Common Factor Analysis Versus Principal Component Analysis: Differential Bias in Representing Model Parameters?

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The aim of the present article was to reconsider several conclusions by Velicer and Jackson (1990a) in their review of issues that arise when comparing common factor analysis and principal component analysis. Specifically, the three conclusions by Velicer and Jackson that are considered in the present article are: (a) that common factor and principal component solutions are similar, (b) that differences between common factor and principal component solutions appear only when too many dimensions are extracted, and (c) that common factor and principal component parameters are equally generalizable. In contrast, Snook and Gorsuch (1989) argued recently that principal component analysis and common factor analysis led to different, dissimilar estimates of pattern loadings, terming the principal component loadings biased and the common factor loadings unbiased. In the present article, after replicating the Snook and Gorsuch results, an extension demonstrated that the difference between common factor and principal component pattern loadings is inversely related to the number of indicators per factor, not to the total number of observed variables in the analysis, countering claims by both Snook and Gorsuch and Velicer and Jackson. Considering the more general case of oblique factors, one concomitant of overrepresentation of pattern loadings is an underrepresentation of intercorrelations among dimensions represented by principal component analysis, whereas comparable values obtained using factor analysis are accurate. Differences in parameters deriving from principal component analysis and common factor analysis were explored in relation to several additional aspects of population data, such as variation in the level of communality of variables on a given factor and the moving of a variable from one battery of measures to another. The results suggest that principal component analysis should not be used if a researcher wishes to obtain parameters reflecting latent constructs or factors.

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The choice between common factor analysis and principal component analysis as a model for representing the correlations among a set of measured variables is often presented as simply that—a choice—with few serious implications for the representation and interpretation of data (Velicer & Jackson, 1990a). Both forms of analysis result in reduced rank representations of the relations among a set of measured variables. Moreover, on the basis of Monte Carlo studies, some researchers (e.g., Velicer, 1977; Velicer, Peacock, & Jackson, 1982) have claimed that common factor analysis and principal component analysis provide very similar solutions for a given set of data. Such findings support the contention that the decision to use common factor analysis or principal component analysis is a choice, with no clear advantages associated with either model. However, I will argue that there are important differences between the common factor and principal component models, differences lending differential support for one model over the other.

The most widely used type of component analysis—indeed, the most widely used method of factor or component analysis—is principal component analysis (see Lilly, Hoaglin, & Anderson-Kulman, 1989, cited in Velicer & Jackson, 1990b); as a result, the aim of the present article is to compare and contrast common factor analysis and principal component analysis. To avoid repetition, I will use the shortened term component analysis in the remainder of this article when referring to principal component analysis. However, I note here explicitly that results presented in this article apply only to principal component analysis and may not generalize to other forms of component analysis, such as image component analysis.

Recently, Velicer and Jackson (1990a) summarized seven major issues involved in the choice of analytic model. The first four issues related to analyses of empirical data; here, Velicer and Jackson claimed that (a) component analysis and common factor analysis provide similar representations of data; (b) differences between component and common factor representations appear only when too many components or factors are extracted; (c) common factor solutions are prone to improper solutions, such as Heywood cases, whereas component solutions are not; and (d) component analysis is a more efficient procedure than common factor analysis, requiring smaller amounts of computer time and resources to arrive at a solution.

The remaining three issues presented by Velicer and Jackson (1990a) were broader, more theoretical ones. These issues were that (e) common factor analysis is subject to factor score indeterminacy, whereas component analysis is not; (f) common factor analysis is a confirmatory technique, whereas component analysis is an exploratory technique, and confirmatory
methods have greater scientific merit than exploratory ones; and (g) common factor analysis is a latent variable procedure, component analysis is a manifest variable procedure, and common factor parameters therefore should have greater generalizability to variables not included in the analysis relative to parameters from component analysis. Across the seven data analytic and theoretical issues, Velicer and Jackson concluded that component analysis seems to have properties that are either equal to or superior to those of common factor analysis.

Complete consensus with Velicer and Jackson (1990a) on the seven issues has not been achieved, however. The Velicer and Jackson (1990a) article was followed by 10 comments. Some comments were rather complimentary (Schönemann, 1990; Steiger, 1990), others were mixed (Bookstein, 1990; McArdle, 1990), and still others were more negative (Bentler & Kano, 1990; Gorsuch, 1990; Loehlin, 1990; Mulaik, 1990; Rozeboom, 1990; Widaman, 1990). However, virtually all of the comments called for some qualification of the conclusions stated by Velicer and Jackson (1990a; see Velicer & Jackson, 1990b, for a response to these comments).

Moreover, recent evidence challenges the Velicer and Jackson (1990a) conclusion that common factor analysis and component analysis provide rather similar representations of data (cf. Bentler & Kano, 1990; Lohmöller, 1989; Widaman, 1990). For example, Snook and Gorsuch (1989) compared common factor and component representations of simulated data that were drawn, using Monte Carlo techniques, from populations with specified parameters. Using population factor loadings as a criterion, Snook and Gorsuch concluded that nonzero, defining component loadings were significantly more biased and more variable than were nonzero common factor loadings, which were rather unbiased. Snook and Gorsuch argued that bias in component loadings was negatively related both to the level of communality and to the total number of observed variables in the analysis. That is, the bias in estimation of population loadings by component analysis was lessened as communality increased and as the total number of observed variables in the analysis increased. These findings provide a systematic substantiation of observations by other researchers, such as Comrey and his associates (1978; 1988; Lee & Comrey, 1979) and Gorsuch (1988).

The primary aim of the present study is to investigate further the possibility of differential representations of model parameters by common factor analysis and component analysis. First, the manner in which common factor analysis and component analysis represent correlations among observed variables will be discussed, shedding light on the mathematical basis for the differential population loadings defined by the two methods. Further,
considering the more general oblique factor case leads to the conclusion that one concomitant of the larger pattern loadings defined by component analysis is a systematically lower estimate of the correlations among dimensions relative to those from common factor analysis. Next, several analyses are undertaken to demonstrate the differences in population parameters represented by the component analysis and common factor analysis models. These analyses include (a) a replication of the Snook and Gorsuch (1989) study, using population data as opposed to simulated sample data, (b) an extension of the Snook and Gorsuch study designed to test directly their claim that differences in pattern loadings are an inverse function of the number of observed variables in an analysis, and (c) an examination of the moderating effects of several other aspects of population data, such as variation in level of communality and the factorial composition of the battery of measures. In concluding, the impact of these findings on several of the conclusions offered by Velicer and Jackson (1990a) will be discussed.

