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Revealed Meta-Preferences: Axiomatic Foundations of Normative Assessments in the Capability Approach

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Abstract

This paper explores the possibility of defining a non-utilitarian normative standard for assessments of welfare and deprivation. The paper contributes to this end in three ways. First, the paper formalises a key aspect of Prof. Sen's critique of the assumption of consistent utility maximisation in the revealed preference theory. Secondly, the paper explores alternative formulations of the axiom of revealed preferences that are consistent with Prof. Sen's critique and proposes a set of intuitive assumptions to characterise the relation between observed choices and underlying preferences in the absence of consistent utility maximisation. Finally, we use these to construct two alternative normative ranking rules that can be used in non-utilitarian welfare economics.

Keywords: Revealed preferences, Rational choice, Utility maximisation, Capability approach

JEL classification: D11, D46, D70

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Acronyms

ANO **Anonymity axiom**

ARM **Axiom of Revealed Meta-Preferences**

ARP **Axiom of Revealed Preference**

IND **Independence axiom**

ING **Indifference between Non-Consensual Groups axiom**

MON **Monotonicity axiom**

PBR **Probability-Based Ranking**

The Oxford Poverty and Human Development Initiative (OPHI) is a research centre within the Oxford Department of International Development, Queen Elizabeth House, at the University of Oxford. Led by Sabina Alkire, OPHI aspires to build and advance a more systematic methodological and economic framework for reducing multidimensional poverty, grounded in people's experiences and values.

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> "It is a mistake to set up physics as a model and pattern for economic research. But those committed to this fallacy should have learned one thing at least: that no physicist ever believed that the clarification of some of the assumptions and conditions of physical theorems is outside the scope of physical research." (Ludwig von Mises, 1949, p. 4)

If, as George Stigler has argued, Adam Smith's *Wealth of Nation* constitutes "a stupendous palace erected on the granite of self-interest" (Stigler, 1982, p. 136), then this paper will be concerned with tectonics. More precisely, we shall be looking at one of the key pieces of theoretical engineering upholding the contemporary edifice of neoclassical welfare economics, namely the axiom known under the term of revealed preference theory (Samuelson, 1938; 1948). This axiom plays a key role in linking the common variables that furnish economic theory (e.g. prices, markets, goods) with the normative concepts imported from utilitarian ethics (e.g. utility, efficiency, maximisation). As such, it constitutes a precondition for allowing neoclassical economics to make normative claims regarding, for instance, the efficiency of market outcomes or the stability of price equilibria. However, as Prof. Sen has argued on more than one occasion, this theory rests on presuppositions regarding human behaviour that are neither self-evident nor devoid of normative implications (Sen, 1977; Sen, 1997; Sen, 2002, p. 21; Sen, 1980).

Despite his pointed critique of some of the key tenets of neoclassical welfare economics, however, Prof. Sen has so far refrained from proposing a comprehensive normative alternative, insisting instead on the role of public discussion in settling the difficult normative issues involved in economic judgements (Sen, 2004 a). This may have left his so-called capability approach vulnerable to criticism, as some people have questioned the theoretical grounding of various indices, such as the Human Development Index and the more recent Multidimensional Poverty Index, which have claimed theoretical indebtedness to the work of Prof. Sen (Ravallion, 2010, p. 3). In particular, Ravallion claims that the weights chosen by these indices do not rest on the sort of theoretical justification that allows neoclassical economists to claim that "market prices are defensible weights on quantities in measuring national income" (Ravallion, 2010, p. 3). This paper will seek to address the – in our opinion – legitimate question raised by Martin Ravallion, and in so doing, hopefully contribute with a small but important theoretical component required to rebuild from sounder foundations an edifice that could one day hope to rival the internal coherence and methodological solidity achieved by neoclassical welfare economics.

The precise question posed by this paper is "what alternative standard should we use to make normative claims in economics if we accept Sen's criticism of the revealed preferences theory?" In particular, we will focus on the implications of relaxing the assumption of self-interested utility maximisation in the so-called revealed preference theory. Based on this analysis, we will then explore possible alternative formulations of the revealed preference theory, which can provide a stepping-stone for devising a non-utilitarian normative standard. Finally, we will use this reformulated revealed preference theory to propose two alternative normative standards that can be used in normative economic assessments that are based on non-utilitarian normative premises. An immediate and obvious application of such a standard could be in the construction of multidimensional indices of welfare and deprivation that do not rely on monetary estimates of welfare (Stiglitz, Sen, & Fitoussi, 2008; Alkire & Santo, 2010).

1. Sen's Critique of the Revealed Preference Theory

Revealed Preference Theory

The justification for using market prices as normative weights in assessments of welfare and deprivation has been formalised in the first welfare theorem (Arrow & Debreu, 1954), which links the positive description of the General Equilibrium (Walras, 1874) with the normative concept of Pareto-efficiency (Pareto, 1906)[i]. The use of Pareto-optimality allows the first welfare theorem to circumvent classical economics' reliance on the informationally demanding –and unobservable – notion of cardinal utility, which had been central to Smith's original formulation of the "invisible hand" argument (Smith, 1776, p. 400), on which this line of thought is based. As Samuelson pointed out, however, despite its significant weakening of Smith's original normative claims, it is not clear that the neoclassical formulation would meet the minimum empiricist requirements of verifiability typically demanded by scientific methodology (Hicks, 1956, p. 6):

> Consistently applied, however, the modern criticism [of classical utility theory] turns back on itself and cuts deeply. For just as we do not claim to know by introspection the behaviour of utility, many will argue we cannot know the behaviour of ratios of marginal utilities or of indifference directions. Why should one believe in the increasing rate of marginal substitution, except in so far as it leads to the type of demand functions in the market which seem plausible? (Samuelson, 1938, p. 61)

In order to address this flaw, Samuelson proposed a key piece of theoretical engineering which has come to be known under the term of revealed preference theory. As long as individuals behave consistently, Samuelson argued, we could infer from their choice of x over y, certain features of the underlying preference relation between x and y. Hence, while utility itself may be unobservable, sufficient fragments of it can be indirectly observed to allow us to make the type of ordinal claims needed for normative welfare assessments of Pareto-efficiency (Little, 1949; Houthakker, 1950). Later Samuelson, building on Little (Little, 1949), went on to show how indifference curves could be approximated through the iterative comparison of binary preference relations over choice bundles (Samuelson, 1948), thus paving the way for a reconstruction of a theory of consumer behaviour based solely on observable choices.

Here, we will build on the notation used by Sen (1971) and Arrow (1959) to formalise the argument. Let X be the set of all possible options and K the set of all non-empty and finite subsets, S, of X. The non-empty "choice set" $c_i(S)$ represents elements of any given subset S of K that have been chosen by individual i. Henceforth, we will write i's choice set over S simply as c_i for short. We will not make any particular assumptions about the functional form (or absence thereof) of c_i, and will treat it simply as a subset of S in our analysis. The decision maker i's preferences are represented by transitive, reflexive and binary relations, R_i^U, defined over the elements of X contained in S. Let $N = \{i|i = (1, \ldots, n)\}$ be the set of all individuals holding preferences R_i^U. Then $\Gamma_N^U = \{R_1^U, \ldots, R_n^U\}$ represents the set of all possible preference relations over X.

The preference for x over y is designated by $xR_i^U y$, meaning that "x is at least as good as y according to the preference relation R_i^U". The superscript U indicates that preference relations are based on utility maximising calculus. Hence, $xR_i^U y$ is equivalent to saying that x confers i with at least as much utility as y, and can be interpreted in marginalist terms, if x and y represent units of x and y for given quantities. The asymmetric and symmetric elements of R_i^U will be designated, respectively, by P_i^U and I_i^U.

