Multidimensional Poverty Measurement without the Strong Focus Axiom

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- for the estimation of multidimensional poverty,
- for the estimation of intertemporal poverty,
- for the estimation of income poverty when income sources are not perfect substitutes.

An axiomatic approach of poverty measurement:

- to highlight the link between ethical and mathematical properties,
- to avoid policy bias.

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- the identification step,
- the aggregation step.

Outline I

- The identification of the poor
- The axiomatic framework
- Multidimensional poverty measurement
- Concluding remarks

First part

The identification of the poor

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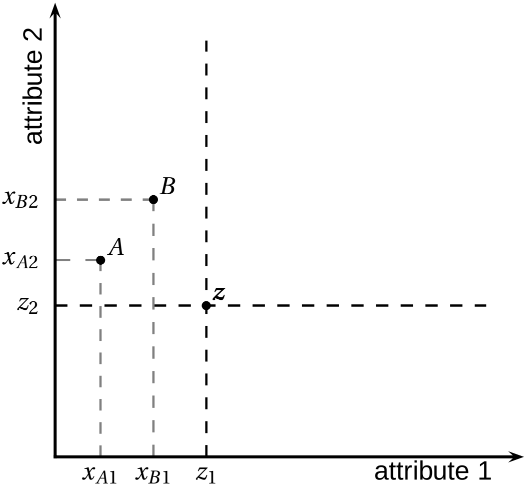
Alkire & Foster's (2007) "intermediate" identification approach: an individual is deemed poor if he is deprived with respect to a certain number of attributes (weights allowed).



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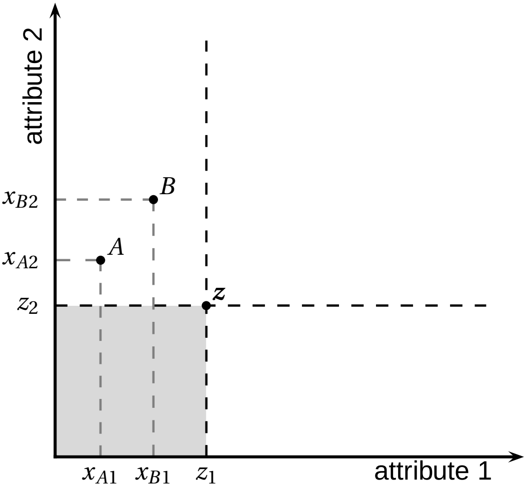

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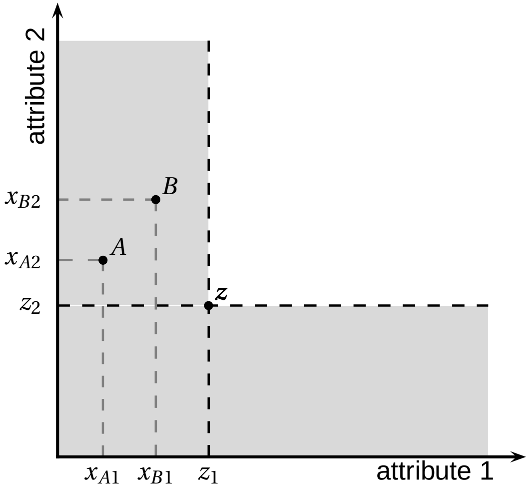

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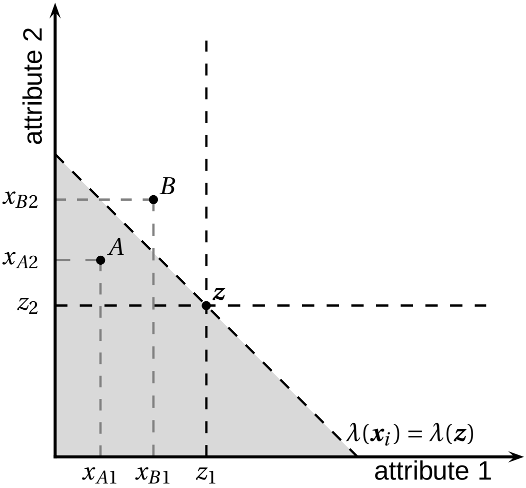