Models Representing Observed Variables in Reduced Dimensionality

A general formula for representing the relationship between two vectors \( \mathbf{i} \) and \( \mathbf{j} \) is the inner product of the two vectors, \( \mathbf{IP}_{ij} \), which is defined as:

\[
\text{IP}_{ij} = l_i \cdot l_j \cdot \cos(\alpha_{ij}),
\]

where \( l_i \) and \( l_j \) are the lengths of vectors \( \mathbf{i} \) and \( \mathbf{j} \), respectively, and \( \cos(\alpha_{ij}) \) is the cosine of \( \alpha_{ij} \), the angular separation between vectors \( \mathbf{i} \) and \( \mathbf{j} \). If vectors \( \mathbf{i} \) and \( \mathbf{j} \) represent observed variables \( i \) and \( j \), respectively, and the lengths of vectors \( \mathbf{i} \) and \( \mathbf{j} \) represent the standard deviations of variables \( i \) and \( j \), respectively, then Equation 1 above can be used to represent \( \sigma_{ij} \), the covariance between the two variables, as:

\[
\sigma_{ij} = \sigma_i \cdot \sigma_j \cdot \cos(\alpha_{ij}),
\]

where \( \sigma_i \) and \( \sigma_j \) are the standard deviations of variables \( i \) and \( j \), respectively, and other symbols are as defined above.

If variables are standardized to unit variance, and thereby to unit length, Equation 2 simplifies to the following:

\[
r_{ij} = 1 \times 1 \times \cos(\alpha_{ij}) = \cos(\alpha_{ij}),
\]
where $r_{ij}$ is the Pearson product moment correlation between variables $i$ and $j$ and equals the cosine of the angular separation between unit-length vectors $\mathbf{i}$ and $\mathbf{j}$, and all other symbols are as defined above. Equation 3 may be used to represent each element in a matrix of correlations among $n$ observed variables.

**Common Factor Analysis Model**

Common factor analysis is one of the two procedures commonly used to obtain a reduced-dimensional representation of a set of observed variables. Common factor analysis was developed to express the variance shared among $n$ observed variables as a function of $p$ underlying common factors. The common factor model may be expressed in matrix notation in the following manner:

\[
\mathbf{R} - \mathbf{U}^2 \cong \mathbf{F}_f \mathbf{F}_f' = \mathbf{P}_f \mathbf{\Phi}_f \mathbf{P}_f' = \mathbf{R}_f^*.
\]

where $\mathbf{R}$ is an $(n \times n)$ matrix of correlations among the observed variables, $\mathbf{U}^2$ is an $(n \times n)$ diagonal matrix of estimates of unique variance, $\mathbf{F}_f$ is an $(n \times p)$ unrotated factor matrix with $p < n$, $\mathbf{P}_f$ is an $(n \times p)$ rotated factor pattern matrix, $\mathbf{\Phi}_f$ is a $(p \times p)$ matrix of factor intercorrelations, $\mathbf{R}_f^*$ is an $(n \times n)$ matrix of reproduced correlations among the $n$ variables, and the subscript $f$ designates the matrices as derived from a common factor analysis. Equation 4 implies that $\mathbf{R}$, reduced by estimates of unique variance (or, equivalently, with communality estimates on the diagonal), is approximated by the $p$-dimensional unrotated factorization $\mathbf{F}_f \mathbf{F}_f'$, which may be rotated orthogonally or obliquely into $\mathbf{P}_f \mathbf{\Phi}_f \mathbf{P}_f'$. The unrotated and rotated factor representations produce the identical matrix $\mathbf{R}_f^*$ of reproduced correlations, with represented communalities on the diagonal. The $j$th diagonal element of $\mathbf{R}_f^*$ is the common variance of the $j$th variable that is represented by the factor solution, and this element may be denoted $h_{j0}^2$, to indicate that it represents variance from a common factor solution. Also, the square root of the $j$th diagonal element of $\mathbf{R}_f^*$, $h_{j0}$, is the length of the vector representing the $j$th observed variable.

**Component Analysis Model**

The second procedure that is often used for obtaining a reduced-dimensional representation of a set of observed variables is component analysis. There is one major contrast between the common factor and component models: Whereas common factor analysis represents only the
variance shared among a set of observed variables, component analysis represents the total variance of a set of observed variables in an economical, reduced-dimensional form. As with common factor analysis, the component analysis of a set of \( n \) observed variables may be obtained in \( p \) dimensions, with \( p < n \). A standard matrix equation for component analysis is:

\[
\mathbf{R} \cong \mathbf{F}_c \mathbf{F}_c^\prime = \mathbf{P}_c \mathbf{\Phi}_c \mathbf{P}_c^\prime = \mathbf{R}_c^*.
\]

where \( \mathbf{R} \) is as defined above, the matrices \( \mathbf{F}_c, \mathbf{P}_c, \mathbf{\Phi}_c \), and \( \mathbf{R}_c^* \) have the same order and serve the same function as the matrices \( \mathbf{F}, \mathbf{P}, \mathbf{\Phi}, \) and \( \mathbf{R}_c^* \), respectively, from Equation 4, and the subscript \( c \) designates the matrices in Equation 5 as deriving from a component analysis. In Equation 5, the matrix \( \mathbf{R} \) itself, with unities on the diagonal, is approximated by the \( p \)-component solution \( \mathbf{F}_c \mathbf{F}_c^\prime \), which may be rotated orthogonally or obliquely into \( \mathbf{P}_c \mathbf{\Phi}_c \mathbf{P}_c^\prime \). Paralleling the common factor results discussed above, the \( j \)th diagonal element of \( \mathbf{R}_c^* \), denoted \( h_{ij}^2 \), is the proportion of total variance of the \( j \)th variable represented by the component solution, and the square root of this element, \( h_{ij} \), is the length of the component vector representing the \( j \)th observed variable.

There is considerable similarity, at least on the surface, in the matrix representations of the common factor analysis and component analysis models. In both their unrotated and rotated forms (cf. Equations 4 & 5), common factor analysis and component analysis lead to similar general forms of representation of the relations among a set of observed variables. In addition, component analysis may be viewed as a limiting, special case of common factor analysis, in which the unique variance in matrix \( \mathbf{U}^2 \) in Equation 4 vanishes. Thus, as \( \mathbf{U}^2 \) approaches zero, the pattern loadings and correlations among dimensions defined by the common factor and component models approach identity (cf. Lohmöller, 1989). Further comparisons between the common factor and component analysis models have been presented by many authors (e.g., Velicer & Jackson, 1990a). Such considerations, together with the arguments of Velicer and Jackson that common factor analysis and component analysis appear to provide similar representations of empirical data, have led many, including Velicer and Jackson, to conclude that component analysis is a suitable method to use when attempting to estimate coefficients in reduced-dimensional models. In the remainder of this article, several issues will be discussed that bear on whether component analysis is, in general, a suitable method of representing parameters in latent variable models.
Mathematical Basis for Differential Population Estimates Of Model Parameters By Common Factor and Component Analysis

In this section, only population factor pattern matrices with congeneric patterns of loadings are considered. A congeneric pattern of loadings means that each indicator for a given common factor has a nonzero loading only on the common factor for which it serves as indicator and has loadings of exactly zero on the remaining common factors. Population matrices with congeneric patterns were used to ensure clarity of the presentation and conformed to the population matrices used in several recent studies, including those by Velicer and Jackson (1990a) and Snook and Gorsuch (1989). To add clarity with regard to elements in correlation matrices, the correlations between indicators for a given common factor were termed "intrafactor indicator correlations." Complementarily, the correlations between indicators for different common factors were termed "interfactor indicator correlations."