<u>The Axiom of Revealed Preference (ARP)</u> then states in its original, weak, form:

$$\forall x, y \in X \text{ and } \forall i \in N\text{: } x \in c_i \text{ and } y \in S \backslash c_i \Rightarrow \neg \left(y R_i^U x\right)$$

(1)

Plainly put, if both x and y are part of i's opportunity set, but only x is chosen, we can conclude, at the least, that i does not prefer y to x.

In order to create this direct relation between observable choices and underlying preferences and utility, the revealed preference theory is forced to impose the simplifying, but implicit, assumption of a single motivational source, namely that of self-interested utility maximisation. Hence utility appraisals are consistently translated into preference orderings, which are consistently translated into choices. In practice, however, Sen argues that although self-interest maximisation is undoubtedly a most powerful driving force of human conduct, it is but one of the factors that motivate action, alongside things like commitment, compliance with moral rules, religious principles or social norms.

There have been attempts to circumvent this line of criticism by expanding the definition of preferences to include anything sought by the individual, be it for religious reasons, moral reasons, pathological desires or some other form of "want" (Hicks, 1939). Such an expansion of the definition of preferences, however, would quickly empty the term of its normative content by equating it with the descriptive notion of "behaviour" (Harsanyi, 1997). As Sen has pointed out, from a normative point of view, "[t]he rationale of the revealed preference approach lies in this assumption of revelation" (Sen, 1973, p. 244), and not in "dropping off the last vestiges of the utility analysis", as Samuelson had initially claimed (Samuelson, 1938, p. 62). Indeed, if behaviour didn't reveal anything about underlying preferences, and if preference said nothing about utility, price ratios would not tell us anything about marginal rates of substitution and market equilibria would thus tell us nothing about Pareto-optimality.

Plural Motivations

In this subsection, we will look at what happens to ARP when we relax the implicit assumption of consistent and exclusive utility maximising behaviour. For the purpose of the present argument, we will make a distinction between two fundamental types of motivations described by Sen, namely (1) self-centred motivations, R_i^U which can include altruism, sympathy, etc., and (2) reasoned motivations, which we denote by $R_i^M \in \Gamma_N^M = \{R_1^M, \ldots, R_n^M\}$ [ii]. Sen refers to these as "preferences over preferences" or " 'metarankings', reflecting what [an individual] would like his preference to be" (Sen, 1983, p. 25).

The former type of motivations are idiosyncratic in the sense that they are defined by a person's particular interests, inclinations, desires, fears, etc., with regards to her *own* utility (e.g. in the case of sympathy, a person's utility is negatively affected by the sight of someone else's suffering). The latter type of motivations is here defined as those that allow us to disregard our own particular circumstances and act, as Sen calls it, "as if" we are maximising someone else's utility. The latter is derived from our capacity for universalisation, that is, the faculty of reason to abstract from our own circumstances in order to act in accordance with a law or a moral principle[iii]. Sen describes this as the capacity of individuals to "suspen[d] calculations geared towards individual rationality" (i.e. self-interest maximisation) and to "behave as if they are maximizing a different welfare function from the one that they actually have" (Sen, 1973, p. 252). It is this capacity to see a problem from someone else's perspective and act in the interest of the common good that allows individuals to avoid suboptimal outcomes in problems such as the prisoners' dilemma, as well as numerous other social problems, such as taxation, provision of public goods, etc. (Sen, 1973, p. 250), and even, arguably, in many market transactions (Sen, 1991).

While we do not exclude the possibility that different individuals' idiosyncratic preferences may coincide, due to sympathy or convergence of interests, for instance, we must recognise the possibility that they may differ due to divergence of interests and tastes. Therefore, we define R_i^U as:

Definition 1: $\Gamma_N^U = \{R_i^U \mid \exists i, j \in N, i \neq j, s.t. R_i^U \neq R_j^U\}$.

By contrast, meta-preferences, R_i^M, will be defined here by their convergence across individuals within a group. Without going as far as claiming that the latter motivations must concord with a universal law of reason, we will define them as the ability of individuals, who are part of a group, to take a bird's eye view on a situation, and recognise that there is a common interest that transcends their individual interests. This formulation will allow for reasoned disagreements between groups of individuals, which is important to Sen (Sen, 2004 b), while at the same time recognising the possibility of existence of universal truths (see corollary 1).

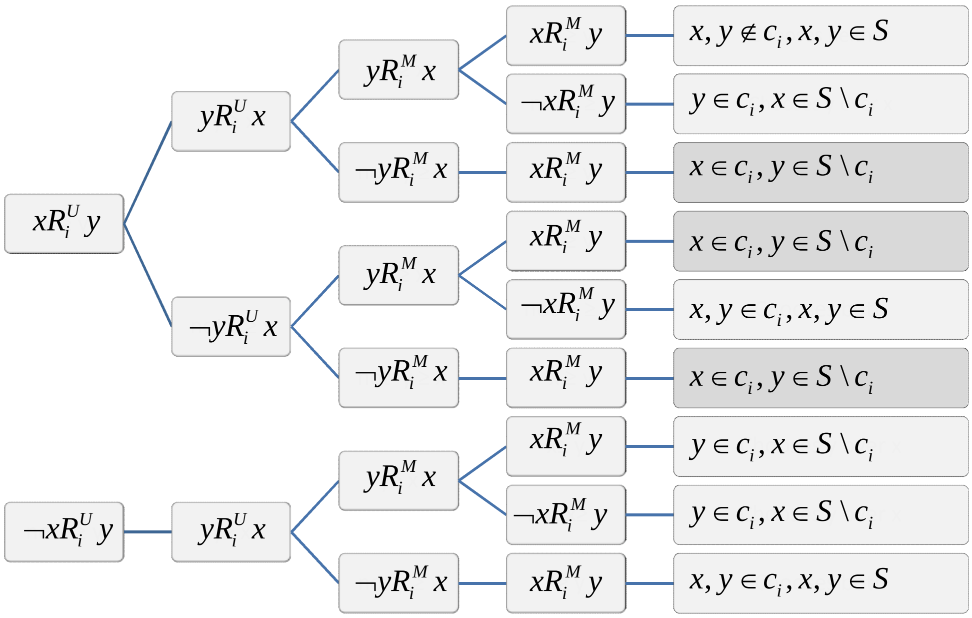
Let Q be the set of all non-empty subsets of N and let $I \in Q$ designate a group of individuals $i = (1, \dots, n)$:

Definition 2: $\Gamma_N^M = \{R_i^M \mid \forall I \in Q\colon\ i, j \in I \ \Rightarrow \ R_i^M = R_j^M\}$.

We will assume that R_i^U and R_i^M each, independently of each other, satisfy the normal properties of completeness, reflexivity and transitivity over X.

Once we admit the possibility of plural motivations, the technical trick of revealed preferences, which allowed us to draw inference about underlying utility from observable behaviour, ceases to be effective, since observed behaviour could equally well have been caused by moral commitment, norm following or even self-sacrifice, neither of which tell us anything about a person's level of utility – from which all value must ultimately be derived in the utilitarian framework. The figure below indicates various possible outcomes that can result from a combination of utility- and meta-preferences over the outcomes x and [iv].

Figure 1: Event tree associated with a utility and meta-preference over x and y



In this example, we have assumed that an individual will choose both x and y in cases where she has strict preferences for both simultaneously, and will choose neither if she is indifferent both in terms of utility and reasoned preferences. While it is possible to conceive of other plausible combinations of preference relations to outcomes, the precise form of the relation will not matter here. Instead, we are interested in showing that, in the presence of dual motivations it will no longer be possible to infer underlying preference relations with certainty from observed choices, as was done in ARP, since each choice pattern will be associated with, not one, but several plausible combinations of underlying preference relations.

2. Reformulating the Revealed Preference Theory

Modified Axioms of Revealed Preferences

Depending on the assumptions we choose about the form of underlying decision-making mechanism, we will still be able to make certain weaker claims based on observed choices. .

