Fuzzy Set Theory and Multidimensional Poverty Analysis

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A brief literature review on fst and poverty/inequality analysis

- A few papers on fst-based approach to poverty and inequality measurement (Basu, 1987 and Ok, 1995 on inequality. Shorrocks & Subramanian 1994; Chakravarty & Roy, 1995; Cheli & Lemmi, 1995 on poverty)

- A relatively good number of cross-sectional and longitudinal multidimensional poverty analysis, largely based on a standard view but also on CA, makes use of fuzzy methodologies

- Lemmi and Betti (eds.), 2006 “Fuzzy set Approach to Multidimensional Poverty Measurement”: many empirical applications, some methodological papers

Some typical problems facing in a multidimensional setting

1. Not only quantitative but also qualitative variables
   - ⇒ (Q-squared approach)

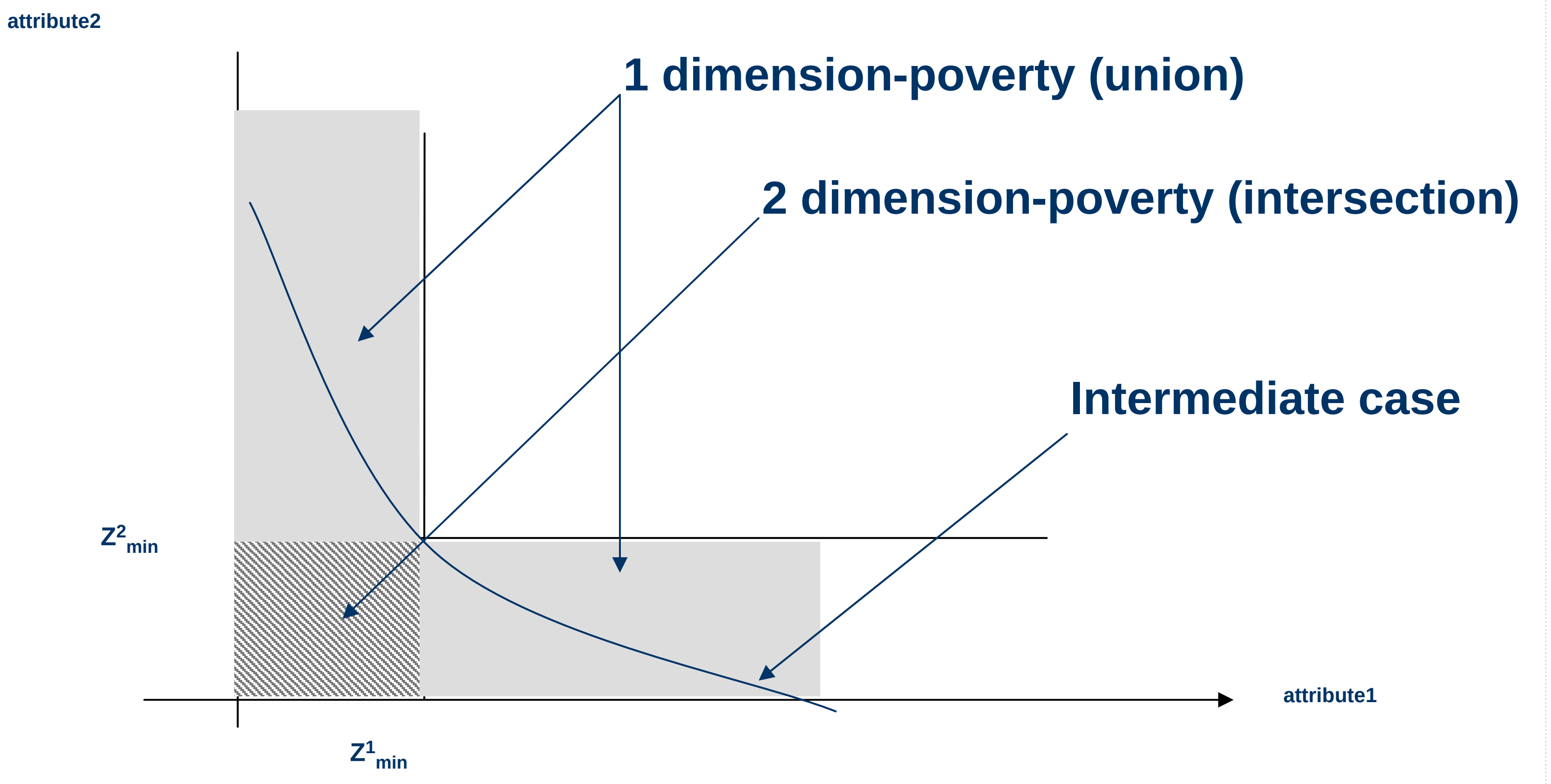
2. Multidimensional poverty lines

3. Poverty/well-being comparisons in a plurality of evaluative spaces
   - ⇒ dominance approach (Atkinson&Bourguignon, 1982,1987; Duclos et al, 2006)

4. Aggregation across multiple well-being/poverty dimensions