Finally, in this section, reference is made several times to Figure 1 (next page). In Figure 1, a two-dimensional space is presented, with unrotated Principal Axes I and II as a reference frame. Four vectors representing observed variables are presented; Vectors A and B are indicators of one factor, and Vectors C and D are indicators of a second factor. The solid part of each vector denotes the part of each observed variable represented by a common factor or component solution; the dashed part reflects the vector extended out to unit length, with the unit circle drawn. Axes I' and II' represent optimally rotated oblique axes, with each axis passing through one of the clusters of vectors.

Representing Pattern Loadings

It is possible to derive expressions providing the mathematical basis for the difference in population pattern loadings defined by component and common factor models for certain special cases. Consider the case of a population factor model having a single common factor with \( n = m \) indicators (i.e., \( n = m \)), in which the \( m \) population pattern loadings, \( \lambda \), are identical to one another, and the \( m \) unique factors have equal variance and are mutually uncorrelated. In this case, Widaman (1990) showed that the pattern loading represented by common factor or component analysis from population data, \( \hat{\lambda} \), may be calculated as:

\[
\hat{\lambda} = \left[ \lambda^2 + \left( \Delta h^2 / m \right) \right]^{1/2}
\]
Figure 1
Geometric representation of four test vectors (lettered A-D) in a two-dimensional space, with the unrotated dimensions (I and II) and the optimally rotated oblique dimensions (I' and II').

where $\Delta h^2$ is the signed difference between the true and estimated communalities (calculated as estimated communality minus true communality), and other terms are as defined above.

Formulae similar to Equation 6 have been presented by other researchers (e.g., Bentler & Kano, 1990; Lohmöller, 1989), and a similar formula is implicit in Velicer et al. (1982). These researchers derived their formulae to calculate the population component loading assuming that the population is characterized by given common factor loadings. However, Equation 6 appears to be more general than previous formulae, as it applies to common factor analysis as well as component analysis. Further, Equation 6 provides the mathematical basis for the results obtained by Snook and Gorsuch (1989). In the single factor case, component analysis attempts to reproduce the unities on the diagonal as well as the lower intrafactor indicator correlations off the diagonal. The unities on the diagonal must overestimate population communality, so $\Delta h^2 > 0$, and the represented component loading $\hat{\lambda}_c$ will be larger than the population loading $\lambda$ that generated the data. The deviation of component loadings from population pattern loadings should be related to at least two conditions: level of communality and the
number of indicators for the factor. First, the deviation should decrease as the level of communality increases, because $\Delta h^2$ decreases as the discrepancy between the unity on the diagonal and the true communality decreases. Second, the deviation between component and population loadings should decrease as the number of indicators $m$ increases; this occurs because the number of off-diagonal elements increases at a much faster rate than the number of diagonal elements, lowering the influence of the diagonal elements in the fitting of parameters. Both of these trends were evident in the results of the Snook and Gorsuch study (cf. Comrey, 1978, 1988; Lee & Comrey, 1979) and are represented explicitly in Equation 6.

In contrast to component analysis, the common factor analysis of a set of data should lead to a fairly accurate representation of the parameters generating the data. In the single factor case with equal loadings, if communalties are estimated optimally, then $\Delta h^2 = 0$, and each factor loading $\hat{\lambda}$ would be represented at its population value of $\lambda$. Of course, if communalties are over- or underestimated, this will lead to positive or negative values of $\Delta h^2$, respectively, leading to corresponding over- or underrepresentation of the population factor loading $\lambda$. Thus, Equation 6 explicitly places a correct and heavy emphasis on the optimal estimation of communalties — only if communalties are optimally estimated will common factor loadings be accurately represented. Also, there may be a consistent bias associated with given methods of communality estimation. For example, the squared multiple correlation (SMC) of a variable with the $(n - 1)$ remaining variables in the analysis is a commonly used method of communality estimation that has attractive theoretical properties, such as providing a lower bound estimate of communality. But, if the SMC consistently underestimates the true communality, then the represented pattern loading $\hat{\lambda}$ will be smaller than the population factor loading $\lambda$. However, the inaccuracy in representing parameters of a common factor solution should always be rather less than would occur in a component solution, because commonly used communality estimates are virtually always more accurate estimates of communality than is unity, the value used in component analysis. Findings consistent with these conclusions were reported by Snook and Gorsuch (1989) and Widaman (1990).

Given the preceding results, a formula for the difference in pattern loadings under nonoptimal and optimal estimation of communalties is obtained as the simple difference between the respective population values, or:

\[
(7) \quad \text{DIFF}_{\text{load}} = [\hat{\lambda}^2 + (\Delta h^2/m)]^{1/2} - \lambda,
\]
where $\text{DIFF}_{\text{load}}$ is the difference in loadings under nonoptimal and optimal communality estimation, and all remaining terms are as defined above. Equation 7 shows that the difference in loadings may be positive, zero, or negative, and that the difference is directly related to $\Delta h^2$, the discrepancy between nonoptimal and optimal estimates of communalities.

At least three additional formulas may be useful in representing the effects of nonoptimal estimates of communality. These three formulas are:

\begin{align*}
(8) \quad \text{DIFF}_\text{var} &= [\lambda^2 + (\Delta h^2/m)] - \lambda^2 = \Delta h^2/m, \\
(9) \quad \text{PDIFF}_{\text{load}} &= \text{DIFF}_{\text{load}}/\lambda = [1 + (\Delta h^2/m\lambda^2)]^{1/2} - 1, \\
(10) \quad \text{PDIFF}_\text{var} &= \text{DIFF}_\text{var}/\lambda^2 = \Delta h^2/(m\lambda^2),
\end{align*}

where $\text{DIFF}_\text{var}$ is the difference in variance explained by pattern loadings under nonoptimal and optimal estimation of communality, $\text{PDIFF}_{\text{load}}$ is the proportional difference in factor loadings under nonoptimal and optimal communality estimation relative to the population factor loading, $\text{PDIFF}_\text{var}$ is the proportional difference in variance explained by loadings under nonoptimal and optimal communality estimation relative to the true value, and all other terms are as defined above.