Figure 1, for instance, allows us to make the following claim, which is a close but weaker relative to the ARP:

$$\forall x, y \in X \text{ and } \forall i \in N\text{: } x \in c_i \text{ and } y \in S \backslash c_i \Rightarrow \neg \left(yP_i^U x \vee yP_i^M x\right)$$

(2)

The axiom now states that if i is observed to choose x over y, he cannot possibly have a *strong* preference for y over x. Given the uncertainty we now face about underlying preference relations, it may be more appropriate to represent these claims as probabilities. We will therefore refer to inferred motivational sources as events, with ψ_i^U and ψ_i^M describing the events $yP_i^U x$

and $yP_i^M x$, respectively and χ_i^U and χ_i^M describing the events $xP_i^U y$ and $xP_i^M y$. In this perspective, axiom (2) can be reformulated as :

$$\forall x, y \in X \; and \; \forall i \in N: \; p\left(\psi_i^U \cup \psi_i^M | x \in c_i \; and \; y \in S \backslash c_i\right) = 0$$

(3)

Henceforth, for notational simplicity, we will write the probability that a given event has occurred, given observation that option x is chosen over y (i.e. given $x \in c_i \; and \; y \in S \backslash c_i$) simply as $p(\cdot\,|c_i)$.

In order to characterise the relation between observed choices and underlying preferences in the absence of exclusive utility maximising behaviour, we will assume that the following intuitive properties, which are consistent with definitions 1 and 2, are satisfied:

$\forall i, j \in N$, $i \neq j$,and $\forall x, y \in X$:

1. Coherence: $p\left(\psi_i^U \cup \psi_i^M | c_i\right) = 0$
2. Completeness: $p\left(\chi_i^U \cup \chi_i^M | c_i\right) = 1$
3. Uncertainty: $p\left(\chi_i^U | c_i\right) = p(\chi_i^M | c_i)$
4. Anonymity: $p\left(\chi_i^U | c_i\right) = p\left(\chi_j^U | c_j\right)$
5. Independence: $p\left(\chi_i^U \cap \chi_j^U | c_i \cap c_j\right) = p\left(\chi_i^U | c_i\right) \times p\left(\chi_j^U | c_j\right)$

Property 1 is a re-statement of axiom (3). Property 2 states that a given act, c_i, must have been motivated either by utility or by reason, or both. Property 3 captures the fact that, with no additional information about underlying preferences, we cannot know whether a given behaviour is motivated by self-interested utility-based considerations or by moral concerns. Property 4 states that the probability that a given act is motivated by selfish motives is equal across all individuals (i.e. there are no *a priori* more virtuous individuals). Property 5, finally, makes the point that idiosyncratic motivations are independent across individuals[v].

Axiom of Revealed Meta-Preferences

In the non-utilitarian perspective advocated by Prof. Sen[vi], it is not underlying levels of utility that constitute a legitimate source of value, but the reasoned assessment that individuals make of their own preferences, i.e. their metarankings (Sen, 2002). Indeed, utility-based preferences may be determined by culture, upbringing, genetics or other factors over which the individual has no control (Nussbaum, 2003) and they may be adaptive, in the sense of being moulded by prejudice, fear or simply resignation to an unjust fate (Sen, 1984)[vii]. The question that is of interest for the purpose of operationalisation of the capability approach is thus whether meta-preferences, R_i^M, can be, if not observed, at least inferred from looking at choices c_i. in the same way as Samuelson's revealed preference theory had allowed us to infer unobservable utility-preferences from observed choices?

At the individual level, the answer to this question is negative, since we do not exclude the possibility of a convergence between idiosyncratic and moral preferences due to chance or sympathy. This implies that a same behaviour c_i, e.g. charitable giving, could be caused by either self-interested motives (e.g. attempting to secure loyalties or relieving one's own displeasure at the sight of poverty) or moral motivations (e.g. a social obligation towards other less fortunate members of the group), as indicated in property 3. However, at the aggregate level, we argue that it will be possible to infer some properties of underlying meta-preference based on the definitions adopted here. The rationale is simple: as neoclassical economics accurately predicts, the probability of achieving a convergence of interests over a large group of individuals is typically small. Looking at the problem from the opposite angle, therefore, and accepting the existence of reasoned meta-preferences, we can conclude that in the event that individuals actually do reach agreement, it is likely that something other than the simple pursuit of self-interest must have been at work.

This idea is captured in the following lemma, which follows from definitions 1-2 and properties 1-5 (See Annex 1 for a proof of Lemma 1):

Lemma 1: $\forall I, A \in Q,\ \ s.t.\ \ 1 < |I| < \infty\, and\ \ A \neq ,$ with individual preference defined as 1-2, and satisfying properties 1-5: $I \supset A \Rightarrow p(\chi_I^M | c_I) \geq p(\chi_A^M | c_A)$.

We use the notation χ_I^U to describe the event that $xR_i^U y$ occurs simultaneously for all $i \in I$. In other words it describes the case in which there is a concordance of interests. For a group $I \in Q$ of n individuals, $i = (1, \dots, n)$, we thus have $p(\chi_I^U) = p\left(\bigcap_{i=1}^n \chi_i^U\right)$. Similarly, on the output side, instead of looking at choice sets, we are now looking at consensus sets, c_I, which we define simply as the non-empty intersection between individual choice sets: $c_I \equiv \bigcap_{i=1}^n c_i \neq$. The set of all possible non-empty and finite consensus choice sets, c_I, held by groups $I \in Q$ is called Z (this includes individual choice sets). The cardinality of a consensus, c_I, achieved by a group I of individuals $i = (1, \dots, n)$, is designated by $|I|$. At the moment we do not make any assumptions about the way that consensus is reached, other than the fact that each individual, $i \in I$ who counts towards $|I|$, has equal influence over the consensus c_I. This formulation can therefore accommodate all possible decision-making mechanisms, ranging from absolute dictatorship (in which case we would have $|I| = 1$), to unanimity rule (in which case $|I| = n$, i.e. the consensus size is equal to the number of individuals in the population). In the most common case of absolute majority rule, we have $|I| = \frac{1}{2}n + 1$.

In the limit, as $|I|$ tends to ∞, we can think of a hypothetical consensus involving an infinite number of individuals with all possible combinations of preference relations. Definition 1 implies that the probability of utility-based interests converging over an infinite number of individuals is 0. Consequently, such a consensus would, by construction, need to coincide with a universal law of reason since that is the only law that any reasonable individual could voluntarily be expected to subject themselves to, regardless of their personal circumstances, inclinations and interests. This idea is represented by the following corollary, which follows from Lemma 1 (see Annex 2 for a proof of Corollary 1):

Corollary 1: $\forall I \in Q,\ \ 1 < |I| < \infty,$ with individual preference defined as 1-2, and satisfying properties 1-5: $\lim_{|I| \to \infty} p(\chi_I^U \cup \chi_I^M | c_I) = p(\chi_I^M | c_I) = 1$.

This analysis allows us to propose the following general axiomatic formulation, which we will call the Axiom of Revealed Meta-Preferences (ARM):

$$\forall x, y \in X \text{ and } \forall I \in Q: \ p(\chi_I^M | c_I) = f(\cdot, |I|)$$

(4)

Where $f(\cdot, |I|)$ is a monotonically non-decreasing function of $|I|$. The axiom of revealed meta-preferences simply states that the probability that an observed consensus, c_I, has been generated by a meta-preferences will be a function of the number of individuals reaching consensus. An outcome x over which there is a consensus among $|I|$ members of a group I will thus be said to be revealed meta-preferred to outcome y with probability $p(\chi_I^M | c_I)$. In this perspective, the case covered by the original revealed preference axiom becomes a special case of this general axiom, in which $|I| = 1$. In this case, we know from property 3 that $p(\chi_i^M | c_i) = p(\chi_i^U | c_i)$, meaning that the observed choice has equal probabilities of having been caused by either motive. Other special cases that have been discussed above, include the case in which $|I| \to \infty$, in which case corollary 1 tells us that $p(\chi_I^M | c_I) = 1$. A final special case is that in which there is no consensus, that is $|I| = 0$. By definition 2, if there is no consensus, we know that individuals have been unable to overcome their divergence of interests in order to agree on a common objective. Hence, in this case, we have $p(\chi_I^M) = 0$. All other cases, such that $1 < |I| < \infty$ are covered by Lemma 1.

