by Bourguignon (1999) is defined using any arbitrary weights on dimensions. However, here we consider the equal weight version of the index without loss of generality.

5.1 Policy Recommendation

Recall that an index that is only sensitive to distribution cannot solve the dilemmatic situation facing the policymaker. Assuming dimensions are perfect substitutes, the new class of indices suggests that the government must target the person who is the poorest in terms of overall achievement. If the dimensions are perfect complements, then the policymaker should target the person with the least achievement in any dimension and assist that person to improve the most deprived dimension. Consider our example based on three persons, three dimensions. Again assume that the dollar increases achievement in any dimension at any level by the same amount. Suppose the second stage aggregation is strictly distribution sensitive. If the dimensions are perfect substitutes, then the third person should receive the assistance. On the contrary, if the dimensions are perfect complements, assistance should go to the second person to improve his level of education. For a detailed discussion on policy recommendation, see Seth (2009).

Table 2: The second achievement matrix

	Health	Education	Income
Person 1	90	90	90
Person 2	80	20	80
Person 3	30	40	40

6. Empirical Illustration

In this section, we provide an empirical illustration to demonstrate how interregional ranking might be reversed if interdimensional interaction is incorporated in the human development index. With this illustration, we also contrast the results obtained by applying the proposed index with the existing indices. We use the sample data from the Mexican population census for the year 2000. It is the same dataset used by Foster, López-Calva and Székely (2005). The sample dataset contains information on education and income for 9,727,387 individuals from almost 2.27 million households. The dataset does not contain adequate information on individual health for calculating the life expectancy rate or child mortality rate. We pursue the same approach as Foster, López-Calva and Székely, and use the municipality-level infant mortality or infant survival rates data as proxy. Although the infant mortality variable does not capture individual-level inequality across persons, it captures, at least, health inequality at the municipality level.

The individual- and household-level incomes are not comparable to that of the standard income measure used in the HDI. To solve this problem, we apply the same two-step approach used by Foster, López-Calva and Székely. In the first step, we estimate the per capita income for each household from the sample. In the second step, we raise these by factors equal to the ratio of the state-level GDP per capita for the year 2000 (provided by the National Statistical Institute), to the state-level census per capita income. The inflated per capita income for each household is then normalized with respect to a minimum of 100 pesos[18] and a maximum of 226,628 pesos.[19] The education variable is constructed by combining the literacy and enrolment status of individuals. The literacy rate for each individual is

[18] Following the methodology of the UNDP, we take the logarithm of income and, accordingly, restrict the lower bound of income to a positive value.

[19] This value is equivalent to USD 40,000 that is applied by the UNDP as an upper limit of per capita GDP. We use a deflator from the 2002 *Human Development Report*.

estimated by the ratio of the number of literate persons over 14 years of age to the total number of persons in the household over 14 years of age. The enrolment rate of each individual is estimated by the ratio of the number of persons attending school within the age group 6 – 24 years to the total number of persons within the same age group in the household. The education index is estimated by a weighted average of the literacy rate and the enrolment rate. According to the standard HDI methodology, we attach two-thirds weight to the literacy rate and one-third weight to the enrolment rate. The literacy rate and the enrolment rate are normalized between zero and one.[20]

Finally, the municipality-level infant mortality rate is used as a proxy for the individual-level health. As Foster, López-Calva and Székely point out, the health variable makes the estimation of the HDI biased since it does not capture the intramunicipality inequality on child mortality rate. However, in this paper, we are still going to assume that in every municipality all individuals have the same health status and thus use the variable to estimate the state-level association-sensitive human development indices. The child mortality rate is normalized between zero and 100.[21] We have access to infant mortality rates for 2,441 municipalities. We apply different human development indices to the data to calculate the level of human development for the different states of Mexico.

In Table 3, we depict the state-wise scores on human development applying the traditional HDI, the Foster, López-Calva-Székely index, and the proposed index. For the purpose of this example, we assume the dimensions to be substitutes rather than complements. The first column lists 32 states of Mexico. The second column reports the state-wise scores applying the traditional HDI with $\alpha = \beta = 1$. Districto Federal ranks first with a score of 0.840, whereas Chiapas ranks last (32nd) with a score of 0.623. The third column reports the human development score applying the Foster, López-Calva-Székely index with $\alpha = \beta = -2$. Again, Districto Federal ranks first, but with a score of 0.479, whereas Chiapas ranks last (32nd), but with a score of only 0.160. In the final column, we report the human development score for different states applying the association-sensitive index with $\alpha = -3$ and $\beta = -1$. Dimensions are assumed to be substitutes since $\alpha < \beta$. Similar to the earlier two approaches, Districto Federal ranks first, but this time Guerrero ranks last.

