# Time decompositions of the adjusted headcount ratio

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- The adjusted headcount ratio, M0, has nice decomposability properties, e.g. by regions, groups, etc.
- In this class we will explore the nice decomposability properties of $\Delta\%M0$ and its components: $\Delta\%H$ and $\Delta\%A$.
- This material is based on Apablaza, Ocampo and Yalonetzky (2010), and on Apablaza and Yalonetzky (2011).

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- Then we will discuss more decompositions relevant to panel datasets.
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- We will finish with some remarks on comparability from Apablaza, Ocampo and Yalonetzky (2010).

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$$\Delta\%_a M^0(t) \equiv \frac{M^0(X^t; Z) - M^0(X^{t-a}; Z)}{M^0(X^{t-a}; Z)}$$

# Basic decomposition of M0

$$\Delta\%_a M^0(t) = \Delta\%_a H(t) + \Delta\%_a A(t) + \Delta\%_a H(t)\Delta\%_a A(t)$$

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- or if $k < D$ it is possible that $\Delta\%_a H(t) = 0$ and $\Delta\%_a A(t) \neq 0$.
- As k goes from 1 to D, H decreases and A increases "mechanically". Hence as k increases toward D, it is more likely to find higher $\Delta\%_a H(t)$ and lower $\Delta\%_a A(t)$.

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In turn:

$$H^i(X_i^t, Z) = \frac{1}{N_i^t} \sum_{n=1}^{N} I(c_n \geq k) I(n \in i)$$

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$$\Delta\%_a H(t) = \sum_{i=1}^{G} r_i(t-a)[\Delta\%_a \psi_i^t + \Delta\%_a H^i(X_i^t, Z) + \Delta\%_a \psi_i^t \Delta\%_a H^i(X_i^t, Z)]$$

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And $A_d(X^t, Z) \equiv \frac{\sum_{n=1}^{N^t} I(c_n \geq k \wedge x_{nd}^t \leq z_d)}{N(t)H(t)}$

Then:

$$\Delta\%_a A(t) = \sum_{d=1}^{D} s_d(t-a)[\Delta\%_a \theta_d A_d(X^t, Z)] = \sum_{d=1}^{D} s_d(t-a)[\Delta\%_a A_d(X^t, Z)]$$

Then:

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Because, by construction, $\Delta\%_a \theta_d = 0$

In practical comparisons, we divide the changes by the year-gaps to improve comparability.

# The countries

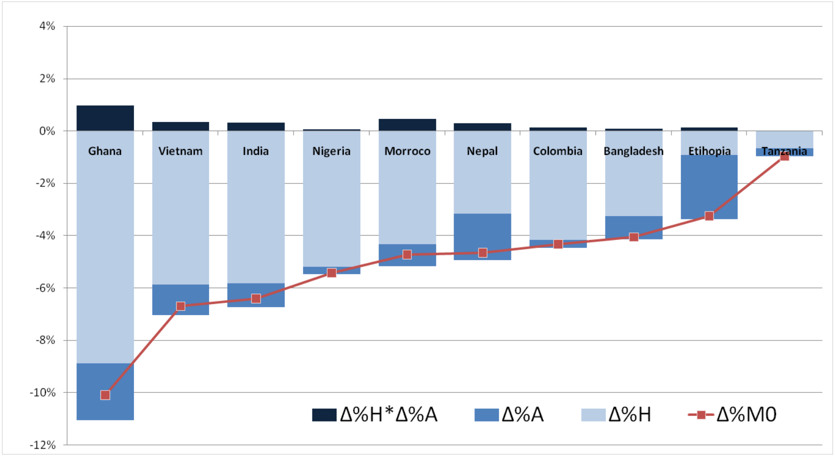
| Country | Years |
|---|---|
| Bangladesh | 2004-2007 |
| Colombia | 1995-2005 |
| Ethiopia | 2000-2005 |
| Ghana | 2003-2008 |
| India | 1999-2005 |
| Morocco | 1992-2004 |
| Nepal | 2001-2006 |
| Nigeria | 1999-2003 |
| Tanzania | 2005-2008 |
| Vietnam | 1997-2002 |