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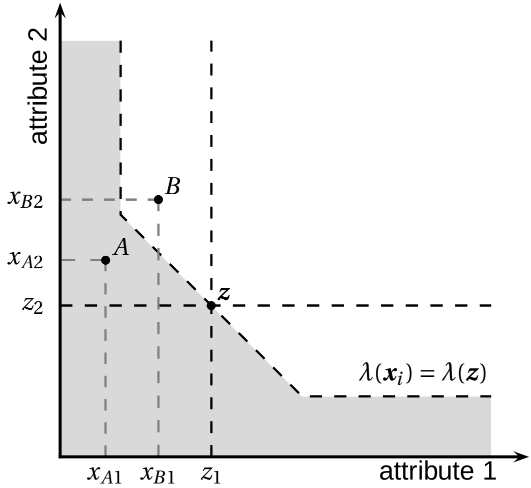

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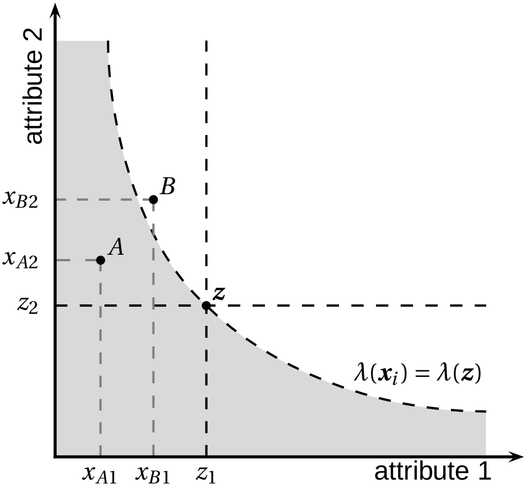

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$$\varphi^W(\boldsymbol{x}_i, \boldsymbol{z}, \lambda) := \begin{cases} 1 & \text{if } \lambda(\boldsymbol{x}_i) < \lambda(\boldsymbol{z}), \\ 0 & \text{otherwise,} \end{cases}$$

with λ being a well-being function such that $\frac{\partial \lambda}{\partial x_{ij}} \geqslant 0$ $\forall j \in \{1, \ldots, m\}$.

Second part

An axiomatic framework for multidimensional poverty measurement

The focus axiom with unidimensional settings

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Focus axiom: any improvement for a non-poor does not change the level of poverty, other things being equal.

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FOC_W and FOC_S are equivalent with the "intersection" approach. FOC_S is not consistent with all poverty domains that may be used with the "well-being" approach of poverty identification since FOC_S entails the use of identification functions based on the number of deprivations.

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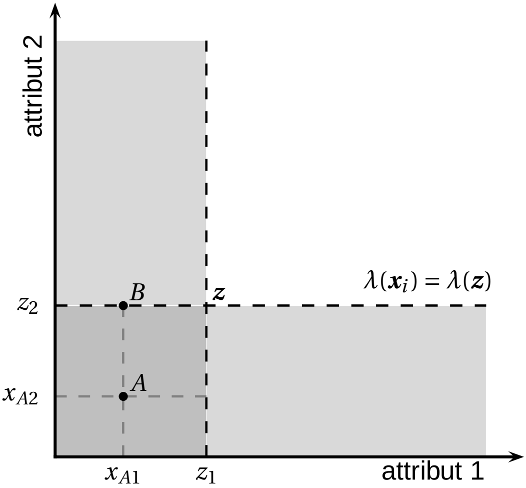
the “substitution” approach: “surpluses” in some dimensions can compensate deprivations in other dimensions in terms of well-being,

the “variable needs” approach: some poverty lines are determinated by deprivation levels observed with respect to other attributes.



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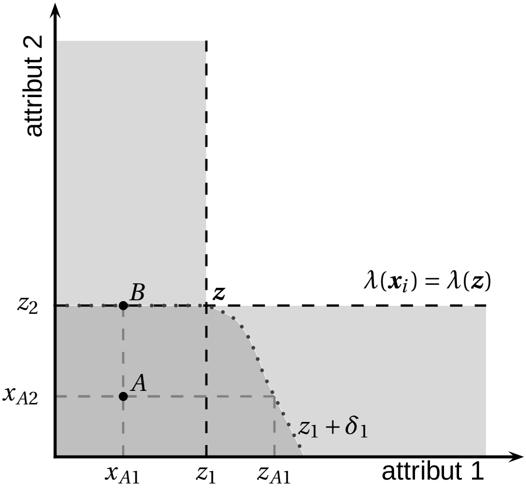