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3. Normative Meta-Rankings

Probability-Based Ranking
At the moment, the axiom of revealed meta-preferences does not make any normative claim. It simply states that broader consensuses are more likely to have been reached through reasoned agreement. If, like Sen, we consider that value stems from reason rather than from utility, it is logical to make the normative value of a given choice directly proportional on the probability that it is caused by R_i^M rather than R_i^U. We do this by turning the probabilistic logic of Lemma 1 into a normative rule, using the following axiom:

Probability-Based Ranking (PBR): $\forall I, J \in Q \text{ and } \forall c_I, c_J \in Z$:

1. $c_I \succ c_J \Leftrightarrow p(\chi_I^M | c_I) > p(\chi_J^M | c_J)$
2. $c_I \sim c_J \Leftrightarrow p(\chi_I^M | c_I) = p(\chi_J^M | c_J)$

(5)

Where $\succcurlyeq$, meaning "is at least as good as", describes the normative ranking of social states– not to be confused with the positive description of individuals' actual preferences over those states, noted R_i^U and R_i^M. Hence, whereas $x R_I^M y$ indicated that a group I meta-preferred x to y, $x \succcurlyeq y$ indicates that it is also, and for that reason, preferable from a normative point of view (with asymmetric and symmetric elements of $\succcurlyeq$ noted $\succ$ and $\backsim$, respectively).

The PBR axiom captures Sen's (Sen, 2004 a) notion of normative validation through a process of inter-rational testing – e.g. through public discussion or reasoned argument (see also (Popper, 1980, p. 111)). Although we can never reach certainty that one normative standard is universally and unconditionally superior to the other, the logic developed in this paper implies that the international consensus reached about fundamental human rights, for instance, will carry more normative weight than, say, the norms that govern a small isolated community or tribe. This is because the former involves agreement between a large number of different individuals with widely divergent views, cultures and interests, whereas in the latter case it is more likely that agreement may have been facilitated by a strong convergence of interests and experiences. By the same logic, a national constitution should carry more normative weight than national legislation, insofar as it is protected by additional procedural guarantees (e.g. two thirds majority or double majority) that mean that a broader consensus (i.e. a more thorough vetting process) would be needed to reach agreement on the former than on the latter.

This idea is captured by the following proposition, which follows from definitions 1-2 and properties 1-5 (see Annex 3 for a proof of proposition 1)[viii]:

***Proposition 1**: If $\succcurlyeq$ is a binary transitive and reflexive ranking over Z satisfying PBR, and if properties 1-5 hold, then* $\forall c_I, c_J \in Z \;\; and \;\; \forall I, J \in Q,\; s.t. 1 < |I|, |J| < \infty : \;\; |I| > |J| \;\Rightarrow c_I \succcurlyeq c_J$.

A potentially controversial implication of proposition 1 is that the decisions of a large democracy, such as India, would, assuming comparable political systems, automatically carry more normative force than that of almost any other country. While this may seem like an uncomfortable conclusion, it is a logical consequence of the normative standard that relies on a process of inter-rational validation that gives equal weight to the views of each individual. To be convinced of this, it suffices to consider the alternative of a population-neutral normative standard, which would give equal weights to the consensus of Lichtenstein as to that of the United States. One way of avoiding these two extremes would be to devise a more restrictive ranking rule, contained in corollary 2, that only allows us to compare groups that have been able to agree on a common (higher) standard (the proof for corollary 2 is provided by the first step of the proof for proposition 1):

***Corollary 2**: If $\succcurlyeq$ is a binary transitive and reflexive ranking over Z satisfying PBR, and if properties 1-5 hold, then* $\forall c_I, c_A \in Z \;\; and \;\; \forall I, A \in Q : \;\; I \supset A \;\Rightarrow c_I \succcurlyeq c_A$.

Such a ranking rule would thus not allow us to compare, for instance, Denmark and the United States on their fulfilment of the right to healthcare implied by Art. 12.2.d of the international covenant of social, economic and cultural rights, since the United States is one of a handful of countries in the world that have not ratified the international covenant (although it is a signatory since 1977). By focusing on substets, rather than on numbers, we introduce more flexibility in the way that normative standards are defined and used. When judging each others' actions, members of the tribe or the village may be content with, and indeed prefer, doing so in accordance with their own set of rules and norms that they have all consented to rather than having to refer to abstract concepts of universal human rights. However, when comparing conflicting tribal laws, we will need to appeal to higher order consensus, such as a national legislation or international human rights. Under the current set of assumptions, however, the more restrictive ranking provided by corollary 2 would only be a special case of the more general ranking provided by proposition 1.

Consensus-Based Ranking
Finally, we propose an alternative normative ranking rule that captures the rationale of the above discussion, but presents the advantage of being based entirely on the external and thus observable properties of the consensus reached between different agents. Needless to say that the normative power of this ranking ultimately is derived from the assumed link of consensuses to underlying meta-preferences, much in the same way as the normative power of choice in the neoclassical framework was derived from its assumed link to underlying utility. By avoiding references to unobservable properties of the individual decision-making mechanism, however, we make it possible to centre the axiomatic discussion on the desirable political properties of a consensus-based ranking. We propose the four following simple axioms to characterise our ranking:

Anonymity (ANO):

$$\forall i, j \in N:\ c_i \backsim c_j$$

(6)

Monotonicity (MON):

$$\forall I, A \in Q, \qquad s.t.\ \ c_I, c_A \neq \ :\ I \supset A \Rightarrow\ c_I \succ c_A$$

(7)

Independence (IND):

$$\forall I, J \in Q, and\ \forall \quad \in Q \backslash (I \cup J), \qquad s.t.\ c_I \cap c \quad \neq , c_J \cap c \quad \neq : c_I \succcurlyeq c_J \Rightarrow c_I \cap c \quad \succcurlyeq c_J \cap c$$

(8)

Indifference between Non-Consensual Groups (ING):

$$\forall I, A \in Q, \qquad s.t. I \supset A \quad and\ \forall \quad \in Q \backslash A:\ c_A \cap c \quad = \quad \Rightarrow c_A \cap c \quad \sim c_I \cap c$$

(9)

The Anonymity axiom (ANO) states that the choice of any given individual has the same normative force as that of any other individual. The Monotonicity axiom (MON) states that the consensus reached by any given group of individuals has more normative force than the consensus reached by any of its subgroups. The Independence axiom (IND) states that a ranking between two consensuses will not be affected by the inclusion of an additional individual who agrees with both. These axioms echo fairly closely the axioms proposed by Pattanaik and Xu (Pattanaik & Xu, 1990) to characterise a cardinal ranking rule. However, they exclude cases in which individuals are unable to reach consensus. To complete the ranking we therefore need to add a fourth axiom, to deal with those cases. We do this by using the Indifference between Non-Consensual Groups axiom (ING), which states that all groups that have been unable to reach consensus will have the same normative force regardless of their size.

Based on these four axioms, we are able to construct a ranking rule that is dependent solely on the number of individuals reaching consensus (see Annex 4 for a proof of theorem 1):

Theorem 1: *If* $\succcurlyeq$ *is a binary, reflexive and transitive ranking over* Z, *satisfying ANO, MON, IND and ING, then* $\forall\, I, J \in Q$ *and* $\forall c_I, c_J \in Z : c_I \succcurlyeq c_J \Leftrightarrow |I| \geq |J|$.