# The variables

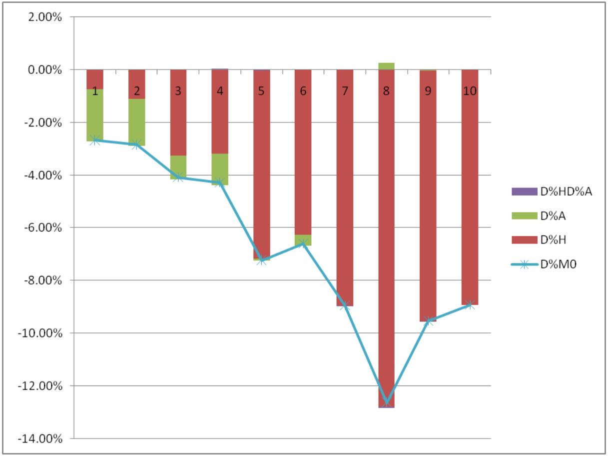
| Variable | B | C | E | G | I | M | Ne | Ni | T | V |
|---|---|---|---|---|---|---|---|---|---|---|
| Years school | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Enrollment | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Child mortality | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Nutrition | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | x | x |
| Electricity | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Toilet | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Water | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Floor | ✓ | ✓ | ✓ | ✓ | x | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cooking | ✓ | ✓ | ✓ | ✓ | ✓ | x | ✓ | x | ✓ | x |
| Asset | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

B=Bangladesh; C=Colombia; E=Ethiopia; G=Ghana; I=India
M=Morocco; Ne=Nepal; Ni=Nigeria; T=Tanzania; V=Vietnam

# Decomposition of M0 for 10 countries and k=3



# The impact of the choice of k: the case of Bangladesh



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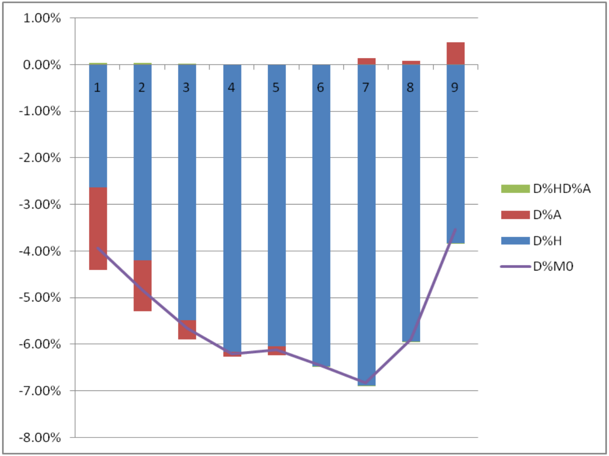

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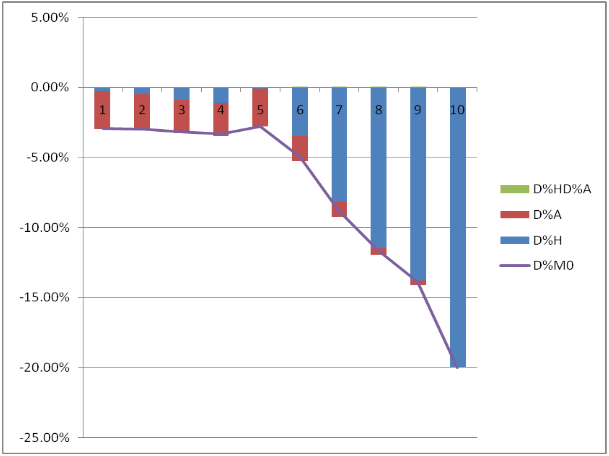