   - ⇒ union vs intersection approach (Atkinson, 2003)
   - ⇒ correlation between dimensions (substitutes vs complements attributes)

5. Aggregation across individuals or households

Intersection, union and intermediate poverty definitions (B&C 2003, Duclos et al, 2006)



Fuzzy sets methodologies: membership functions

Simply a generalization of the crisp set theory (dichotomous criteria; poor/not poor): individual membership function μ_{i}, (i= 1…., n individuals; j=1, …, k attributes) that takes the form

[2] $$\mu_{ij} : R^{j}_{+} \rightarrow [0, 1]$$

where [0,1] is the interval of real numbers from 0 to 1, inclusive.

- $\mu_{ij} = 0$ if the element does not belong to the set P of poor people (he/she is surely not poor)
- $\mu_{ij} = 1$ if the element completely belongs to P (totally poor)
- $0 < \mu_{ij} < 1$ if he/she partially belongs to P (otherwise)

Nb: $\sim P = 1 - \mu_{ij}(x)$ W(**X**)= 1- P(**X**) with **X** is a vector of attributes

Example 1 - Trapezoidal-shaped membership functions

Trapezoidal-shaped mf

f(x; a,b)=

0 if $x \leq a$

(x-a)/(b-a) if $a \leq x \leq b$

1 if $x \geq b$

(1st case: totally unachieved)