As Bentler and Kano (1990) noted, it is possible to derive formulae such as Equation 6 assuming that the data may be represented with a single factor having $m$ indicators and that all $m$ indicators have equal loadings on the factor. These results should also generalize to multiple dimensions if the interfactor indicator correlations are zero. But, if loadings vary on a factor or if the multiple factors are correlated, appropriate formulae are not available (Bentler & Kano, 1990), due to the resulting complexity of the analytic task. Results of analyses of population data are presented below for such situations that are not amenable to analytic treatment.

**Representing Intercorrelations Between Dimensions**

We turn now to the way in which common factor and component analysis represent correlations between dimensions. Consider vectors $i$ and $k$ that denote variables $i$ and $k$, respectively, that are indicators for different factors in a two-factor space. If a congeneric test solution is appropriate for the data, an optimal oblique rotational solution would pass one dimension through each of two clusters of vectors representing observed variables, one cluster containing vector $i$ and the other cluster containing vector $k$. If each of the two clusters consists of a tight cluster of vectors, then the cosine of
the angle between the rotated factors should be approximately equal to the
average of the cosines of the angles between all pairs of vectors for different
factors.

This proposition can be illustrated using Figure 1, in which rotated Axes
I* and II* are each passed through a cluster of two vectors. The cosine of the
angle between the two rotated axes should be approximated by the average
of the cosines of the angles between each indicator for Axis I* paired with
each indicator for Axis II*. In Figure 1, this entails four pairs of vectors:
vector pairs (A,C), (A,D), (B,C), and (B,D). But, the cosine of the angle
between a pair of vectors is equal to the correlation between the vectors only
if the vectors are extended to unit length (cf. Equation 3). The unit-length
extended vectors A through D are shown by the dashed portions of each
vector in Figure 1.

The inner product formula may be used to represent the reproduced
correlation between variables i and k. This leads to the following equation
based on a factor analytic representation of the data:

\[
(11) \quad r_{\alpha(i)}^* = \hat{\lambda}_{\alpha(i)} \hat{\lambda}_{\alpha(k)} \cos(\alpha_{\alpha(ik)}),
\]

where \( r_{\alpha(i)}^* \) is the reproduced correlation between variables i and k, which
are indicators for different factors, \( \hat{\lambda}_{\alpha(i)} \) and \( \hat{\lambda}_{\alpha(k)} \) are the lengths of the vectors
i and k, respectively, \( \cos(\alpha_{\alpha(ik)}) \) is the cosine of the angle between vectors
i and k, and the \( f \) subscript denotes that elements were derived from a
common factor analysis. The corresponding reproduced correlation based
on a component analysis of the data is:

\[
(12) \quad r_{\alpha(i)}^* = \hat{\lambda}_{\alpha(i)} \hat{\lambda}_{\alpha(k)} \cos(\alpha_{\alpha(ik)}),
\]

where all terms are as defined above except for the \( c \) in the subscript to
denote that the elements derive from a component analysis.

The average of the interfactor indicator cosines between vectors derived
from a factor or component analysis is a reasonable estimator of the proper
magnitude of the correlation between obliquely rotated dimensions. To
estimate the proper correlation between oblique factors or components, one
may solve Equations 11 and 12 for the cosine of the angle between vectors
i and k as:

\[
(13) \quad \cos(\alpha_{\alpha(ik)}) = \frac{r_{\alpha(i)}^*}{\hat{\lambda}_{\alpha(i)}} \hat{\lambda}_{\alpha(k)}, \text{ and}
\]

\[
(14) \quad \cos(\alpha_{\alpha(ik)}) = \frac{r_{\alpha(i)}^*}{\hat{\lambda}_{\alpha(i)}} \hat{\lambda}_{\alpha(k)}.
\]
where Equations 13 and 14 provides estimates based on common factor and component representations, respectively, and all terms are as defined above.

A more efficient way of computing estimates of the cosines among all vectors in an analysis is to solve for them simultaneously using the following:

\[
C_f = H_f^{-1} R_f H_f^{-1}, \text{ or }
\]

\[
C_c = H_c^{-1} R_c H_c^{-1},
\]

where \(C_f\) and \(C_c\) are matrices of cosines among all vectors representing measured variables based on common factor and component analyses, respectively, \(H_f\) and \(H_c\) are diagonal matrices containing lengths of vectors represented using common factor analysis and component analysis, respectively, and the remaining terms as defined above. Equivalently, the diagonal elements of \(H_f\) and \(H_c\) are the square roots of the diagonal elements of \(R_f^*\) and \(R_c^*\), respectively. As a result, matrices \(C_f\) and \(C_c\), based on common factor analysis and component analysis, respectively, have unit diagonals and contain the correlations among extended vectors for all observed variables in the analysis, which are equal to the cosines of the angles among the unextended vectors. The average of the interfactor correlations among extended vectors contained in \(C_f\) and \(C_c\) provide the proper estimator of the magnitude of the correlation between optimally rotated oblique common factors and components, respectively. Importantly, all elements in the \(C_f\) and \(C_c\) matrices are invariant under rotation of axes.

If a common factor analysis is appropriate for the data, the pattern loadings and the lengths of vectors for observed variables accurately represent the population parameters that generated the data if communalities are estimated optimally, as discussed above. If these conditions hold, then the interfactor indicator cosines among extended vectors are accurate estimators of the population correlation between factors. But, if communalities are over- or underestimated, this will lead to over- or underrepresentation of pattern loadings, respectively; this will result in under- or overrepresentation, respectively, of the population correlation between factors (cf. Eq. 13).

Contrast the preceding with results based on component analysis. Assuming that the reproduced correlations among indicators for different factors are approximately equally represented by common factor analysis and component analysis, or \(r_{f(ik)}^* = r_{c(ik)}^*\), then Equations 13 and 14, taken together with implications of Equation 6 (i.e., that component loadings for
a given data set will be larger than common factor loadings estimated from the same data), imply that:

\[ |\cos(\alpha_{c(ik)})| < |\cos(\alpha_{f(ik)})|, \]

where all terms are as defined above. Thus, one concomitant of the larger loadings obtained in component analysis, embodied in Equation 6, is a reduction in the magnitude of the cosines between extended vectors based on component analysis relative to those based on common factor analysis, as shown in Equation 17. Thus, the intercorrelations between optimally rotated oblique components will be smaller than the comparable estimates obtained using common factor analysis. Moreover, the degree of reduction of the component intercorrelations is inversely related to the level of population communality of the indicators. As population communality decreases, component loadings become systematically larger than common factor loadings, and component intercorrelations become systematically smaller than common factor intercorrelations.

Above, the mathematical bases for differential representations of model parameters by common factor and component analysis were presented. In the remainder of the article, the influence of the analytic models on parameters represented in systematically varied conditions is explored. Then, the implications of these results for theory regarding common factor analysis and component analysis and for applications of the two techniques are discussed.