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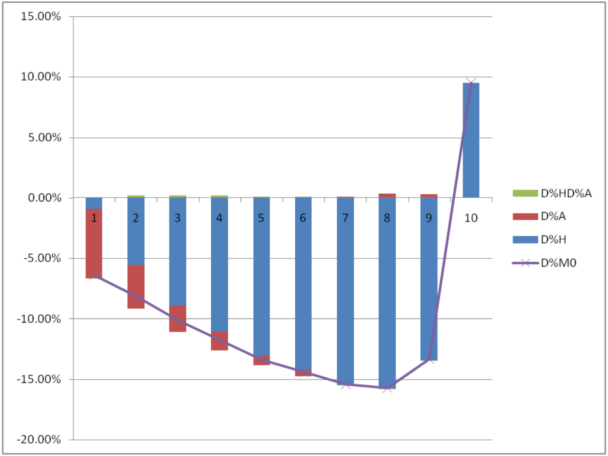
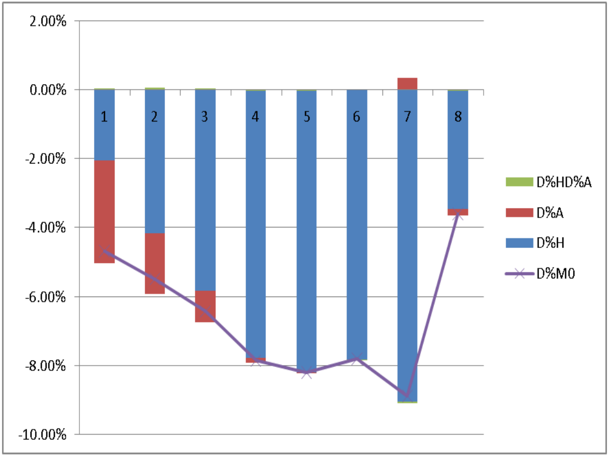
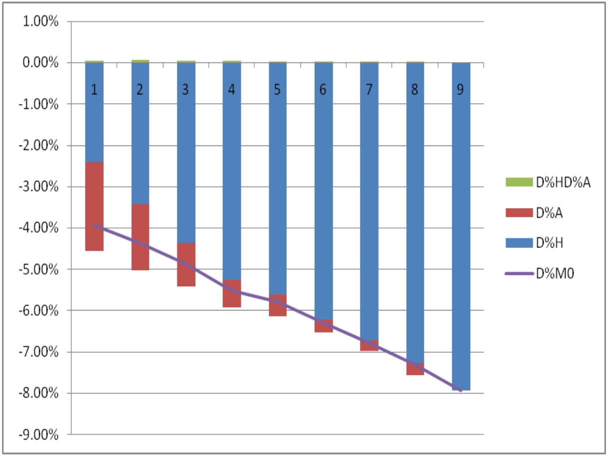


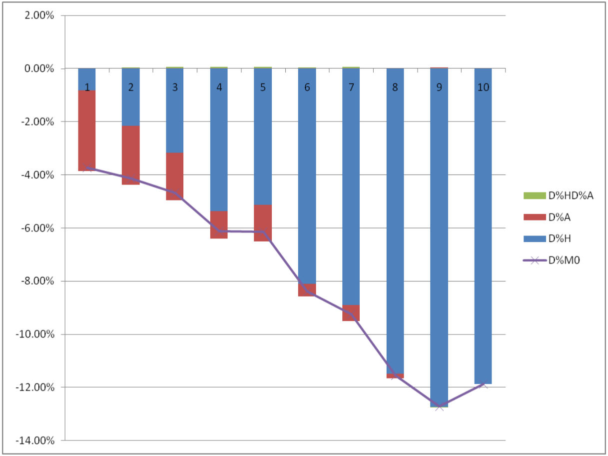
# The impact of the choice of k: the case of India



# The impact of the choice of k: the case of Morocco



# The impact of the choice of k: the case of Nepal



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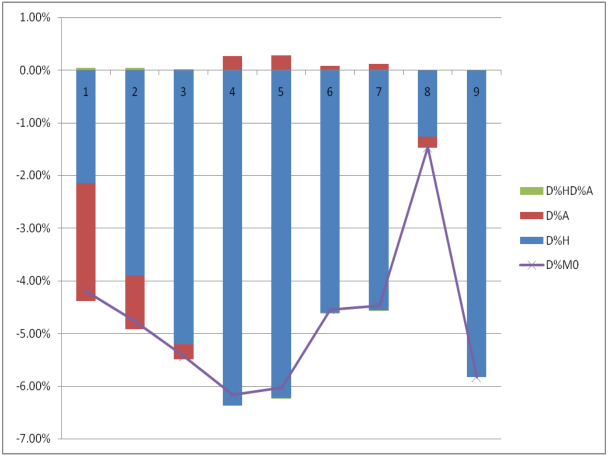