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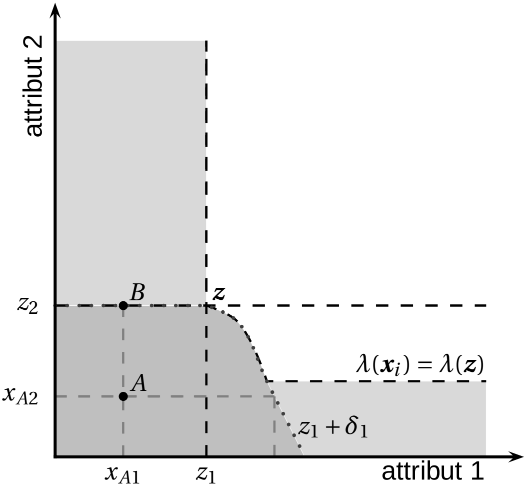

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An intermediate axiom between FOC_W and FOC_S:

Extended strong focus (FOC_E): increasing the level x_{ij} of the jth attribute of person i does not change poverty if $x_{ij} \geqslant z_j + \delta(\boldsymbol{x}_{i,-j})$.

with δ_j such that $\delta_j(\boldsymbol{x}_{i,-j}) \leqslant 0$, $\forall \boldsymbol{x}_{i,-j} \geqslant \boldsymbol{z}_{-j}$, and $\delta_j(\boldsymbol{x}_{i,-j}) = 0$, $\forall \boldsymbol{x}_{i,-j} = \boldsymbol{z}_{-j}$

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Monotonicity (**MON**): any increase in the level of all attributes of a poor person reduces poverty.

Restricted strong monotonicity ($\mathbf{MON}_R$): any increase for a poor person of the level of an attribute inside its substitution space reduces poverty.

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- Transfers that change the marginal distributions of the attributes:
  - Simple transfer (**TRA**),
  - Non ambiguous transfer (**TRN**),
  - Transfer in the sense of Schur (**TRS**),
  - Independent transfer (**TRI**).

Third part

Multidimensional poverty measurements without FOC_S

General expression for Θ_m (I)

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A poverty measure Θ_m complying with **FOC$_{\boldsymbol{E}}$**, **MON**, **MON$_{\boldsymbol{R}}$**, **CON**, **NDZ**, **SUC**, **ANO** and **POP** is of the form (Tsui, 2002):

$$\Theta_m(\boldsymbol{X}, \boldsymbol{z}) = \xi\left(\frac{1}{n}\sum_{i\in\boldsymbol{P}}\theta(\boldsymbol{x}_i, \boldsymbol{z}), \boldsymbol{z}\right)$$

with:

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with:

- ξ being a continuous and increasing function,
- $\boldsymbol{P}$ being the set of the poor defined by some identification function $\varphi(\boldsymbol{x}_i,\boldsymbol{z})$,
- θ being a continuous function on $\mathscr{P}$ and $\mathbb{R}^m_{++}\backslash\mathscr{P}$, such that $\frac{\partial\theta}{\partial x_{ij}} < 0\ \forall x_{ij} < z_j + \delta(\boldsymbol{x}_{i,-j},\boldsymbol{z})$, $\boldsymbol{x}_i \in \mathscr{P}$, $\partial\theta/\partial x_{ij} = 0$ otherwise, and $\frac{\partial\theta}{\partial z_j} \geqslant 0$.

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ATD: if and only if $\theta(\boldsymbol{x}_i, \boldsymbol{z}) = m^{-1}\sum w_j\theta(x_{ij}, z_j)$ with $\sum_{j=1}^m w_j = m$.

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TRS: if and only if θ is convex.

TRI: if and only if $\partial^2\theta/\partial x_{ij}^2 \geqslant 0$ and Θ_m satisfies **ATD**.

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$$\Theta_m^{BC}(\boldsymbol{X}, \boldsymbol{z}) := \frac{1}{n} \sum_{i=1}^{n} \left(\sum_{j=1}^{m} w_j \left(1 - \frac{x_{ij} \wedge z_j}{z_j} \right)^{\beta} \right)^{\frac{\alpha}{\beta}},$$