Ranking under both the proposed index and the Foster, López-Calva-Székely index differs from that of the traditional HDI ranking and discount for the existing inequality. It can be shown that the proposed index takes into account changes in association among dimensions, whereas the other two indices do not. We choose a state at random and order the dimensions so that the interdimensional association increases. We choose Tabasco and order the dimensions by a sequence of association increasing transfers until no further association-increasing transfer is possible. Table 4 summarizes the post-transfer human development scores of the different approaches for Tabasco.

Note that the development scores of Tabasco that are based on the traditional HDI and the Foster, López-Calva-Székely index are the same as in the pre-transfer situation. As earlier, Tabasco ranks 22nd in terms of the traditional HDI and 15th in terms of the Foster, López-Calva-Székely index. However, Tabasco's rank drops in terms of the association-sensitive index since $\alpha < \beta$ and dimensions are assumed to be substitutes. Previously, Tabasco scored 0.254 and ranked 14th. The post-transfer score drops to 0.244 and Tabasco is relegated to the 15th place. Therefore, Tabasco is penalized for increasing interdimensional association. The overall impact of inequality is caused solely by the first form of inequality if dimensions are assumed to be independent of each other and we use $\alpha = \beta$.

---

[20] For the households with no persons aged between 6 and 24 years, the literacy rate receives a weight equal to one, whereas for the households with no person older than 14 years, the enrolment rate receives a weight equal to one.

[21] Child mortality is measured as the number of children not surviving per 1,000 births.

Table 3: State-wise ranking for the different human development indices

States	Human development index (W_A)		Foster, López-Calva, Székely index (W_{FLS})		Proposed index (Ω)	
	$(\alpha = 1, \beta = 1)$		$(\alpha = -2, \beta = -2)$		$(\alpha = -3, \beta = -1)$	
Aguascalientes	0.790	(6)	0.418	(4)	0.333	(4)
Baja California	0.797	(3)	0.370	(7)	0.299	(7)
Baja California Sur	0.795	(4)	0.391	(5)	0.317	(5)
Campeche	0.759	(14)	0.281	(17)	0.238	(16)
Coahuila de Zaragoza	0.793	(5)	0.423	(3)	0.337	(3)
Colima	0.778	(9)	0.302	(14)	0.251	(15)
Chiapas	0.623	(32)	0.160	(32)	0.157	(31)
Chihuahua	0.782	(8)	0.283	(16)	0.232	(19)
Distrito Federal	0.840	(1)	0.479	(1)	0.368	(1)
Durango	0.738	(18)	0.322	(11)	0.270	(10)
Guanajuato	0.719	(23)	0.251	(25)	0.218	(24)
Guerrero	0.654	(30)	0.163	(31)	0.156	(32)
Hidalgo	0.700	(26)	0.222	(26)	0.199	(26)
JaliSCo	0.772	(11)	0.315	(12)	0.258	(13)
México	0.762	(13)	0.337	(9)	0.277	(9)
Michoacán de Ocampo	0.696	(28)	0.214	(28)	0.193	(28)
Morelos	0.752	(16)	0.268	(19)	0.229	(21)
Nayarit	0.729	(21)	0.252	(23)	0.219	(23)
Nuevo León	0.814	(2)	0.443	(2)	0.346	(2)
Oaxaca	0.638	(31)	0.172	(30)	0.164	(30)
Puebla	0.707	(25)	0.219	(27)	0.197	(27)
Querétaro Arteaga	0.755	(15)	0.278	(18)	0.233	(17)
Quintana Roo	0.777	(10)	0.326	(10)	0.267	(11)
San Luis Potosí	0.716	(24)	0.258	(21)	0.223	(22)
Sinaloa	0.751	(17)	0.268	(20)	0.232	(18)
Sonora	0.790	(7)	0.386	(6)	0.309	(6)
Tabasco	**0.719**	**(22)**	**0.296**	**(15)**	**0.254**	**(14)**
Tamaulipas	0.771	(12)	0.349	(8)	0.287	(8)
Tlaxcala	0.736	(19)	0.309	(13)	0.258	(12)
Veracruz de Ignacio dlL	0.698	(27)	0.213	(29)	0.193	(29)
Yucatán	0.732	(20)	0.252	(24)	0.217	(25)
Zacatecas	0.690	(29)	0.253	(22)	0.23	(20)
Mexico	0.744		0.260		0.233	