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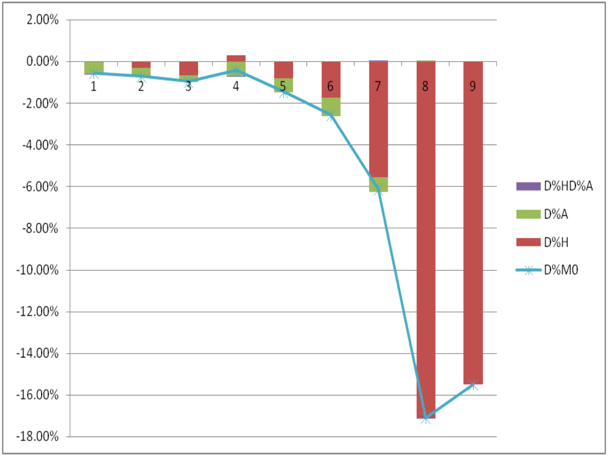

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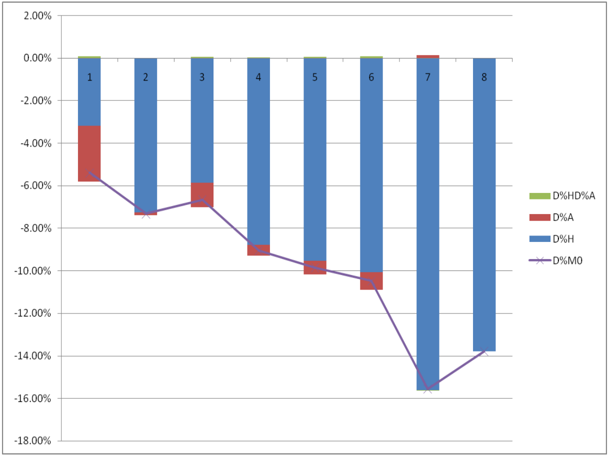

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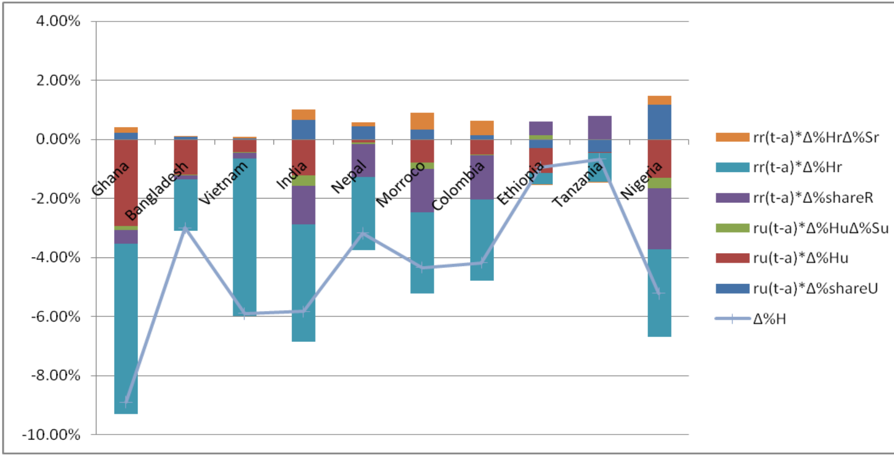
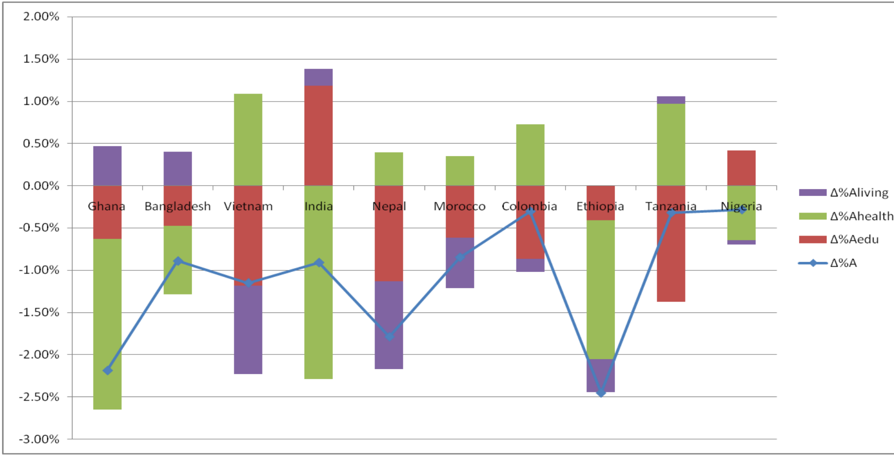