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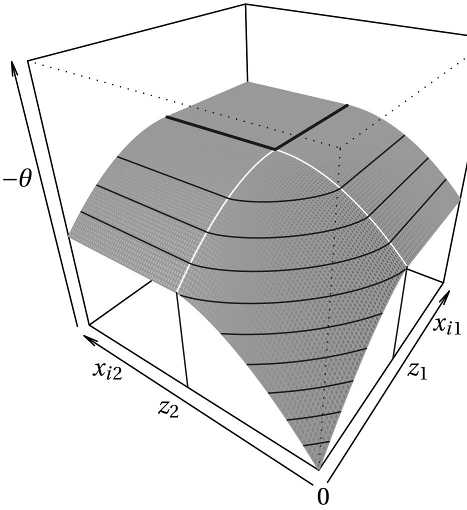
with $\alpha \geqslant 1$ and $\beta \geqslant 1$. β stands for the degree of substituability between the different attributes and α for the aversion to extreme poverty. The measure complies with **FOC$_S$**, **MON**, **MON$_R$**, **CON$_S$**, **NDZ**, **SUD**, **ANO**, **POP**, **SCI**, **NOR**, **TRA** and **TRS**, and suits a "union" approach of poverty identification.



Note: $w_1 = w_2 = 0.5$, $\beta = 1.5$ and $\alpha = 2$.

Figure 3: The individual poverty function in Bourguignon & Chakravarty (2003).

Bourguignon & Chakravarty (2003) and other approaches of poverty identification

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Generalization of Bourguignon & Chakravarty (2003) with other approaches of poverty identification:

$$\Theta_{m\varphi}^{BC}(\boldsymbol{X},\boldsymbol{z}) := \frac{1}{n}\sum_{i=1}^{n}\varphi^{W}(\boldsymbol{x}_i,\boldsymbol{z},\lambda)\left(\sum_{j=1}^{m} w_j\left(1-\frac{x_{ij}\wedge z_j}{z_j}\right)^{\beta}\right)^{\frac{\alpha}{\beta}}.$$

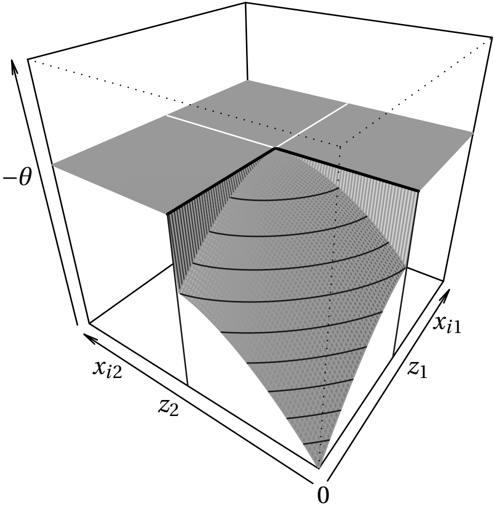

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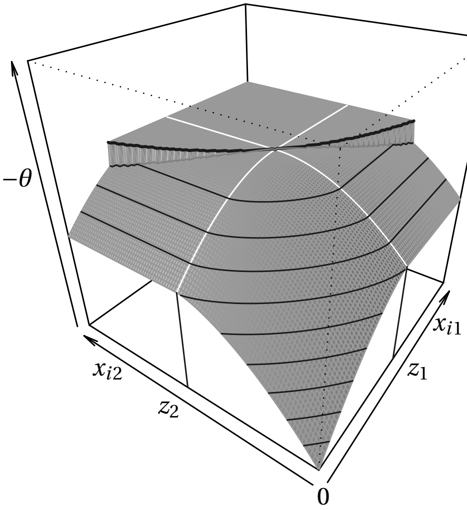
Figure 4: The individual poverty function in Bourguignon & Chakravarty (2003) with the “intersection” approach.



Note: $w_1 = w_2 = 0.5$, $\beta = 1.5$, $\alpha = 2$ and $\varphi(\boldsymbol{x}, \boldsymbol{z})$ based on $\lambda(\boldsymbol{x}_i) = \left(x_{i1}^{1/4} + x_{i2}^{1/4}\right)^4$.

Figure 5: The individual poverty function in Bourguignon & Chakravarty (2003) with the "well-being" approach.

