Fuzzy Set Theory and Principal Component Analysis: complementary aggregation solutions

José Manuel Roche

J.M.Roche@sussex.ac.uk

PhD Student at the University of Sussex, UK Research Fellow at CISOR, Venezuela

General interest:

Monitoring inequality in capabilities between social groups

Technical problem:

How to measure and aggregate qualitative data?

Methodological proposal:

A combined use of Fuzzy Set Theory (FST), and Principal Component Analysis (PCA)

Complementary solutions

Their combined use contributes with empirical evidence and operational solutions for the design of synthetic indices

Core of the proposal



An illustrative application



What difference does it make? Does it matter?

Selected Indicators Sewage system Water Electricity Fuel Floors Roofs Walls Housing Overcrowding Index





Indicator transformation and fuzzy set standardization

Variables recoding (standarization '71, '81, '90, 2001)

Rearrangement of categories in a meaningful ordinal scale

Assign a fuzzy set membership function $\mu_A : X \rightarrow [0, 1]$

Unrotated, Varimax-rotated common components matrix

2nd

	Unrotated			VARIMAX-rotated		
	Component			Component		
	1	2	3	1	2	3
Sewage	0.734	0.120	-0.010	0.518	0.418	0.331
Water	0.565	0.435	0.144	0.695	0.100	0.190
Electricity	0.420	0.529	0.138	0.687	-0.014	0.061
Fuel used for cooking	0.401	0.495	-0.088	0.620	0.147	-0.087
Floors	0.752	-0.208	-0.310	0.226	0.752	0.297
Roofs	0.597	-0.312	-0.595	0.018	0.897	0.070
Walls	0.692	-0.228	0.345	0.258	0.250	0.721
Housing Overcrowding Index	0.495	-0.513	0.513	-0.064	0.101	0.870

Extraction Method: Principal Component Analysis. 3 components extracted. VARIMAX: Rotation converged in 4 iterations. Oblimin: Rotation converged in 9 iterations.





Identify the best fuzzy set aggregator that capture the underlying variables

Fuzzy Set Aggregator

It could capture the component structure

Less obscure than the PCA

Housing Structure

	Strong	Weak	Arithmetic
	Intersection	Intersection	Average
Component 2 VARIMAX	0.610	0.784	0.700

Housing Services

	Strong	Weak	Arithmetic
	Intersection	Intersection	Average
Component 1 VARIMAX	0.654	0.655	0.869

Recomputing the set of indicators in other data set



Monitoring progress

4th

Assessing inequality

Overall housing adequacy Specific area

Recomputing the set of indicators in other data set

Monitoring progress and inequalities at a local level Detection of vulnerable groups



CONCLUSIONS

The methodological proposal is enriched by the strengths of both techniques

Principal Component Analysis

- Identifying the underlying or latent variables
- Empirical evidence to support aggregation decisions

Fuzzy Set Theory

- Expressing the initial indicators in terms of degree of achievements
- Arithmetic procedures to capture the underlying variables