<u>or</u>

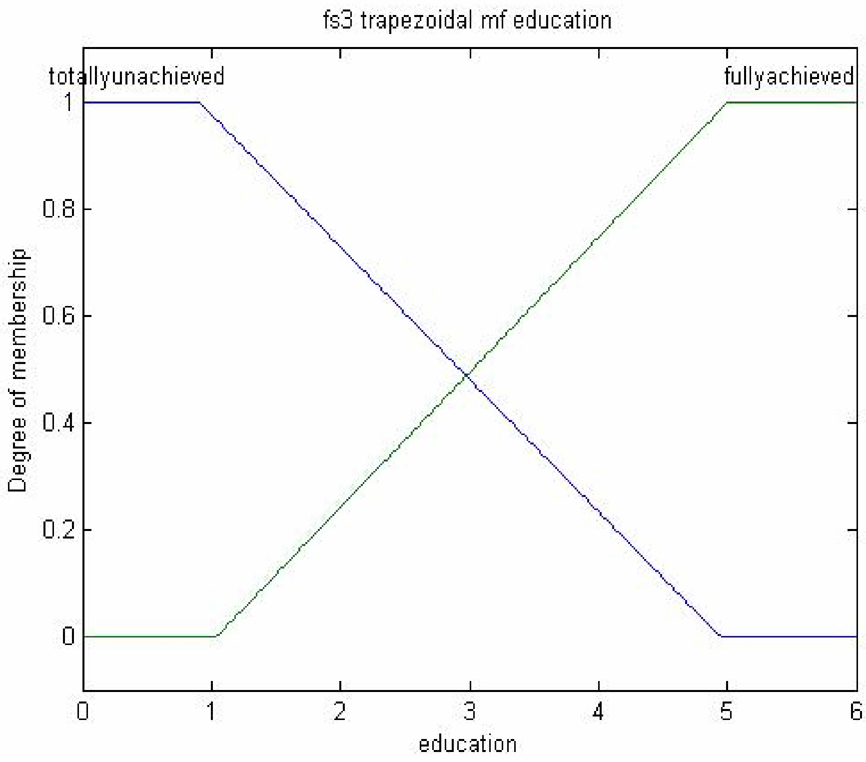
1 if $x \leq b$

(b-x)/(b-a) if $b \leq x \leq a$

0 if $x \geq a$

(2° case: fully achieved)

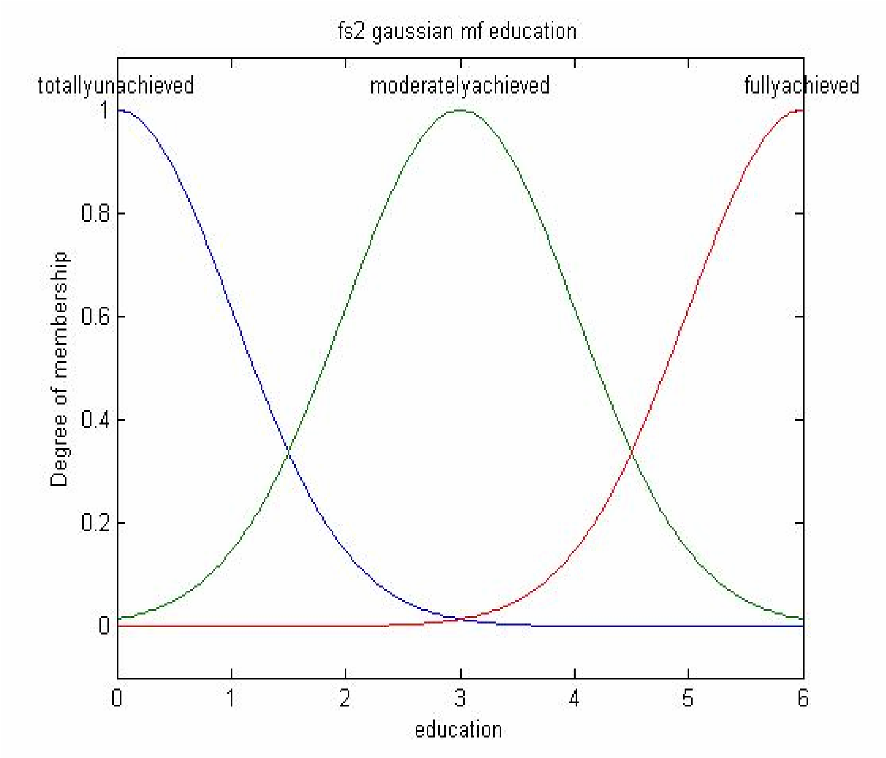
Nb: HDI general formula!



Example 2 – Gaussian curve membership functions

Gaussian curve mf

$$f(x;\sigma,c) = e^{\frac{-(x-c)^2}{2\sigma^2}}$$



Fuzzy sets methodologies: aggregation operators

Being A and B two poverty/well-being attributes

- **Max-min operators**
  - standard (or strong) intersection
    $\mu_{A \cap B} = \min[\mu A, \mu B]$
  - standard (or strong) union
    $\mu_{A \cup B} = \max[\mu A, \mu B]$
- **Product operators**:
  - weak union (or algebraic sum)
    $\mu_{A+B} = [\mu A + \mu B - \mu A \cdot \mu B]$
  - weak intersection (or algebraic product)
    $\mu_{A \cdot B} = [\mu A \cdot \mu B]$