# Decomposition of A for k=3



# More general results for $\Delta\%_a M0$

Consider now the censored headcount, $CH_d(t)$:

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Then: $A_d = \frac{CH_d(t)}{H(t)}$ and $M^0(t) = \sum_{d=1}^{D} \theta_d CH_d(t)$

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$$\Delta\%_a M^0(t) = \sum_{d=1}^{D} s_d(t-a)\Delta\%_a CH_d$$

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$$\Delta\%_a H(t) = P[c_n^t \geq k | c_n^{t-a} < k][\frac{1 - H(t-a)}{H(t-a)}] - P[c_n^t < k | c_n^{t-a} \geq k]$$

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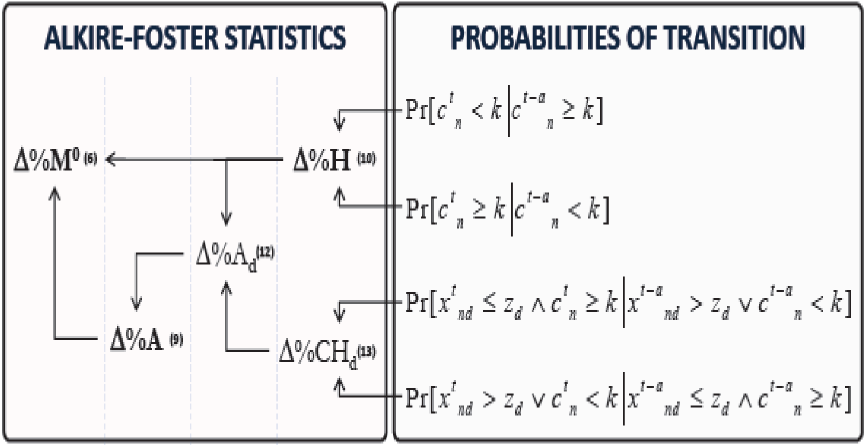
$$\Delta\%_a CH(t) = P[c_n^t \geq k \wedge x_{nd}^t \leq z_d | c_n^{t-a} < k \vee x_{nd}^{t-a} > z_d][\frac{1-CH(t-a)}{CH(t-a)}]$$

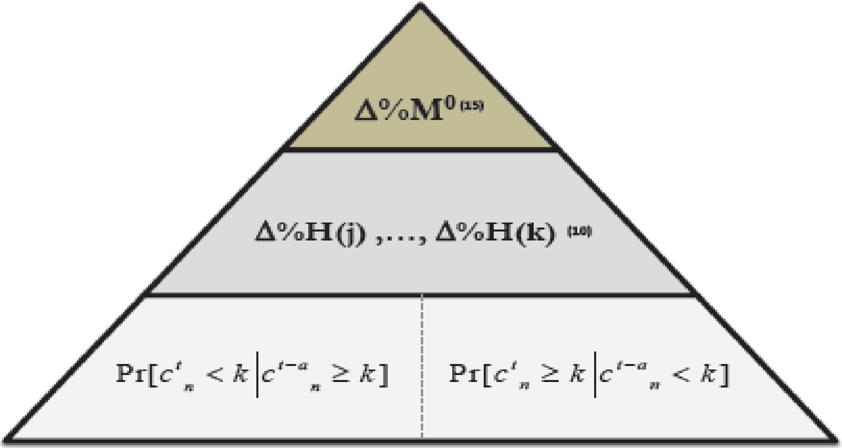
$$-P[c_n^t < k \vee x_{nd}^t > z_d | c_n^{t-a} \geq k \wedge x_{nd}^{t-a} \leq z_d]$$