Method

Data

Error-free population data were analyzed in the present study to explore design variables that lead to differences in model parameters. In previous studies (e.g., Snook & Gorsuch, 1989), the data analyzed were simulated sample data generated through Monte Carlo methods. Only error-free population data were used in the present study to ensure that sampling variability would neither obscure results nor be available as an alternative explanation for trends noted.

Second, only population factor matrices having congeneric loading patterns were employed. As noted above, population factor matrices with very clean simple structure were used to ensure the clarity of the presentation. Moreover, use of congeneric loading patterns conforms to the pattern matrices used in a number of Monte Carlo studies, including those by
Velicer and Jackson (1990a), Velicer (1977), Velicer et al. (1982), and Snook and Gorsuch (1989).

Population factor pattern and factor intercorrelation matrices were constructed for a number of conditions. For population models with orthogonal factors, certain procedures of both Velicer and Jackson (1990a) and Snook and Gorsuch (1989) were followed. To produce matrices that differed systematically in level of communality, population factor loadings of .8, .6, and .4 were used to represent high, medium, and low levels of common variance, respectively. Then, two sets of analyses were performed. The first set of analyses was designed to replicate the Snook and Gorsuch results, in which equal population factor loadings and equal numbers of indicators per factor were used in each matrix. Keeping the number of factors constant at 3, population factor loading matrices were constructed in which each factor had 3, 6, or 12 indicators; this led to factor pattern matrices with 9, 18, and 36 observed variables. As Snook and Gorsuch (1989) noted, using 3 indicators per factor meets minimum identifiability conditions (Anderson & Rubin, 1956), whereas the use of 6 and 12 indicators per factor leads to moderately and highly overidentified factors, respectively. This first set of analyses comprised a $3 \times 3$ design formed by the orthogonal crossing of three levels of common variance (.8, .6 and .4) and three levels of number of indicators per factor, or $m$ (3, 6, and 12). In the resulting nine conditions, a single population correlation matrix was computed using Equation 4; to represent correlations among indicators for orthogonal factors, the matrix of factor intercorrelations, $\Phi_f$, was an identity matrix. These nine population matrices were then analyzed as described below.

The second set of analyses using orthogonal population factor matrices was designed to extend the study by Snook and Gorsuch (1989), who attributed the lessening of the bias in component loadings to the number of observed variables. But, as argued above, differences between component and common factor loadings appear to be inversely related to the number of indicators per factor. To examine these competing claims in more detail, the number of indicators per factor was crossed with the number of observed variables. The resulting $3 \times 3 \times 3$ design was formed by the orthogonal crossing of three levels of common variance (loadings of .8, .6, and .4), three levels of number of indicators per factor (3, 6, and 12), and three levels of number of observed variables (24, 48, and 96). For each of the 27 conditions, a single population correlation matrix was computed using Equation 4, using an identity matrix for $\Phi_f$. The 27 population matrices were then analyzed as described below.
To investigate the influence of level of communality on estimates of factor intercorrelations, population pattern matrices exhibiting high, medium, and low levels of communality were constructed. To maintain comparability with preceding analyses, identical loadings of .8, .6, and .4 were used to represent the high, medium, and low levels of communality, respectively. Population factor correlation matrices had .50 correlations among all factors. Population correlation matrices among observed variables were constructed using Equation 4 and analyzed as described below.

For the remaining demonstrations of contextual effects on estimates of model parameters, each factor had a systematic mixture of nonzero population loadings of .8, .6, and .4. This reflected a reasonable expectation that some indicators for each factor may have a high saturation with the factor, whereas other indicators will have moderate or low levels of saturation with the factor, respectively. For each of these matrices, hyperplanar loadings on each factor were specified as precisely zero in the population, and factor intercorrelations were fixed at .50. Once again, population correlation matrices were constructed using Equation 4 and analyzed as discussed below.

Analyses

All component and factor analyses were performed using the PROC FACTOR program in the Statistical Analysis System (SAS Institute, 1985) package. Principal axes extraction of factors was used for all analyses; retaining unities on the diagonal led to the estimation of principal components, whereas using some form of communality estimate resulted in the estimation of principal factors, or common factors. In all component and common factor analyses, the correct number of population components or factors were extracted and rotated.

All common factor analyses were performed twice, using two different estimates of communality. Analyses were performed once with SMCs as communality estimates. SMCs are often recommended as noniterative estimates of communality because in theory they provide lower bound estimates of communality. Analyses were performed a second time using iterative estimates of communality, using the SMC as initial estimate (Widaman & Herringer, 1985). A rather conservative stopping criterion was used in the present study, specifically that iterations should continue until the largest change in any communality estimate from one iteration to the next was less than .0001. Under no conditions were more than 30 iterations required before the convergence criterion was satisfied.
Finally, all component and factor matrices were rotated twice, once orthogonally and once obliquely. The orthogonal rotation used was varimax, which always led to an appropriate resolution of orthogonal population data in the present study. The oblique rotation used was the Harris-Kaiser orthoblique (Harris & Kaiser, 1964) rotation, employing the independent cluster solution ($e = 0.0$), appropriate for the population model generating the data.

**Results**

*Effects of Variation in Pattern Loadings and Dimensional Obliquity On Common Factor and Component Parameter Values*

In the present section, common factor and component representations of data are investigated in what may be considered the four cells of a $2 \times 2$ design, defined by the dimensions of *equal versus varied loadings on factors* and *orthogonal versus oblique factors*. All analytical expressions provided in the present article as well as others (e.g., Bentler & Kano, 1990; Lohmöller, 1989; Widaman, 1990) are appropriate to only one of the four cells — equal loadings on orthogonal factors. The analyses in this section will demonstrate whether identical or similar trends arise if these restrictions are lifted.

*Equal Pattern Loadings on Orthogonal Factors*

*Replication of the Snook and Gorsuch (1989) Study*

The first task was to replicate the Monte Carlo results regarding differences in pattern loadings reported by Snook and Gorsuch (1989) and to verify the correctness of Equation 6. Here, the data analyzed consisted of the nine population matrices, one matrix per cell from the $3 \times 3$ design formed by the crossing of three levels of communality and three levels of number of observed variables. These nine matrices were analyzed using component and common factor analysis. The mean nonzero rotated factor loading and the difference between mean and population nonzero loadings for each matrix are presented in Table 1, along with comparable results from Snook and Gorsuch based on Monte Carlo simulated data.

The results based on component analysis are presented in the top half of Table 1. These results revealed clear trends in nonzero component loadings, as (a) at each level of number of observed variables, the difference between component and population loadings decreased as communality increased,
Table 1

<table>
<thead>
<tr>
<th>No. of observed vars.</th>
<th>No. of factors</th>
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<th>Mean loading Δ_{pop}</th>
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<td>.263</td>
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<td>W-SMC</td>
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</table>

Note. The data presented are the mean nonzero loading across all factors and the difference (Δ_{pop}) between the mean nonzero loading and the population value.