Table 4: Post-transfer development scores of Tabasco under different methods

State	HDI		W_{FLS}		Ω	
	$(\alpha = 1, \beta = 1)$		$(\alpha = -2, \beta = -2)$		$(\alpha = -3, \beta = -1)$	
Tabasco	0.719	(22)	0.296	(15)	0.244	(15)

Next, with an application on Tabasco, we show that the Bourguignon index is not consistent with respect to the inequality aversion parameter. In Table 5, we report the inequality level according to both indices and for different combinations of α and β. For both the Bourguignon class of indices and the proposed class of indices, α is the inequality aversion parameter. A society that is more averse towards inequality across persons should choose a lower value of α. We fix the other parameter β at 0.95 and vary α. It is evident from Table 5 that the level of inequality according to the Bourguignon class of indices increases initially and then falls. Therefore, the inequality aversion parameter fails to reflect society's aversion towards inequality in a consistent manner. On the other hand, the inequality index based on the proposed class of indices has a consistent relationship with respect to its inequality aversion parameter. A society chooses a lower value of α if the society is more averse towards inequality across persons.

Table 5: Bourguignon index and the inequality aversion parameter (α) with β = 0.95

Value of α	**0.9**	**0.7**	**0.5**	**0.3**	**0.1**	**-0.1**	**-0.3**
I_{WB}	0.0025	0.004	0.0043	0.0035	0.0015	-	-
I_{Ω}	0.0028	0.0057	0.0086	0.0117	0.0148	0.018	0.0212

7. Conclusion

In this paper, we propose a class of human development indices that is sensitive to two forms of multidimensional inequality: the distribution-sensitive inequality and the association-sensitive inequality. The first form deals with the spread of the distributions concerned, whereas the second is concerned with the correlation among dimensions. The proposed class of indices is based on generalized means and the achievements are first aggregated across dimensions and then across persons. This particular order of aggregation is appropriate because the indices that first aggregate across persons and then across dimensions, are not sensitive to interdimensional interactions. It is further shown in the paper that the class of path independent human development indices is excluded from the set of strictly association-sensitive indices. Consequently, the distribution-sensitive human development indices proposed by Hicks (1997) and Foster, López-Calva and Székely (2005) that aggregate first across persons and then across dimensions, leave both indices completely insensitive to association in the strict sense. The proposed class of indices can be thought of as a generalized version of the Foster, López-Calva-Székely class of indices with different parameters for inequality aversion and degree of substitution.

The objective of this paper has been to show why the consideration of interdimensions interaction is important and how a class of association-sensitive human development indices can solve some of the existing puzzles during policy recommendation. We show that consideration of interdimensional interactions makes the policy recommendation more proficient. Sensitivity to association encourages policymakers to determine the proper person and the proper dimension that should receive the marginal assistance. We apply the proposed class of indices to the Mexican census data for the year 2000 and show how the ranking of states is altered by association-increasing transfers. Tabasco ranked fourteenth before association-increasing transfer but relegated to rank fifteen afterwards.

Although the proposed class of indices overcomes some of the shortcomings of the traditional HDI, it is yet to be a perfectly satisfactory index in terms of measuring human development. Further research is required in this area. The proposed class of indices still assumes that the degree of substitution between any two dimensions is the same. It would be more plausible if the proposed class of indices could treat some dimensions as more stronger substitutes than others, or could treat some dimensions as substitutes and the rest as complements.

Second, this paper takes into account only particular types of transfers between persons. In case of distribution-sensitive transfers, we consider only common smoothing that reduces inequality across dimensions simultaneously and with the same bistochastic matrix. In case of association-increasing transfers, we consider only an extreme form of transfer following Boland and Proschan. There could be other types of association-increasing transfers as well (Trannoy 2006). Further research is required in this area that would lead to a class of human development indices, which would be robust to these challenges.

Third, it would be interesting if we could decompose the impact of these two types of inequality. In other words, it would be worthwhile to determine which inequality discounts the level of human development highly for any particular sample dataset containing normalized achievements. This is out of the scope of this paper at this moment and more research is required.

Finally, nothing comes without a trade-off. The class of strictly association-sensitive indices is not decomposable by dimensions. It is difficult to calculate the contribution of each dimension to total human development. Is it a drawback? As Gajdos and Weymark (2005) point out, it depends on what one wants to accomplish with these indices. If the objective is to find only dimensional contributions to the total human development, strict association sensitivity can be demanding. However, if the policy recommendations are of utmost importance, then the omission of association sensitivity while measuring human development is a serious drawback.