Fuzzy sets methodologies: aggregation operators II

- **Bounded difference and bounded sum**

  $\mu_{A\cap B} = \max[0, \mu_A + \mu_B - 1]$

  $\mu_{A\cup B} = \min[1, \mu_A + \mu_B]$

- **Averaging (un-weighted and weighted) operators**

  $$h^\alpha = h(\mu_1, \mu_2, \ldots, \mu_n) = [\mu_1^\alpha + \mu_2^\alpha + \ldots + \mu_n^\alpha]/n]^{1/\alpha}$$

  $$h^\alpha = h(\mu_1, \mu_2, \ldots, \mu_n; w_1, w_2, \ldots, w_n) = [\Sigma w_i\, \mu_i^\alpha]^{1/\alpha}$$

with $\alpha = 1$ for the arithmetic mean, $\alpha = -1$ for the harmonic mean and $\alpha = 0$ for the geometric mean and with $w_i \geq 0$ and $\Sigma w_i = 1$

A simple numerical comparison

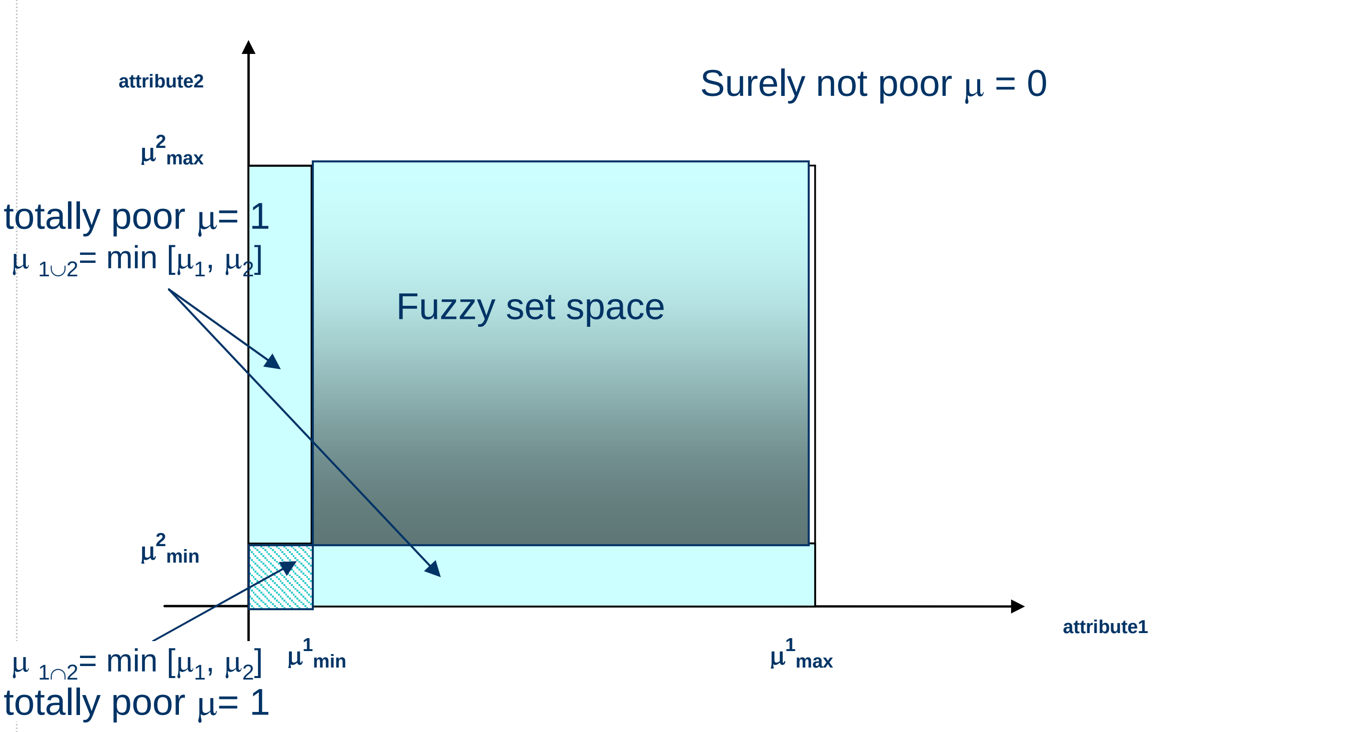
standard approach

individuals	attributes 1	attributes 2	counting approach	standard union	standard intersection
1	1	1	2	1	1
2	0	0	0	0	0
3	1	0	1	1	0
4	0	1	1	1	0

fuzzy approach

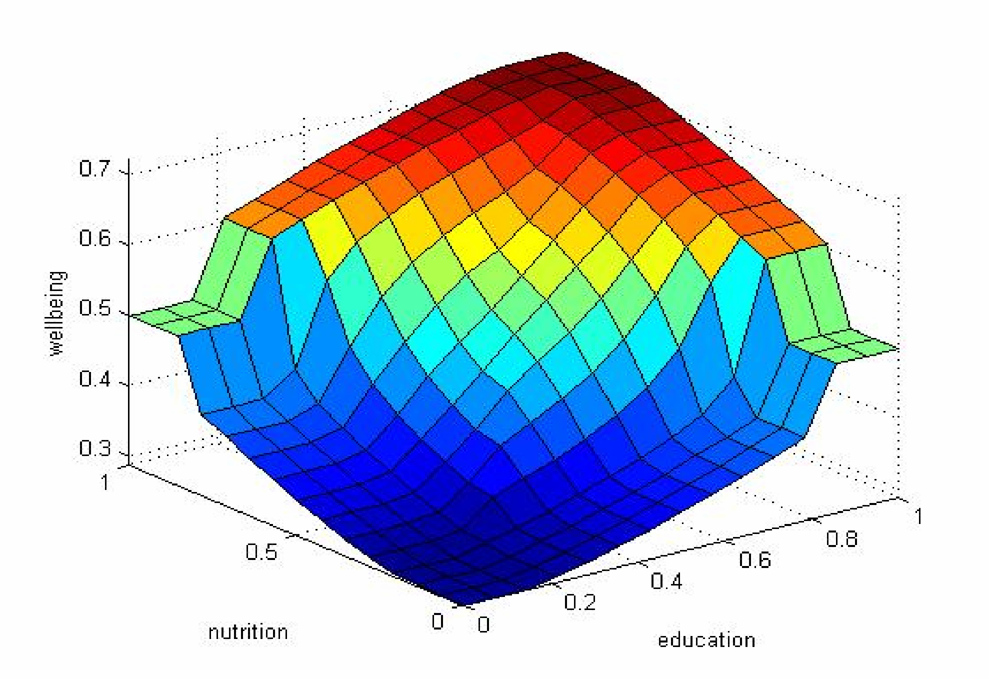
individuals	attributes 1	attributes 2	standard union	standard intersection	weak union	weak intersection	bounded difference	bounded sum
1	0,8	0,7	0,8	0,7	0,94	0,56	0,5	1,0
2	0,2	0,3	0,3	0,3	0,44	0,06	0,0	0,5
3	0,7	0,2	0,7	0,2	0,76	0,14	0,0	0,9
4	0,2	0,8	0,8	0,2	0,84	0,16	0,0	1,0
5	0,0	0,0	0,0	0,0	0,00	0,00	0,0	0,0
6	1,0	1,0	1,0	1,0	1,00	1,00	1,0	1,0