* S & G refers to the Snook and Gorsuch (1989) article, W to the present study, W-Iter to results from the present study using iterated communality estimates, and W-SMC to results from the present study using squared multiple correlations as communality estimates.
and (b) at each level of population communality, the difference decreased as the number of observed variables increased. The reported results are identical to those produced by Equation 6. Moreover, these results revealed very clear replication of the results reported by Snook and Gorsuch (1989). In seven of nine cells, results from Snook and Gorsuch and from the present study differed by less than .01, and estimates differed by only .017 in an eighth cell. In only one cell (i.e., .40 loading and 18 observed variables), the estimates from the two studies differed moderately, .066. This difference may have resulted from sampling variability in the simulated data, as this cell was the most anomalous in their study (see Snook & Gorsuch, 1989, Figure 1, p. 151).

Common factor analysis results are presented in the bottom half of Table 1. Snook and Gorsuch (1989) reported that common factor analysis led to small, inconsistent deviations from population nonzero loadings; nonzero loadings thus were accurately represented. These findings were replicated by the analyses of population data in the present study. As shown in Table 1, iterated communality estimates led to perfect representation of population loadings to three decimal places. Use of SMCs as communality estimates, however, led to a consistent underrepresentation of population loadings, as hypothesized. SMCs are lower bound estimates of communality, so underrepresentation of population loadings is not surprising. The trends in underrepresentation of population loadings by SMCs mirrored those for component analysis: the difference was reduced systematically as communality increased and as the number of indicators increased. Also, comparing corresponding cells in the top and bottom halves of Table 1 reveals that the absolute magnitude of the differences arising from use of common factor analysis with SMCs was always decidedly less than the differences accompanying use of component analysis, also as hypothesized above. Because of the superior accuracy obtained when using iterated communality estimates relative to the use of SMCs, only results using iterated estimates are reported later in this study; SMCs always resulted in poorer representations of population parameters.

In the present study, loadings that were zero in the population were represented at precisely zero by both component analysis and common factor analysis. This result also replicates trends in the Snook and Gorsuch (1989) study, which found only small differences between component analysis and common factor analysis with regard to estimates of zero loadings.

Finally, the similarity of estimates of nonzero loadings in the present study with those from Snook and Gorsuch (1989) provides support for the use of analyses of population data as a basis for examining differences.
arising from use of common factor and component analysis. If results from analyses of population data had departed markedly from those derived from simulated data, further examination of each form of data would have been necessary. However, given the similarity of estimates, use of both types of data would be redundant. As will be shown below, the use of population data results in great clarity in the demonstration of differences arising as a function of various properties of the population data.

Extension of the Snook and Gorsuch (1989) Study

The preceding results must be qualified due to a deficiency in design. Both the Snook and Gorsuch study and its replication, presented in Table 1, shared an important confound: because the number of common factors was held constant at three, the effect of the number of indicators per factor was perfectly confounded with the effect of the number of observed variables. In the present extension of Snook and Gorsuch, a 3 × 3 × 3 design was used, in which the effect of number of indicators per factor (3, 6, and 12) was crossed with the number of measured variables (24, 48, and 96) at each of three levels of communality (loadings of .4, .6, and .8). The features of this design and the results with regard to nonzero pattern loadings represented by component analysis are presented in Table 2 (next page). As shown in Table 2, the use of prescribed levels of indicators per factor and number of observed variables led necessarily to variation across cells in the number of factors. However, if the number of factors influenced the resulting pattern loadings, such effects should be observable in Table 2, given the considerable overlap across cells (e.g., two cells had 4 population factors, three cells had 8 factors, etc.).

Inspection of Table 2 reveals clearly that differences in the nonzero loadings represented by component analysis were related strongly to the number of indicators per factor and were related neither to the number of measured variables in the analysis nor to the number of dimensions. At each level of communality and for a given number of indicators per factor, the mean nonzero loading was totally unaffected by an increase in the number of measured variables and the resulting increase in the number of factors. The results in Table 2 disconfirm the Snook and Gorsuch (1989) conclusion, echoed by Velicer and Jackson (1990b), that the difference between common factor and component loadings is reduced as the number of observed variables increases. Rather, the difference between component and population loadings is an inverse function only of the number of indicators per factor. Of course, these results are perfectly consistent with Equation 6 and with the arguments of Bentler and Kano (1990) and Widaman (1990), among others.
### Table 2

<table>
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<th>Indicators per factor</th>
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<th>Mean loading</th>
<th>$\Delta_{pop}$</th>
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<td>.643</td>
<td>.043</td>
<td>.819</td>
<td>.019</td>
</tr>
</tbody>
</table>

*Note.* The data presented are the mean nonzero loading across all factors and the difference ($\Delta_{pop}$) between the mean nonzero loading and the population value.

Results of the common factor analyses, which used iterated communality estimates, are not reported in Table 2 to conserve space. Common factor analysis resulted in precisely accurate representation of population loadings in all conditions. That is, in every cell at the low, medium, and high levels of communality, nonzero loadings were represented at precisely .40, .60, and .80, respectively, and these results were not influenced by the number of indicators per factor, the number of observed variables, or the number of factors. Once again, these results are consistent with Equation 6.

With regard to zero population loadings, these were represented as precisely zero in all cells by both component analysis and common factor analysis. These results extend the Snook and Gorsuch (1989) findings with regard to accuracy in estimating zero loadings by both component analysis and common factor analysis.
Importance of the Systematic Differences Among Estimates of Pattern Loadings

The importance of the systematic differences in pattern loadings defined by component analysis and common factor analysis is difficult to convey, as there are no widely accepted standards in the field for describing such differences. Some guidance may be obtained from the recent debate between Borgatta and his colleagues (1989; Borgatta, Kercher, & Stull, 1986), Hubbard and Allen (1987; 1989), and Wilkinson (1989a, 1989b). Wilkinson (1989a) showed that somewhat different common factor parameters are defined when using different methods of communality estimation and different computer programs. Velicer and Jackson (1990b) claimed that the differences uncovered by Wilkinson were large enough to influence the interpretation placed on the estimates. The differences between population and estimated factor loadings reported by Wilkinson (1989a) ranged in absolute value up to .109. If differences of this magnitude may affect the interpretation placed on estimates (cf. Velicer & Jackson, 1990b), then the differences between component and population loadings reported in Tables 1 and 2 must be considered important, as many of the differences revealed were greater than .11.

The deviations of component loadings from population factor loadings, obtained using Equation 7, are presented in Figure 2 for all three levels of communality and for numbers of indicators per component ranging from 3 to 50. Let us assume that a difference of .05 or larger between the component and population factor loadings might influence the interpretation of a loading, with deviations less than .05 unlikely to affect interpretation. In the high communality population, 5 or more indicators per factor are needed to represent population factor loadings fairly accurately (i.e., differences .05; see Figure 2). But, to achieve this same level of accuracy would require about 11 indicators per component with variables of moderate communality and about 20 indicators per component if the variables had low communality, numbers of indicators per component that are rarely encountered in empirical research.