It is easiest to think of I and J as sets of individuals (e.g. electorates). However, since the consensus-based ranking is not directly dependent on the properties of individual motivational structures, it is also possible to think of a different unit, such that I and J representing sets of countries, for instance. This would be relevant for the cases discussed under proposition 1, where it may be necessary to give equal weight to countries regardless of their population, as is currently the case in international human rights legislation, for instance.

Before concluding, we should note that by taking the existing consensus as the starting point of the assessment, and thus disregarding the decision-making process, we are implicitly imposing a number of assumptions that will need to be clarified. In particular, we have assumed that the decisions reached reflect a genuine consensus in the sense that no individual has been subjected to any form of coercion, intimidation or deceit. We may also need to add assumptions regarding the nature of the decision makers. In particular, we need to assume that no individual has more power than any other and can influence the decisions of his followers. Finally, we are, of course, assuming that everyone has access to the same level of information and has sufficient information to make free and informed decisions. All of these assumptions may be optimistic and none may in fact ever be met in reality. However, they are not fundamentally less plausible than the basic set of assumptions needed to prove the fundamental theorems of welfare economics. In fact these assumptions echo quite well the ones that characterise the Walrasian general equilibrium, such as perfect and competitive markets with perfect information and no monopolies. As such, they may provide an adequate basis for exploring the conditions under which these ideals may be realised and the reasons for which they fail to do so.

4. Conclusion

The assumption that individuals consistently act to maximise their own utility has become so central to normative and positive economic theory that it has come to be seen as an integral and inalienable part of economic theory in the eyes of many of its supporters as well as its detractors[ix]. As Stigler put it: "We [economists] believe that man is a utility maximising animal (…) and to date we have not found it informative to carve out a section of his life in which he invokes a different goal of behaviour" (Stigler, 1982, p. 26).

In this paper, we have tried to show that there is nothing intrinsically "economic" about this assumption, any more than there is any inherent reason why market equilibria should be considered "efficient" when they maximise utility – or balance marginal utilities – rather than, say, when they fulfil social and economic rights[x]. The conclusions reached in this paper suggest that the relaxation of this unverified axiom could have far-reaching implications for the normative conclusions reached in economic assessments, and could allow us to envisage a welfare economics based on non-utilitarian premises. This analysis has immediate and practical implications for the way in which we can and should measure welfare and deprivation.

First of all, the argument developed in this paper allows us to discard the use of market prices as acceptable normative weights in such assessments. Market prices are the product of an iterative process involving a very large number of decisions taken by consumers, individually, based on tastes, needs and perhaps even moral concerns (and/or producers individually, based on production costs, etc.). At no point does this process require individuals to reach out and seek compromise with other consumers who have different interests and inclinations. Consequently, market prices do not tell us anything about people's reasoned assessments of the various options they face. From the perspective of non-utilitarian framework, such as the one proposed by Prof. Sen, this thus undermines the normative justification for using measures, such as GDP or income poverty, which effectively rely on market prices to weigh different goods or dimensions of well-being.

Let us now consider another measure that assigns relative monetary values to different goods, namely a public budget that has been voted by a democratically elected parliament. Unlike market prices, a budget will require lengthy discussions and a process of inter-rational validation in order to agree on which priorities the community should invest in and in what proportions. The weights provided by public spending ratios in a national budget could therefore, unlike market prices, constitute a valid normative standard in the construction of an aggregate index.

It is possible to think of other standards that may carry information about weights. The Millennium Declaration, for instance, describes an implicit weighting system, as it defines eight equally important goals, each of which is composed of a number of targets, which in turn are measured by various indicators. The Universal Declaration of Human Rights also makes reference to weights and even to marginal rates of substitution when it states, for instance, that all rights are indivisible and inalienable (implying zero elasticity of substitution between rights). More importantly, it also implicitly assigns a weight of zero to a large number of rights, namely those that are not included in the declaration.

Which of these standards should be used in constructing a normative index of welfare will depend on the purpose and the scope of the assessment. A national budget may, for instance, provide an adequate standard for a punctual assessment of national policies, whereas a more demanding standard, such as the Universal Declaration, may be required when making a comparison between various countries and over several years. Finally, an assessment geared specifically towards development issues, may be content with a more focused normative standard, such as the Millennium Declaration.

Annex 1: Proof of Lemma 1
Let $A, I \in Q$ be two non-empty groups of individuals, such that $A \subset I$ and $1 < |I| < \infty$. Further, let $B = I \backslash A$.

$A \neq$ implies $|A| > 0$, whereas $A \subset I$implies $|B| = |I| - |A| > 0$, given that $1 < |I| < \infty$. There is therefore at least one individual in each subgroup.

Furthermore, since $B = I \backslash A$, we can use definition 1 to infer that $\chi_A^U \cap \chi_B^U \subseteq \chi_A^U$, given that there is at least one pair of individuals, i and j, $i \neq j$, such that $R_i^U \neq R_j^U$.

Consequently, $p(\chi_A^U \cap \chi_B^U) \leq p(\chi_A^U)$.

(10)

By contrast, since $A, B \in I$, definition 2 implies that $\chi_A^M \cap \chi_B^M = \chi_A^M = \chi_B^M$, since $R_i^M = R_j^M$ for all $i, j \in I$.

Consequently, $p(\chi_A^M \cap \chi_B^M) = p(\chi_A^M)$

(11)

Let us now turn to the conditional proabilties $p(\chi_A^M \cap \chi_B^M | c_A \cap c_B)$ and $p(\chi_A^M | c_A)$:

By definition of a conditional probability, we have:

$$p(\chi_A^M \cap \chi_B^M | c_A \cap c_B) = \frac{p(\chi_A^M \cap \chi_B^M \cap c_A \cap c_B)}{p(c_A \cap c_B)}$$

(12)

And:

$$p(\chi_A^M | c_A) = \frac{p(\chi_A^M \cap c_A)}{p(c_A)}$$

(13)

Property 2 combined with property 1 implies that $\chi_i^M \subseteq c_i$, since it means that c_i must have been caused by either χ_i^M or χ_i^U and cannot have been caused by any of the other motives, ψ_i^M or ψ_i^U.

Consequently: $\chi_A^M \cap c_A = \chi_A^M$ and thus $p(\chi_A^M \cap c_A) = p(\chi_A^M)$.

We can therefore rewrite (12) and (13) as:

$$p(\chi_A^M \cap \chi_B^M | c_A \cap c_B) = \frac{p(\chi_A^M \cap \chi_B^M)}{p(c_A \cap c_B)}$$

(14)

And:

$$p(\chi_A^M | c_A) = \frac{p(\chi_A^M)}{p(c_A)}$$

(15)

We know from (11) that : $p(\chi_A^M \cap \chi_B^M) = p(\chi_A^M)$. The magnitudes of (14) and (15) will therefore depend on $p(c_A \cap c_B)$ and $p(c_A)$.

Since $B = I \backslash A$, it follows from the definition of $c_I \equiv \bigcap_{i=1}^{|I|} c_i$ that $c_A \cap c_B \subseteq c_A$.