# Decomposition of Alkire-Foster statistics based on transition probabilities



# Two final results

I:



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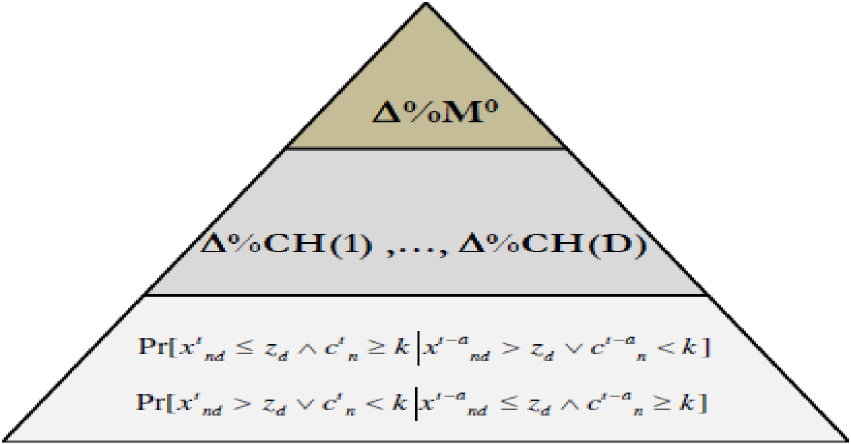


# The Young Lives dataset

We use the three waves: 2002, 2006/7, 2010.

Table 1: Sample Characteristics

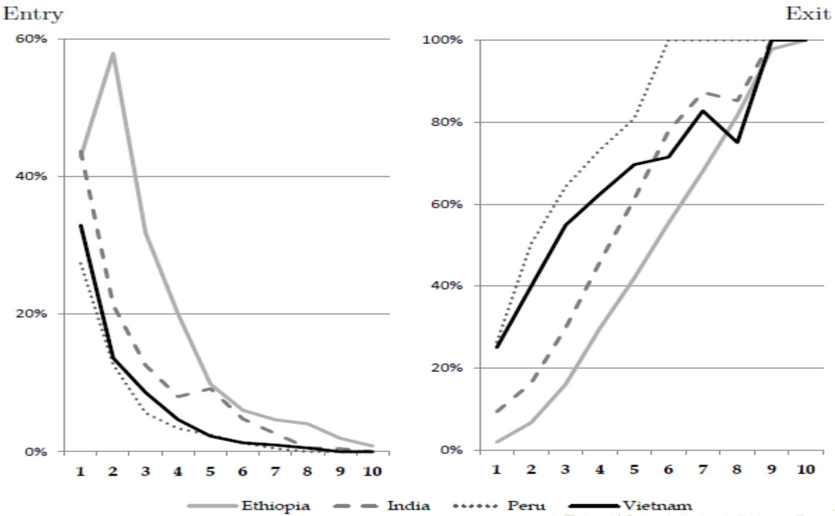
| | Wave | Original sample | Selected Sample | Mean Age | % Females | % rural |
|---|---|---|---|---|---|---|
| Ethiopia | 1 | 1000 | 868 | 7.88 | 49.1% | 61.2% |
| | 2 | 980 | 868 | 12.05 | | 60.7% |
| | 3 | 973 | 868 | 14.56 | | 59.7% |
| Andhra Pradesh | 1 | 1008 | 944 | 7.98 | 50.6% | 75.6% |
| | 2 | 994 | 944 | 12.32 | | 74.8% |
| | 3 | 975 | 944 | 14.72 | | 57.1% |
| Peru | 1 | 714 | 660 | 7.93 | 47.0% | 26.1% |
| | 2 | 685 | 660 | 12.31 | | 40.3% |
| | 3 | 678 | 660 | 14.44 | | 23.6% |
| Vietnam | 1 | 1000 | 957 | 7.97 | 50.4% | 80.6% |
| | 2 | 990 | 957 | 12.25 | | 69.3% |
| | 3 | 974 | 957 | 14.73 | | n.a. |