Intersection, union and intermediate fuzzy poverty



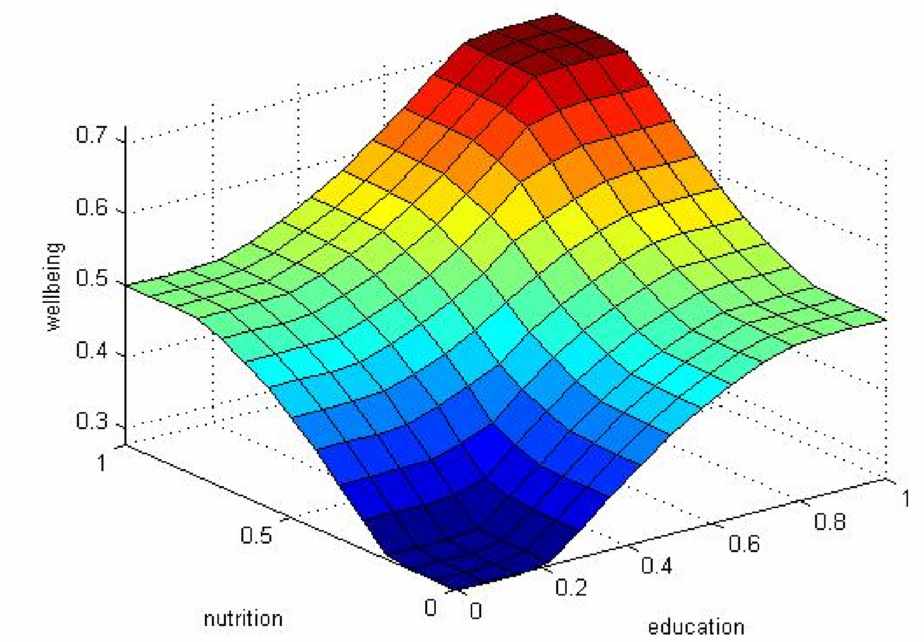
Well-being surfaces based on different mf and rules of aggregation I

- trapezoidal mf
- Standard intersection (and/min) operator



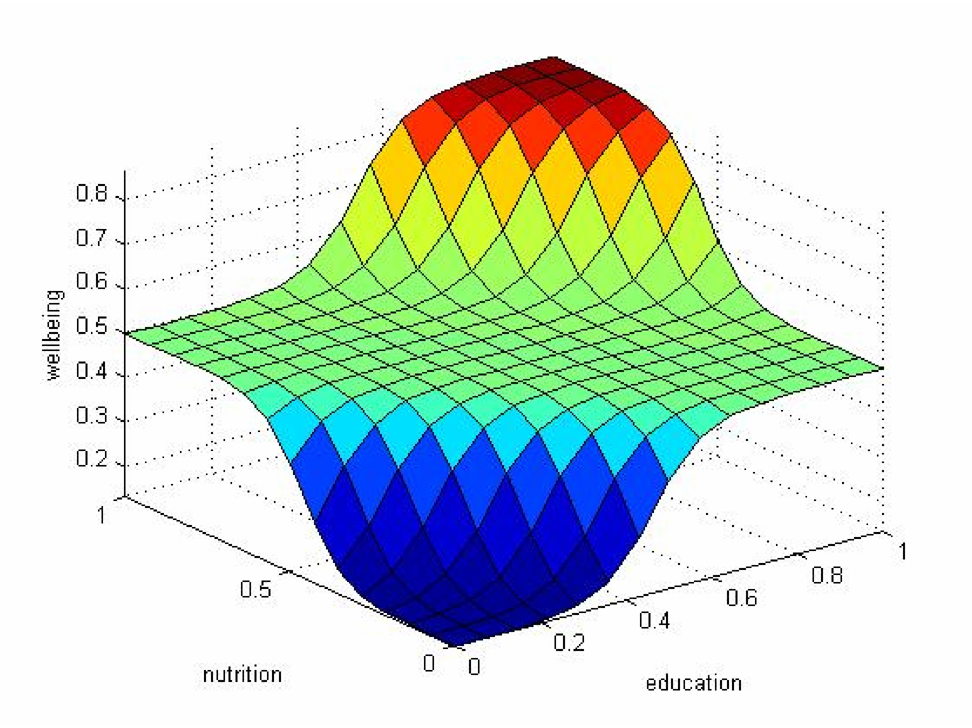
Well-being surfaces based on different mf and rules of aggregation II