Consequently:

$$p(c_A \cap c_B) \leq p(c_A)$$

(16)

Combining equations (14) and (15) with results (11) and (16), finally, we get:

$$p(\chi_A^M \cap \chi_B^M | c_A \cap c_B) = \frac{p(\chi_A^M \cap \chi_B^M)}{p(c_A \cap c_B)} \geq \frac{p(\chi_A^M)}{p(c_A)} = p(\chi_A^M | c_A)$$

(17)

By construction of χ_I^M and c_I, finally, $B = I \backslash A$ implies $\chi_A^M \cap \chi_B^M = \chi_I^M$ and $c_A \cap c_B = c_I$. We can therefore rewrite (17) as:

$$p(\chi_I^M | c_I) \geq p(\chi_A^M | c_A)$$

(18)

Annex 2: Proof of Corollary 1

By the general property of additivity of probabilities, we have:

$$p(\chi_I^U \cup \chi_I^M | c_I) \quad = p(\chi_I^U) + p(\chi_I^M) - p(\chi_I^U \cap \chi_I^M)$$

(19)

By construction of χ_I^M and c_I, we can rewrite equation (19) in its generalised for the case of a group I of cardinality $|I|$ as:

$$p(\chi_I^U \cup \chi_I^M | c_I) \quad = p\left(\bigcap_{i=1}^{|I|} \chi_i^U | c_i\right) + p\left(\bigcap_{i=1}^{|I|} \chi_i^M | c_i\right) \ - p\left(\bigcap_{i=1}^{|I|} (\chi_i^U \cap \chi_i^M) | c_i\right)$$

(20)

Let us now consider what happens to (20) when n tends to ∞:

From definition 2, first, we know that R_i^M is equal for all $i \in I$, which means that $\bigcap_{i=1}^{\infty}(\chi_i^M) = \chi_i^M = \chi_I^M$.

Consequently, $\lim_{|I|\to\infty}\left[p\left(\bigcap_{i=1}^{|I|} \chi_i^M | c_I\right)\right] = p(\chi_I^M | c_I)$.

Definition 1, on the other hand, tells us that $\bigcap_{i=1}^{\infty}(\chi_i^U) =$, since $R_i^U \neq R_j^U$ for some $i, j \in I$.

Hence, as $|I|$ tends to infinity, $\lim_{|I|\to\infty}\left[p\left(\bigcap_{i=1}^{|I|} \chi_i^U | c_I\right)\right] = p() = 0$

And $\lim_{|I|\to\infty}\left[p\left(\bigcap_{i=1}^{|I|}(\chi_i^U \cap \chi_i^M) | c_I\right)\right] = p(\cap \chi_I^M | c_I) = p() = 0$.

Consequently, in the limit, equation (20) will boil down to:

$$\lim_{|I|\to\infty} p(\chi_I^U \cup \chi_I^M | c_I) \quad = p() + p(\chi_I^M | c_I) - p(\cap \chi_I^M | c_I)$$

(21)

which is equivalent to:

$$\lim_{|I|\to\infty} p(\chi_I^U \cup \chi_I^M | c_I) \quad = p(\chi_I^M | c_I)$$

(22)

By the property of completeness (property 2), finally, we have:

$$\lim_{|I|\to\infty} p(\chi_I^U \cup \chi_I^M | c_I) = p(\chi_I^M | c_I) = 1$$

(23)

Annex 3: Proof of Proposition 1

Let $I, J \in Q$ be two groups of individuals, such that $|I| > |J|$ and let $A \subset I$ such that $|A| = |J|$.

By Lemma 1, we have:

$$I \supset A \Rightarrow p(\chi_I^M|c_I) \geq p(\chi_A^M|c_A)$$

(24)

By definition of $\succcurlyeq$ and $\geq$, $\succ$ implies $\succcurlyeq$ and $>$ implies $\geq$.

We can thus apply PBR1 to get $p(\chi_I^M|c_I) \geq p(\chi_A^M|c_A) \Rightarrow c_I \succcurlyeq c_A$.

Further, we know that $|A| = |J| > 1$. We can therefore choose two pairs of distinct individuals $(\ \ , i), (j, k) \in N$, such that $\ \ , i \in A, \ \ \neq i,$ and $j, k \in J, \ j \neq k.$

By property 4, we have:

$$\forall i, j \in N: p\left(\chi_i^U|c_i \ \ \right) = p\left(\chi_j^U|c_j \ \ \right) \text{ and } \forall(\ \ , k) \in N: p\left(\chi^U|c \ \ \ \right) = p\left(\chi_k^U|c_k \ \right)$$

(25)

Furthermore, by property3, we have $\forall i \in N$ (including $j,$ and k):

$$p\left(\chi_i^U\right) = p(\chi_i^M)$$

(26)

By property 2, we have $\forall i, j \in N$ (and thus also for and k):

$$p\left(\chi_i^U \cup \chi_i^M\right) = 1 = p\left(\chi_j^U \cup \chi_j^M\right)$$

(27)

By the general property of additivity, (27) is equivalent to:

$$\begin{aligned} p\left(\chi_i^U \cup \chi_i^M|c_i\right) &= p\left(\chi_i^U|c_i\right) + p(\chi_i^M|c_i) - p\left(\chi_i^U \cap \chi_i^M|c_i\right) \\ &= p\left(\chi_j^U|c_j\right) + p\left(\chi_j^M|c_j\right) - p\left(\chi_j^U \cap \chi_j^M|c_j\right) = p\left(\chi_j^U \cup \chi_j^M|c_j\right) \end{aligned}$$

(28)

Solving for $p\left(\chi_i^U \cap \chi_i^M|c_i\right)$, we get:

$$p\left(\chi_i^U \cap \chi_i^M|c_i\right) = p\left(\chi_j^U \cap \chi_j^M|c_j\right) + p\left(\chi_i^U|c_i\right) - p\left(\chi_j^U|c_j\right) + p(\chi_i^M|c_i) - p\left(\chi_j^M|c_j\right)$$

(29)

From (25) and (26), we know that: $p\left(\chi_i^U|c_i\right) = p\left(\chi_j^U|c_j\right)$ and $p(\chi_i^M|c_i) = p\left(\chi_j^M|c_j\right)$.

Consequently, (29) boils down to:

$$p\left(\chi_i^U \cap \chi_i^M|c_i\right) = p\left(\chi_j^U \cap \chi_j^M|c_j\right)$$

(30)

Using the same method, we can show that the same holds for $p(\chi^{U} \cap \chi^{M} | c\ \ ) = p(\chi_k^{U} \cap \chi_k^{M} | c_k)$.

By property 5, we can combine these two pairs to obtain:

$$p(\chi_i^{U} \cap \chi^{U} | c_i \cap c\ \ ) = p(\chi_i^{U} | c_i) \times p(\chi^{U} | c\ \ ) = p(\chi_j^{U} | c_j) \times p(\chi_k^{U} | c_k) = p(\chi_j^{U} \cap \chi_k^{U} | c_j \cap c_k)$$

(31)

Similarly, by using properties 4 and 5, we can add all remaining elements in A and J without affecting the equality, to obtain:

$$p(\chi_A^{U} | c_A) = p(\chi_J^{U} | c_J)$$

(32)

Using property 2, we then have:

$$p(\chi_A^{U} \cup \chi_A^{M} | c_A) = 1 = p(\chi_J^{U} \cup \chi_J^{M} | c_J)$$

(33)

Which is equivalent to:

$$\begin{aligned} p(\chi_A^{U} \cup \chi_A^{M} | c_A) &= p(\chi_A^{U} | c_A) + p(\chi_A^{M} | c_A) - p(\chi_A^{U} \cap \chi_A^{M} | c_A) \\ &= p(\chi_J^{U} | c_J) + p(\chi_J^{M} | c_J) - p(\chi_J^{U} \cap \chi_J^{M} | c_J) = p(\chi_J^{U} \cup \chi_J^{M} | c_J) \end{aligned}$$

(34)

Solving for $p(\chi_A^{M} | c_A)$:

$$p(\chi_A^{M} | c_A) = p(\chi_J^{M} | c_J) + p(\chi_J^{U} | c_J) - p(\chi_A^{U} | c_A) - p(\chi_J^{U} \cap \chi_J^{M} | c_J) + p(\chi_A^{U} \cap \chi_A^{M} | c_A)$$

(35)

From (32), we know that $p(\chi_A^{U} | c_A) = p(\chi_J^{U} | c_J)$. Furthermore, we know from (30) that $p(\chi_i^{U} \cap \chi_i^{M} | c_i) = p(\chi_j^{U} \cap \chi_j^{M} | c_j)$ for all $i, j \in N$, which implies $p(\chi_A^{U} \cap \chi_A^{M} | c_A) = p(\chi_J^{U} \cap \chi_J^{M} | c_J)$.