A second way of characterizing the importance of the deviation of component loadings from population factor loadings is the difference in variance explained by the loadings, provided by Equation 8. Again let us assume that a difference of 5 percent or more in explained variance might affect the interpretation of a loading. For example, this would represent the difference between loadings of .60 and .64, as .60² = .36 and .64² = .41. Differences in the variance explained by component loadings relative to population loadings are presented in Figure 3, for all three levels of
Figure 2
Difference of component loadings from population factor values as a function of level of communality in the population and number of indicators per factor.

Figure 3
Difference in variance explained by component loadings and factor loadings as a function of level of communality in the population and number of indicators per factor.
communality and for numbers of indicators per factor ranging from 3 to 50. Here, to achieve unimportant deviations from population values would require at least 7, 13, and 17 indicators per component in the high, moderate, and low communality populations, respectively. Once again, these are numbers of indicators per component, at least for the moderate and low communality conditions, that are almost never encountered in practice.

The proportional difference between component and population loadings, given by Equation 9, and the proportional difference in variance explained by component and population loadings, from Equation 10, may also be used to characterize the deviations between component and factor pattern loadings. Here, let us say that interpretation of a component loading would be unaffected if it deviated less than 10 percent from its population loading (e.g., if the population loading was .60, and a component loading fell between .54 and .66). The proportional differences between component and population loadings are given in Figure 4. To ensure that component loadings deviate from population factor loadings by no more than 10 percent requires only 3 indicators per component in the high communality population, but at least 9 and 25 indicators per component in the moderate and low communality populations, respectively.

Figure 4
Proportional difference of component loadings from population factor values as a function of level of communality in the population and number of indicators per factor.
For the proportional difference in variance explained, let us again assume that an increase of 10 percent or more in variance explained by a component loading relative to its corresponding population loading might influence interpretation. The proportional differences in variance explained as a function of level of communality and number of indicators per component are presented in Figure 5. Unimportant deviations in explained variance would require at least 6, 18, and over 50 indicators per component in the high, moderate, and low communality populations, respectively.

In summary, it appears that component loadings may be rather poor estimators of population loadings in many situations that arise in practice, deviating sufficiently from population values to influence the interpretation given the results. Although there were some differences across Figures 2-5, approximately 3 to 7 indicators per component are required to provide fairly accurate estimates of population factor loadings when communalities are high, but 10 to 18 indicators per component are needed if communalities are moderate, and 20 to 50 or more indicators per component if communalities are low. Although there are notable exceptions in the research literature, empirical investigations rarely result in more than 3 to 5 high loadings per component, conditions that can lead to substantial differences between represented component loadings and population factor loadings.

Figure 5
Proportional difference in variance explained by component loadings and factor loadings as a function of level of communality in the population and number of indicators per factor.
Varied Pattern Loadings on Orthogonal Factors

The preceding analyses used population data having equal loadings on orthogonal factors; relaxing these restrictions may require a modification of the trends observed in results. The first effect considered was one in which the loadings on each population factor were allowed to vary in magnitude, but the factors were constrained to orthogonality. One example from this condition is presented in Table 3. In the left half of Table 3, the population factor pattern is shown. Factor loadings on each factor varied widely, with one loading of .8, one of .6, and one of .4. A common factor analysis of the population data, using iterated communality estimates and varimax rotation, reproduced the population values presented in Table 3, resulting in accurate representation of the population parameters that generated the data.

The results of a component analysis of the data, followed by varimax rotation, are presented in the right half of Table 3; three major trends should be noted. First, the relative magnitudes of the nonzero loadings on each factor preserve the ranking of the magnitudes of the population values for

<table>
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<th>Population Values</th>
<th>Rotated Components</th>
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<td>Components</td>
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<td>1     2</td>
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<tr>
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<td>U^2</td>
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<tr>
<td>Variable 6</td>
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<td>.000  .642  .588</td>
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Intercorrelations of dimensions

| Dimension 1 | 1.00 | 1.00 |
| Dimension 2 | 0.00 | 1.00 |

Table 3
Population Parameters for Common Factor Model With Varied Loadings on Orthogonal Factors And Results of Varimax Rotation of Two Components
which they are estimators. Second, the loadings are inflated over the population values, but the inflation is not identical to that obtained when population values on each factor were identical. Referring to Table 2, identical population loadings of .8, .6, and .4 led, respectively, to component loadings of .872, .757, and .663, a range of about .21 from highest to lowest. These values are not observed in Table 3, as (a) the highest and lowest defining loadings on each factor were somewhat lower, (b) the middle loading was somewhat higher, and (c) the range from highest to lowest of the defining loadings on each factor was somewhat less, .18, than when population loadings on each factor were identical. This effect may be termed a "homogenization" of component loadings, as the varied component loadings are more similar to one another in magnitude than are (a) the common factor values of which they are poor estimators or (b) their component estimates under equal loadings on each factor. Third, the zero population loadings were represented at precisely zero. These analyses support the conclusion that variation in loadings on each of two or more orthogonal factors leads to a small moderation of the overrepresentation of nonzero loadings by component analysis, but does not affect represented values of zero loadings.

*Equal Pattern Loadings on Oblique Factors*

Next, models having identical loadings on oblique factors were considered. Analyses were performed for the high, medium, and low levels of communality used in preceding analyses (i.e., loadings of .8, .6, and .4, respectively), but only results for the high and low levels are presented here. As shown in the top portion of Table 4, each population factor had identical pattern loadings, and the factor intercorrelations were fixed at .50 for both high and low levels of communality. Common factor analyses of the population correlation matrices, using iterated estimates of communality and orthoblique rotation, led to precise reproduction of the population values, presented in the top half of Table 4. In addition, it is possible to show that the .50 correlations obtained in the orthoblique rotations for both high and low communality data are identical to the .50 interfactor indicator correlations among extended vectors for the high and low communality data based on use of Equations 13 and 15.1

Component analyses of the high and low communality data, followed by orthoblique rotations of two components, were performed; these led to

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1 A demonstration of the application of the equations presented in this article for the estimation of the proper degree of correlation of rotated factors is available from the author by request.
Table 4
Common Factor Model with Identical Loadings on Oblique Factors Under High and Low Levels of Communality: Population Parameters And Results of Common Factor and Component Analyses

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Population parameters

Factor pattern

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Factor intercorrelations

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Results of component analysis

Component pattern

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Component intercorrelations

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MULTIVARIATE BEHAVIORAL RESEARCH
K. Widaman

Tabel 4 (cont.)