Bourguignon & Chakravarty (2003) with the "variable needs" approach

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Generalization of Bourguignon & Chakravarty (2003) so as to suit the "variable needs" approach:

$$\Theta_m^{\boldsymbol{\delta}}(\boldsymbol{X},\boldsymbol{z}) := \frac{1}{n}\sum_{i=1}^{n}\left(\sum_{j=1}^{m} w_j \max\left\{0,\left(1-\frac{x_{ij}}{z_j\left(1+\delta_j(\boldsymbol{x}_{i,-j})\right)}\right)\right\}^{\beta}\right)^{\frac{\alpha}{\beta}},$$

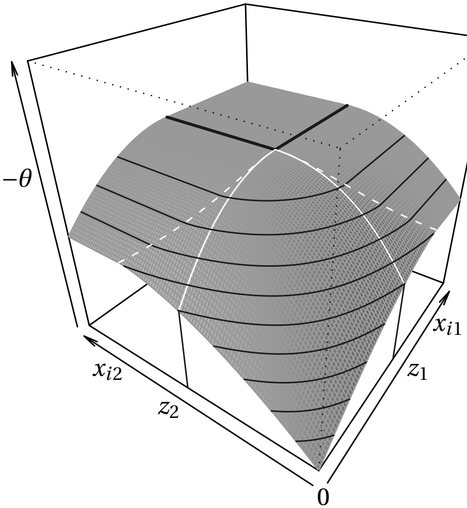
with $\delta_j(\boldsymbol{z}_{-j}) = 0$, $\partial\delta_j(\boldsymbol{x}_{i,-j})/\partial x_k \leqslant 0$ for $x_{ik} < z_k$ and $\partial\delta_j(\boldsymbol{x}_{i,-j})/\partial x_k = 0$ for $x_{ik} \geqslant z_k$, $k \neq j$.

Bourguignon & Chakravarty (2003) with the “variable needs” approach

Generalization of Bourguignon & Chakravarty (2003) so as to suit the “variable needs” approach:

$$\Theta_m^{\boldsymbol{\delta}}(\boldsymbol{X},\boldsymbol{z}) := \frac{1}{n}\sum_{i=1}^{n}\left(\sum_{j=1}^{m} w_j \max\left\{0, \left(1-\frac{x_{ij}}{z_j\left(1+\delta_j(\boldsymbol{x}_{i,-j})\right)}\right)\right\}^{\beta}\right)^{\frac{\alpha}{\beta}},$$

with $\delta_j(\boldsymbol{z}_{-j})=0$, $\partial\delta_j(\boldsymbol{x}_{i,-j})/\partial x_k \leqslant 0$ for $x_{ik}<z_k$ and $\partial\delta_j(\boldsymbol{x}_{i,-j})/\partial x_k = 0$ for $x_{ik} \geqslant z_k$, $k \neq j$. The measure complies with **FOC**$_{\boldsymbol{E}}$, **MON**, **MON**$_{\boldsymbol{R}}$, **CON**$_{\boldsymbol{S}}$, **NDZ**, **SUC**, **ANO**, **POP**, **NOR**, **SUD** and **SCI**.



Note: $w_1 = w_2 = 0.5$, $\beta = 1.5$, $\alpha = 2$ and $\delta_j(\boldsymbol{x}_{i,-j}) = \frac{\psi(x_{i,-j}, z_{-j})}{2}\left(\frac{z_{-j} - x_{i,-j}}{z_{-j}}\right)^3$.

Figure 6: The individual poverty function in Bourguignon & Chakravarty (2003) with the “variable needs” approach.

Bourguignon & Chakravarty (2003) with the "substitution" approach

Bourguignon & Chakravarty (2003) with the "substitution" approach

Generalization of Bourguignon & Chakravarty (2003) so as to suit the "substitution" approach:

$$\Theta_m^{\boldsymbol{\delta}'}(\boldsymbol{X},\boldsymbol{z}) := \frac{1}{n}\sum_{i=1}^{n}\max\left\{0,\sum_{j=1}^{m}w_j\left(\max\left\{0,1+\delta'_j(\boldsymbol{x}_{i,-j})-\frac{x_{ij}}{z_j}\right\}^{\beta}-\delta'_j(\boldsymbol{x}_{i,-j})^{\beta}\right)\right\}^{\frac{\alpha}{\beta}}.$$

with $\delta'_j(\boldsymbol{z}_{-j})=0$, $\partial\delta'_j(\boldsymbol{x}_{i,-j})/\partial x_k \leqslant 0$ for $x_{ik}<z_k$ and $\partial\delta'_j(\boldsymbol{x}_{i,-j})/\partial x_k = 0$ for $x_{ik}\geqslant z_k$, $k\neq j$.