Consequently, (35) reduces to:

$$p(\chi_A^{M} | c_A) = p(\chi_J^{M} | c_J)$$

(36)

We can now apply PBR2, to get: $p(\chi_A^{M} | c_A) = p(\chi_J^{M} | c_J) \Rightarrow c_A \sim c_J$.

Finally, since $\succcurlyeq$ is transitive over Z: $c_I \succcurlyeq c_A \ and \ c_A \sim c_J \Rightarrow c_I \succcurlyeq c_J$.

Annex 4: Proof of Theorem 1.
It is easy to see that a ranking of consensus sets in terms of the number of individuals reaching consensus would be transitive and would satisfy (ANO), (POC), (IND) and (ING).

In order to prove the theorem, we need to prove that for any binary transitive ranking $\succcurlyeq$ satisfying the four axioms:

$$\forall I, J \in Q, \forall c_I, c_J \in Z: |I| = |J| \Rightarrow c_I \sim c_J$$

(37)

And

$$\forall I, J \in Q,, \forall c_I, c_J \in Z: |I| > |J| \Rightarrow c_I \succ c_J$$

(38)

Let us start with a proof of (37) by induction:

Axiom ANO gives us the basis of the induction for the case where $n = 1$, as it tells us that for any individuals $\{i\}, \{j\} \in N: c_i \backsim c_j$. Our inductive hypothesis is that this holds for all n. In order to prove the inductive step, we need to show that this holds true also for $n + 1$. Consider the sets $I, J \in Q$, such that $c_I \neq$ and $c_J \neq$.

Let $|I| = |J| = n + 1$, with $1 \leq n \leq |N|$.

Further, let $\{i\} \in I$ and let $D \subset I$, such that $D = I \backslash \{i\}$.

Since $|I| = n + 1$ and since $|\{i\}| = 1$ by definition of a singleton, it follows that $|D| = n$.

Logically, $\{i\}$ must either be a member of J or not be a member of J. If $\{i\} \in J$, we know, by construction of c_J that $c_i \cap c_J \neq$. However, if $\{i\} \notin J$ it can either be the case that $c_i \cap c_J \neq$ or that $c_i \cap c_J =$. This gives rise to 3 possible cases:

<u>Case 1 ($\{i\} \in J$):</u>

Let $E \subset J$ such that $E = J \backslash \{i\}$.

By construction of c_I and c_J we know that $c_D \neq$ and $c_E \neq$.

Since $|J| = n + 1$ and $|\{i\}| = 1$, it follows that $|E| = n$.

Since we know that $|D| = n = |E|$, we can apply the inductive hypothesis to get $c_D \backsim c_E$.

By applying the IND axiom, we then get: $c_D \cap c_i \backsim c_E \cap c_i$, which,

By construction of D and E is equivalent to $c_I \backsim c_J$.

<u>Case 2 ($\{i\} \notin J$ and $c_i \cap c_J \neq$):</u>

If $\{i\} \notin J$ and $|I| = |J| = n + 1$, then there must be a $\{j\} \in J$, such that $\{j\} \notin I$.

Let $F \subset J$ such that $F = J\backslash\{j\}$.

By construction of c_J we know that $c_F \neq$.

Since$|J| = n + 1$ and $|\{j\}| = 1$ by definition of a singleton, it follows that $|F| = n$.

We can thus apply the inductive hypothesis to get: $c_D \backsim c_F$, and then

Apply IND to obtain: $c_D \cap c_i \backsim c_F \cap c_i$.

By construction of D, this is equivalent to $c_I \backsim c_F \cap c_i$.

Let $\{f\} \in F$.

Since $F \subset J$ it follows from $\{i\} \notin J$ that $\{i\} \notin F$, and from $\{f\} \in F$ that $\{f\} \in J$. Consequently, we have: $|(F\backslash\{f\}) \cup \{i\}| = n$ and $|J\backslash\{f\}| = n$.

By applying the induction hypothesis, we get: $c_{F\backslash f} \cap c_i \backsim c_{J\backslash f}$.

We can then apply IND to obtain: $c_F \cap c_i \backsim c_J$.

We have now shown that $c_I \backsim c_F \cap c_i$ and that $c_F \cap c_i \backsim c_J$.

Since $\succcurlyeq$ is transitive over Z, this implies that: $c_I \backsim c_J$.

<u>Case 3 ($\{i\} \notin J$ *and* $c_i \cap c_J =$):</u>

This is a special case in which, by definition of a non-consensus, we have $n = 0$ for $c_i \cap c_J$. So we need to show that the indifference holds for all $n = 0$ and $n + 1 = 1$.

If it is the case that $c_i \cap c_J =$, then there must be at least one element, $\{j\} \in J$, such that $c_i \cap c_j =$.

By ING, we have: $c_i \cap c_J \sim c_i \cap c_j$

Since we know that $i \in I$, and that $c_i \cap c_j =$ we can also use ING to expand I around i to obtain: $c_i \cap c_j \sim c_I \cap c_j$.

Since $\succcurlyeq$ is transitive on Z, we have: $c_i \cap c_J \sim c_i \cap c_j$ *and* $c_i \cap c_j \sim c_I \cap c_j \Rightarrow c_i \cap c_J \sim c_I \cap c_j$

The choice sets of individuals i and j can be compared using the ANO axiom, which yields: $c_i \sim c_j$. By definition of a singleton consensus set, we have: $c_i \cap c_i \neq$ and $c_j \cap c_j \neq$, which covers the case when $n = 1$.

Proof for (38):

Let $|I| > |J|$.

Now, let $G \subset I$, such that $|G| = |J|$.

By construction of c_I, we know that $c_G \neq$.

By (37), we have: $c_G \sim c_J$.

Further, by MON, we have: $c_I \succ c_G$

Finally, we can apply transitivity of $\succcurlyeq$ to obtain: $c_I \succ c_J$.

QED.

BIBLIOGRAPHY

Alkire, S., & Santo, M. E. (2010). *Acute multidimensional poverty: a new index for developing countries.* New York: UNDP.

Ameriks, K. (2000). *Kant and the fate of autonomy : problems in the appropriation of the critical philosophy.* Cambridge: Cambridge University Press.

Arrow, K. J. (1959). Rational Choice Functions and Orderings. *Economica , 26* (102), 121-127.

Arrow, K. J., & Debreu, G. (1954). Existence of a Competitive Equilibrium for a Competitive Economy. *Econometrica , 22* (3), 265-290.

Becker, G. S. (1976). *The economic approach to human behavior.* Chicago: Chicago University Press.

Bentham, J. (1789). *The principles of morals and legislation.* Oxford: Clarendon Press.

Harsanyi, J. C. (1997). Utilities, preferences, and substantive goods. *Social Choice and Welfare , 14*, 129-145.

Hicks, J. R. (1956). *A revision of demnand theory.* Oxford: Oxford University Press.

Hicks, J. R. (1939). *Value and capita.* Oxford, England: Clarendon Press.

Houthakker, H. S. (1950). Revealed Preference and the Utility Function. *Economica , 17* (66), 159-174.

Jevons, S. W. (1871). *The theory of political economy* (Fourth Edition: 1911 ed.). London: Macmillan and Company, Ltd.

Kant, I. (1781). Critique of Pure Reason (Kritik der reinen Vernunft (1. Aufl. 1781), Prolegomena, Grundlegung zur Metaphysik der Sitten, Metaphysische Anfangsgründe der Naturwissenschaft). In W. d. Gruyte, & E. G. Holger Catherine (Ed.), Akademieausgabe von Immanuel Kants Gesammelten Werken (1969 ed., Vol. IV, pp. 1-238). Berlin: Royal German Academy of Sciences.