- trapezoidal mf
- Standard union (max/or) operator



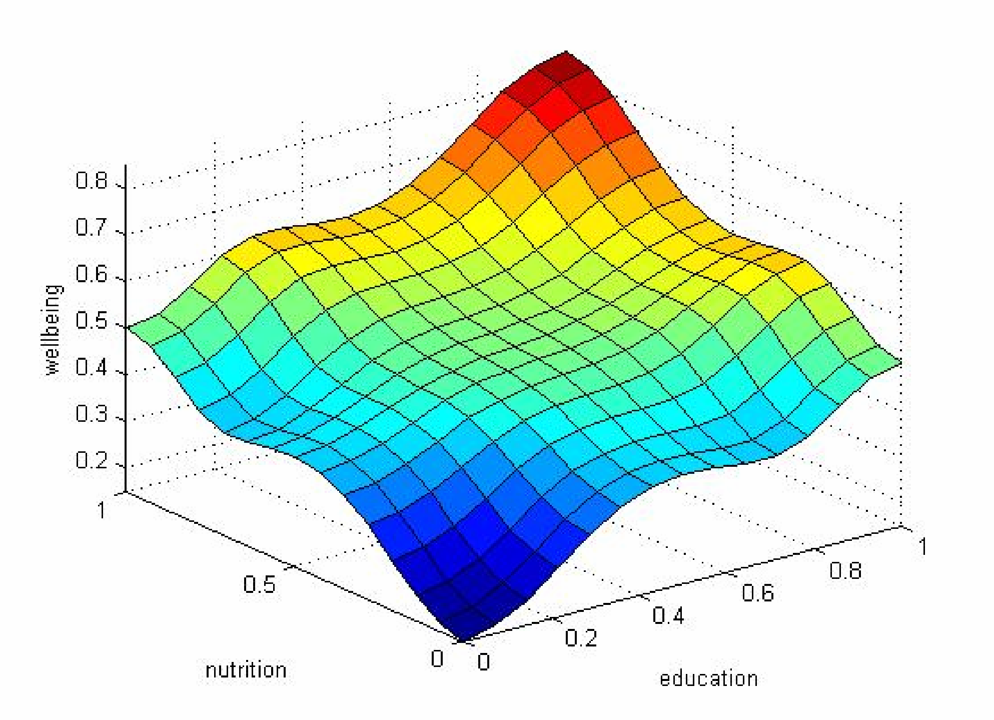
Well-being surfaces based on different mf and rules of aggregation III

- gaussian mf
- weak intersection (and/prod) operator



Well-being surfaces based on different mf and rules of aggregation IV

- gaussian mf
- weak union (algebraic sum/or) operator



Conclusions

- As a generalization of classical set theory, fst is not alternative to the standard approach but it allows to encompass it
- Fst is able to handle vagueness and complexity, strengthening the connection between theory and data analysis;
- Coming back to the previous 5 typical problems of multidimensional setting, the fst allows:
  1. both quant and qual variables using the same approach/methods
  2. multidimensional thresholds for different wellbeing domains, within the same domain, for different individual or subgroup thresholds (representing better the concept of “adequacy”)
  3. It permits to make poverty/well-being comparisons in a plurality of evaluative spaces
  4. It allows to aggregate across domains making use of a plurality of union, intersection and averaging operators each of which satisfies a specific set of properties
  5. recently Chakravarty (2006) reformulates the fuzzy counterpart to a variety of multidimensional poverty indexes, including FGT class of poverty measures

Conclusions II

- it does not represent a sort of mechanical exercise or standard algorithm, rather it requires an interpretative effort in each step, filling better the gap between richness and intrinsic vagueness of the theoretical concepts under examination (capability deprivation, functionings) and their formal and empirical representation