Bourguignon & Chakravarty (2003) with the "substitution" approach

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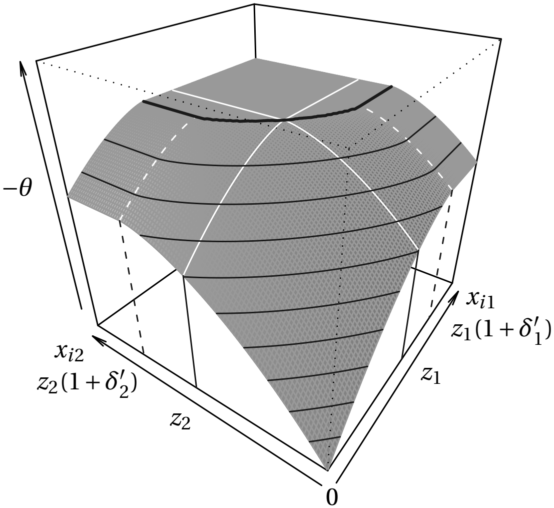
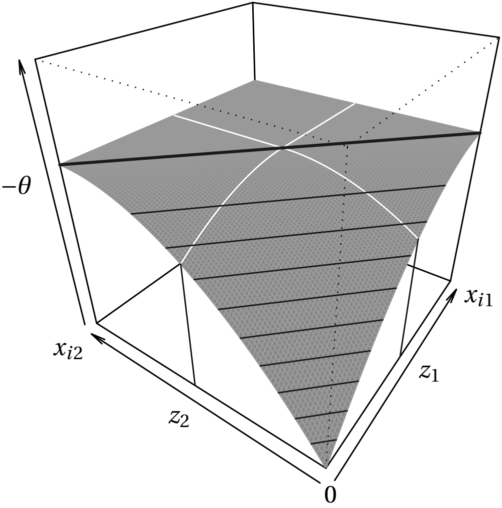
with $\delta'_j(\boldsymbol{z}_{-j})=0$, $\partial\delta'_j(\boldsymbol{x}_{i,-j})/\partial x_k \leqslant 0$ for $x_{ik}<z_k$ and $\partial\delta'_j(\boldsymbol{x}_{i,-j})/\partial x_k = 0$ for $x_{ik}\geqslant z_k$, $k\neq j$. For constant values for δ'_j, the measure complies with **FOC$_E$**, **MON**, **MON$_R$**, **CON$_S$**, **NDZ**, **SUC**, **ANO**, **POP**, **NOR**, **SUD**, **SCI**, **TRA** and **TRS**.



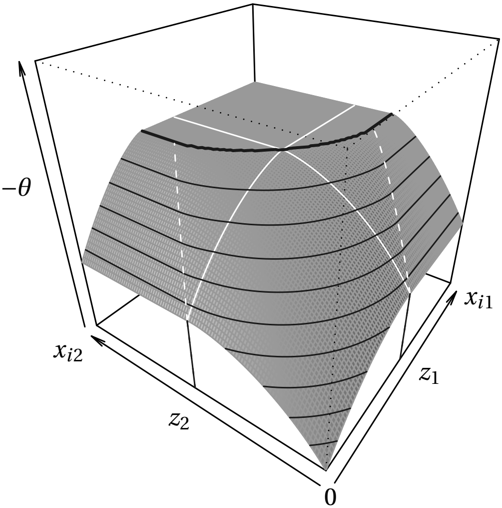
Note: $w_1 = w_2 = 0.5$, $\beta = 1.5$, $\alpha = 2$ and $\delta'_1 = \delta'_2 = 0.5$.

Figure 7: Bourguignon & Chakravarty's (2003) individual poverty function with the "substitution" approach.



Note: $w_1 = w_2 = 0.5$, $\beta = 1$, $\alpha = 2$ and $\delta_1' = \delta_2' = 1$.

Figure 8: Bourguignon & Chakravarty's (2003) individual poverty function with the "substitution" approach.



Note: $w_1 = w_2 = 0.5$, $\beta = 1.5$, $\alpha = 2$ and $\delta'_j(\boldsymbol{x}_{i,-j}) = 1 - \sum_{k \neq j} w_k \frac{z_k - x_{ik} \wedge z_k}{z_k}$.

Figure 9: Bourguignon & Chakravarty's (2003) individual poverty function with the "substitution" approach.

Concluding remarks

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- Identification and aggregation issues cannot be separated with multidimensional poverty in the same manner as with unidimensional poverty,
- The definition of the poverty domain becomes more complicated when slackening the strong focus axiom since substitution effects between the different dimensions have to be taken into account.

The end

Thanks for your attention.