Kant, I. (1790). Critique of Judgment, (Kritik der Urtheilskraft). In W. d. Gruyter, Kants gesammelte Schriften (1902 ed., Vol. 5). Berlin: Royal German Academy of Sciences.

Kant, I. (1786). Review of Gottlieb Hufeland's Principles of Natural Law. Recension von Gottlieb Hufeland's Versuch über den Grundsatz des Naturrechts. In W. d. Gruyter, *Kants gesammelte Schriften* (1963 ed., Vol. 27). Berlin: Royal German Academy of Sciences.

Kant, I. (1798). 'Le conflit des facultés. Der Streit der Facultaten.' In *Kants gesammelte Schriften* (W. d. Gruyter, Trans., 1963 ed., Vol. 7). Berlin: Royal German Academy of Sciences.

Little, I. M. (1949). A Reformulation of the Theory of Consumer's Behaviour. *Oxford Economic Papers , 1*.

Ludwig von Mises. (1949). *Human action.* New Haven, CT: Yale University Press.

Marshall, A. (1890). *Principles of economics: an introductory volume* (8th edition ed.). New York: Macmillan.

Nussbaum, M. (2003). Capabilities as Fundamental Entitlements: Sen and Social Justice. *Feminist Economics , 9* (2), 33-59.

Pareto, V. (1906). *Manual of political economy* (1927 ed.). New York: Kelley.

Pattanaik, P., & Xu, Y. (1990). On Ranking Opportunity Sets in Terms of Freedom of Choice. *Recherches Economiques de Louvain , 56*, 383–390.

Popper, K. R. (1980). *The logic of scientific discovery.* London: Hutchinson.

Ravallion, M. (2010). *Mashup indices of development.* Policy Research Working Paper, World Bank, Washington D.C.

Samuelson, P. A. (1938). A Note on the Pure Theory of Consumer's Behaviour. *Economica , 5* (17), 61-71.

Samuelson, P. A. (1948). Consumption Theory in Terms of Revealed Preference . *Economica , 15* (60), 243-253.

Sen, A. K. (1973). Behaviour and the Concept of Preference. *Economica , 40* (159), 241-259.

Sen, A. K. (1971). Choice Functions and Revealed Preference. *The Review of Economic Studies , 38* (3), 307-317.

Sen, A. K. (1997). *Choice, welfare and measurement.* Cambridge Massachusetts: Harvard University Press.

Sen, A. K. (2004 a). Dialogue, Capabilities, Lists, and Public Reason: Continuing the Conversation. *Feminist Economics , 10* (3), 77 – 80.

Sen, A. K. (2004 b). Elements of a Theory of Human Rights. *Philosophy & Public Affairs , 32* (4), 315-356.

Sen, A. K. (1980). 'Equality of What?' In S. McMurrin (Ed.), *Tanner Lectures on Human Values* (Vol. I). Cambridge: CambridgeUniversity Press.

Sen, A. K. (1991). *Markets and freedoms.* London School of Economics and Political Science, Development Economics Research Programme. London: Suntory-Toyota International Centre for Economics and Related Disciplines.

Sen, A. K. (1987). *On ethics and economics*. Oxford: Blackwell.

Sen, A. K. (1977). Rational fools: a critique of the behavioural foundations of economic theory. *Philosophy and Public Affairs , 6*, 317-344.

Sen, A. K. (2002). *Rationality and freedom*. Cambridge, MA: The Belknap Press of Harvard University Press.

Sen, A. K. (1984). *Resources, values and development*. Oxford: Basil Blackwell.

Sen, A. K. (1983). Liberty and Social Choice. *The Journal of Philosophy 80* (1), 5-28.

Sen, A. K., & Williams, B. (1982). *Utilitarianism and beyond*. New York: Cambridge University Press.

Smith, A. (1776). 'Book IV: Of Systems of Political Economy.' In E. R. Seligman (Ed.), *An inquiry into the nature and cause of the wealth of nations* (1957 ed., Vol. 1). London: J.M. Dent & Sons. Ltd.

Stigler, G. J. (1982). *The economist as preacher, and other essays*. The University of Chicago: Basil Blackwell Publishers.

Stiglitz, J., Sen, A. K., & Fitoussi, J.-P. (2008). The Measurement of Economic Performance and Social Progress Revisited: Reflections and Overview. *Commission on the Measurement of Economic Performance and Social Progress*. Paris.

Walras, L. (1874). 'Elements of Pure Economics – Or the Theory of Social Wealth.' In O. Editions (Ed.). Philadelphia, PA: Homewood, Ill.: Richard D. Irwin Inc.

[i] Walras' description predicts that markets will, under specified conditions, reach a competitive equilibrium when consumers' marginal rates of (utility) substitution between goods have been equalised with producers' marginal rates of transformation for the same goods through the intermediate of market prices. The notion of Pareto-efficiency, on the other hand, defines an outcome as optimal if no one can be made better off without making someone else worse off. Both Walras' positive description of market equilibria and the normative standard of Pareto-optimality grew out of the marginalist critique of classical economics (Jevons, 1871; Marshall, 1890), which had relied on aggregate values, including aggregate utility – defined as "the greatest happiness of the greatest number" (Bentham, 1789).

[ii] It may be worth noting that the rationale for adopting one or the other of these representations of human nature cannot be based on empiricist criteria of "verifiability" since the claim that individuals are driven exclusively by self-interested utility maximisation is as unobservable and unverifiable as the claim that there may be a reasoned

component to human behaviour (Ameriks, 2000, p. 42). Therefore our justification for choosing this particular set of assumptions must be based on logical criteria, which take us far beyond the scope of this paper. The interested reader is referred to Kant's critique of utilitarian and Hobbesian philosophy (Kant, 1798, p. 208; Kant, 1786, p. 1321).

[iii] Formally, Kant defines pure reason as the "faculty of the unity of the rules of understanding under principles" (Kant, 1781, p. A302/B359), that is the capacity for abstraction or universalisation. Judgement, then is the capacity "of thinking the particular as contained under the universal" (Kant, 1790, p. 5:179), that is to link a particular situation to a universal rule.

[iv] Figure 1 excludes the cases which violate the assumption of complete preferences such as: $\neg x R_i^U y$ and $\neg y R_i^U x$, for instance.

[v] This assumption is consistent with definition 1 and includes cases in which the individuals exhibit sympathy or altruism. If i feels sympathy for j, he does not include j's actual utility function in his own, but rather constructs a mental representation of the elements that he thinks will affect j's utility. Actual motivations are thus independent across individuals, even though presumptions about other people's motivations may affect decisions. By contrast, meta-preferences, R_i^M, are defined by the ability that individuals have to achieve a common understanding within a given group. By this definition, the members of this group must therefore be able to hold the exact same meta-rankings.

[vi] Sen's has referred elsewhere to his approach as being part of the "rationalist" tradition, including non-utilitarian thinkers, such as Kant, Rousseau among others (Sen 1999, p.255).

[vii] For a more general critique of utilitarian moral philosophy, see (Sen & Williams, 1982).

[viii] It is important to note that we are ranking choice sets, rather than the options that may be contained in the choices that have been made. Hence, if groups I and J both agree on the public provision of healthcare, for instance, we could have $c_I \succ c_J$ if $|I| > |J|$, which simply means that the normative force of I's preference for healthcare is greater than that of J's preferences for healthcare. The ranking will therefore be transitive over Z but may not be transitive over X.

[ix] Nowhere is this confusion of the discipline for its axioms more evident than in Becker's exploration of the "economics" of issues pertaining to the fields of biology, criminology and even religion (Becker, 1976).

[x] As Prof. A.K. Sen has shown, the reason for which both positive and normative economic theory have embraced utilitarian ethics as well as its descriptive model of human behaviour, is primarily a historic one – namely the fact that many of the founders of modern economic theory, starting with Adam Smith himself, were utilitarian moral philosophers (Sen, 1987).