# Choice of variables

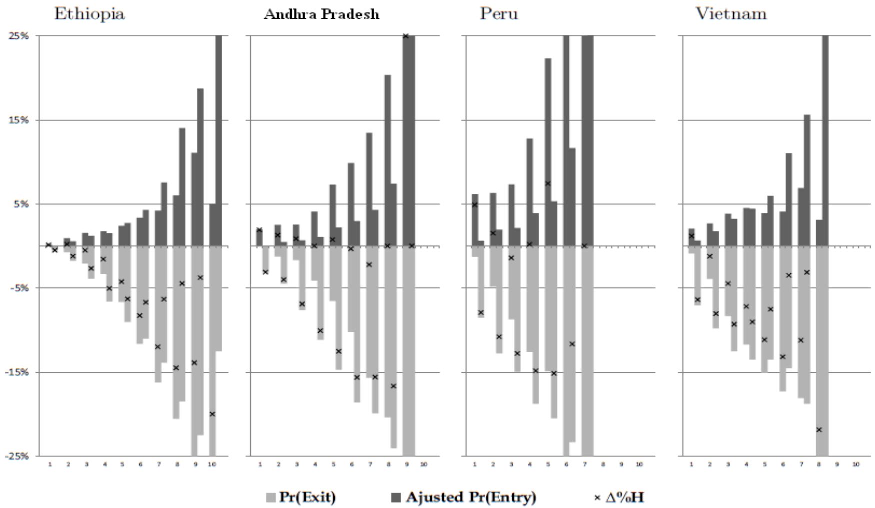
Table 2: Child Poverty Dimensions

| Indicator | Description (threshold) | Weight |
|---|---|---|
| **Child Related** | | |
| Child Labour♣ | Any "commercial" activity before 13 / Light activity from 13 (2 hours per day) | 1/12% |
| School Attendance | No attendance to the school according to National Law | 1/12% |
| Attachment | Any contact with parents mum or dad | 1/12% |
| Nutrition◇ | Less than 2 standards deviations (BMI) | 1/12% |
| **Household Related** | | |
| Electricity | No electricity | 1/12% |
| Cooking Fuel | MDG definition (Branches/ Charcoal/ Coal/ Cow dung /Crop residues / Leaves/ None /Other) | 1/12% |
| Drinking Water | MDG definition (Unprotected/ Well/ Spring/ Pond/ River/ Stream / Canal) | 1/12% |
| Toilet | MDG definition (Forest/ field/ Open place / Neighbours toilet/ Communal pit latrine/ Relative's toilet/ Simple latrine on pond/ Toilet in health post/ Other) | 1/12% |
| Floor | MDG definition (Earth/ Sand) | 1/12% |
| Assets | Less than one (Radio/ Fridge/ Table/ Bike/ Tv/ Motorbike/ Car/ Phone) | 1/12% |
| Overcrowding♠ | 3 or more Individuals per room | 1/12% |
| Child Mortality♡ | Any dead Children in the Household | 1/12% |

# Transition probabilities



# Transition probabilities and $\Delta\%H$



# Concluding remarks on time comparisons with M0 across countries

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- Time spans are different: If the differences are too wild it may be that for one country we observe short-term business cycle fluctuations whereas for the other we observe a medium-term growth trend. Solution: Restrict comparisons to time spans that do not differ too wildly.

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- Time spans are different: It may give undue advantage for, say, poverty reduction to the country with the longest observed span. Solution: Annualize rates of change.
- Time spans are different: If the differences are too wild it may be that for one country we observe short-term business cycle fluctuations whereas for the other we observe a medium-term growth trend. Solution: Restrict comparisons to time spans that do not differ too wildly.
- The years are different: Even when spans are equal, taking year brackets too far apart may affect the meaningfulness of the comparison. (E.g. Kenya in the 1950s with Chile in the 1990s). Solution: Justify your comparisons when the years are different.