References

Alkire, S. and Foster, J. 2007. 'Counting and Multidimensional Poverty Measures', OPHI Working Paper 7, Oxford University, Oxford Poverty & Human Development Initiative.

Anand, S. and Sen, A. K. 2000. 'The Income Component of the Human Development Index', *Journal of Human Development*, 1 (1): 83–106.

Atkinson, A. B. and Bourguignon, F. 1982. 'The Comparison of Multi-Dimensioned Distributions of Economic Status', *Review of Economic Studies*, 49 (2): 183–201.

Boland, P. J. and Proschan, F. 1988. 'Multivariate Arrangement Increasing Functions with Applications in Probability and Statistics', *Journal of Multivariate Analysis*, 25 (2): 286–98.

Bourguignon, F. 1999. 'Comment to "Multidimensioned Approaches to Welfare Analysis" by Maasoumi, E.', in J. Silber (ed.), *Handbook of Income Inequality Measurement*. Boston, Dordrecht and London: Kluwer Academic, pp. 47784.

Bourguignon, F. and Chakravarty, S. R. 2003. 'The Measurement of Multidimensional Poverty', *Journal of Economic Inequality*, 1 (1): 25–49.

Decancq, K. and Lugo, M. A. 2008. 'Measuring Inequality in Well-Being: A Proposal Based on the Multidimensional Gini Index', Mimeo, Katholieke Universiteit Leuven, Faculty of Economics and Applied Economics.

Dutta, I., Pattanaik, P. K. and Xu, Y. 2003. 'On Measuring Deprivation and the Standard of Living in a Multidimensional Framework on the Basis of Aggregate Data', *Economica*, 70 (278): 197–221.

Foster, J. E., López-Calva, L. F. and Székely, M. 2005. 'Measuring the Distribution of Human Development: Methodology and an Application to Mexico', *Journal of Human Development*, 6: 5–29.

Foster, J. E. and Sen, A. 1997. *On Economic Inequality*. New Delhi: Oxford University Press.

Foster, J. E. and Shorrocks, A. F. 1991. 'Subgroup Consistent Poverty Indices', *Econometrica*, 59 (3): 687–709.

Foster, J. E. and Székely, M. 2008. 'Is Economic Growth Good for the Poor? Tracking Low Incomes Using General Means', *International Economic Review*, 49: 1143–72.

Gajdos, T. and Weymark, J. A. 2005. 'Multidimensional Generalized Gini Indices', *Economic Theory*, 26 (3): 471–96.

Hicks, D. A. 1997. 'The Inequality Adjusted Human Development Index: A Constructive Proposal', *World Development*, 25 (8): 1283–98.

Kolm, S.-C. 1977. 'Multidimensional Equalitarianisms', *Quarterly Journal of Economics*, 91: 1–13.

Massoumi, E. 1999. 'Multidimensioned Approaches to Welfare Analysis', in J. Silber (ed.), *Handbook of Income Inequality Measurement*. Boston: Kluwer Academic, pp. 437–77.

Pattanaik, P., Reddy, S. and Xu, Y. 2007. 'On Procedures for Measuring Deprivation and Living Standards of Societies in a Multi-Attribute Framework', Andrew Young School of Policy Studies Research Paper Series, No. 08–02.

Sen, A. 1999. *Development as Freedom*. New Delhi: Oxford University Press.

Seth, S. 2009. 'A Class of Association-sensitive Multidimensional Welfare Indices', OPHI Working Paper 27, Oxford University, Oxford Poverty & Human Development Initiative.

Trannoy, A. 2006. 'Multidimensional Egalitarianism and the Dominance Approach: A Lost Paradise?', in F. Farina and E. Savaglio (eds), *Inequality and Economic Integration*. London: Routledge, pp. 284–302.

Tsui, K.-Y. 1995. 'Multidimensional Generalizations of the Relative and Absolute Inequality Indices: The Atkinson-Kolm-Sen Approach', *Journal of Economic Theory*, 67 (1): 251–65.

Tsui, K.-Y. 1999. 'Multidimensional Inequality and Multidimensional Generalized Entropy Measures: An Axiomatic Derivation', *Social Choice and Welfare*, 16 (1): 145–57.

Tsui, K.-Y. 2002. 'Multidimensional Poverty Indices', *Social Choice and Welfare*, 19: 69–93.

United Nations Development Programme. 2004. *Human Development Report 2004*. Oxford: Oxford University Press.

Weymark, J. A. 2006. 'The Normative Approach to the Measurement of Multidimensional Inequality', in F. Farina and E. Savaglio (eds), *Inequality and Economic Integration*. London: Routledge, pp. 303–28.