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Results of common factor analysis, with SMCs as communality estimates

Factor intercorrelations

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</table>

precisely the same estimates of nonzero component loadings as reported in Table 2 for population matrices with equal loadings on orthogonal factors. In addition, the loadings that were zero in the population were represented at precisely zero in these analyses. These results are particularly important, as Bentler and Kano (1990) concluded that it was difficult to demonstrate analytically the generalizability of formulae, such as Equation 6, to populations with oblique factors. The present analyses support the generalizability of Equation 6 to component analyses in populations with oblique factors if the nonzero population loadings on each factor are equal.

However, it is important to note the low intercorrelations between the components, and the dependence of these correlations on the level of communality of the indicators. For the high communality data, the correlation between the components was .421, somewhat lower than the .50 population value; for the low communality data, the correlation was .182, a drastically lower value. Although one might question the optimality of the obliquely rotated components, the perfectly defined hyperplanes for the
factors suggest that the high and low communality data were rotated optimally. Further, it is possible to show that the interfactor indicator correlations among extended vectors for the high communality data were all .421, precisely the value obtained for the correlation between rotated components; for the low communality data, the interfactor indicator correlations among extended vectors were all .182, once again the value obtained in the oblique rotation of two components.\(^1\) Thus, the basis for the lowered population correlations between oblique components resides in the manner in which the component model represents relations among the observed variables (see Equations 6, 14, and 16).

The influence of component analysis on the represented correlation between rotated dimensions is shown in Figure 6 (next page) for all three levels of communality and for numbers of indicators per factor ranging from 3 to 50, for a population with a correlation of .50 between optimally rotated common factors. Let us assume that a deviation of .05 or more from the population correlation of .50 would be cause for concern; a represented correlation of .45 or less reflects a reduction of 10 percent or more in the magnitude of the correlation and a reduction of approximately 20 percent or more in the shared variance between factors (i.e., \(.45^2/.50^2 = .81\)). For the represented correlation to deviate .05 or less from the population value of .50, about 5 indicators per component would be required in the high communality condition, but approximately 16 and 47 indicators in the moderate and low communality conditions, respectively, as shown in Figure 6. As with preceding analyses of pattern loadings, the numbers of indicators per component required to achieve fairly accurate representation of the population correlation are rarely encountered in practice, especially for the moderate and low levels of communality.

The results of common factor analyses with SMCs as communality estimates are presented in the bottom section of Table 4. These results exhibit two interesting and important trends. First, with regard to pattern loadings, the obtained values were quite similar to those found for identical loadings on orthogonal factors, .772 (versus .770) for the high communality condition and .352 (versus .348) for the low communality condition. This contrasts with results for component analyses of the data, which resulted in identical pattern loadings across the orthogonal and oblique populations (as discussed above). The explanation for this difference resides in the communality estimates (i.e., diagonal values) employed in analyses. For component analyses, diagonal values of unity were employed in both the orthogonal and oblique populations. However, for the common factor analyses, the SMC estimates were, as expected, slightly higher in the oblique population than in the orthogonal population, resulting in slightly
higher pattern loadings in analyses of the oblique population data relative to the orthogonal population data.

Second, as hypothesized on the basis of Equations 6 and 13, underrepresentation of pattern loadings led to corresponding overrepresentation of factor correlations. In the high communality population, the pattern loadings were slightly underrepresented; the obtained values of .772 explained about 93 percent of the variance in each variable relative to the population loading of .80 (i.e., $\lambda^2/\omega^2 = .93$). As a result, the correlation between the two factors was slightly overrepresented, $r = .537$. On the other hand, in the low communality population, pattern loadings were more poorly represented, as the obtained loadings of .352 explained only about 77 percent of the variance reflected by the population loadings of .4 (i.e., $\lambda^2/\omega^2 = .77$). The result was a correspondingly stronger overrepresentation of the correlation between rotated factors, with $r = .645$. The correlations between obliquely rotated factors for the high and low communality data were equal to the interfactor indicator correlations.
among extended vectors, obtained using Equations 13 and 15, for the two data sets, respectively.

The preceding findings, revealing clear interdependence among estimates of pattern loadings and correlations among rotated dimensions, are important contributions of the approach taken in the present article. In prior writings (e.g., Bentler & Kano, 1990), interest centered on the relation between component and common factor pattern loadings, with essentially no consideration of the effects of such estimates on the represented correlations among obliquely rotated dimensions. Here, I argued that Equations 13, 14, 15, and 16 imply that (a) component analysis will lead consistently to underrepresentation of the correlation between oblique dimensions, and (b) common factor analysis will lead to under- or overrepresentation of correlations between oblique dimensions to the extent that communalities are over- or underestimated, respectively. These implications were supported in analyses of population data, demonstrating the dependence of all component and common factor parameters on the diagonal values (i.e., communality estimates) used in analyses.

*Varied Pattern Loadings on Oblique Factors*

As a final condition, the population factors were allowed to be oblique and to have varied loadings on each factor. To demonstrate the influence of unequal loadings on parameters represented by component analysis in an oblique factor model, two population models were proposed, and these are shown in Table 5 (next page). In the upper left corner of Table 5, a population model is shown that had equal .6 loadings on two factors that correlate .5. In the lower left corner of Table 5, a population model is presented that has unequal loadings on oblique factors correlating .5; the unequal loadings are widely spaced and average .6, the same average as in the equal loading population. Common factor analyses with iterated communalities were performed on both sets of population data. In each case, common factor analysis resulted in perfect reproduction of the factor loadings and factor intercorrelations, presented in the left half of Table 5.

The orthoblique rotations of two components for the two conditions are presented in the right half of Table 5. In the equal loading condition shown in the upper right corner of Table 5, the nonzero loadings were represented as identical .757 values, the same value obtained with orthogonal factors (cf. Table 2). The nondefining loadings were represented at exactly zero, their population value. Also, the components were correlated .314, an underestimate of the population value of .5, but identical to the .314
Table 5
Comparison of Common Factor Models with Equal and Varied Loadings on Oblique Factors: Population Parameters and Results of Component Analyses

<table>
<thead>
<tr>
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<th>Population Parameters</th>
<th>Oblique Rotation of Two Components</th>
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</thead>
<tbody>
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<td></td>
<td>Factors</td>
<td>Components</td>
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<td></td>
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<tr>
<td>Dimension 2</td>
<td>.500</td>
<td>1.000</td>
</tr>
</tbody>
</table>
interfactor indicator correlations among extended vectors from the component solution.

The oblique component solution for the unequal loading condition is presented in the lower right corner of Table 5, and it reveals three major contrasts with the equal loading solution. First, the pattern loadings represented by component analysis are inconsistent with those found under populations with identical loadings. As shown in Tables 4 and 5, oblique factors with identical population loadings of .8, .6, or .4 lead to component loadings of .872, .757, or .663, respectively, the values obtained for these conditions with orthogonal factors (cf. Table 2). However, when the oblique factors have one loading each of .8, .6, and .4, the component loadings are represented as .771, .769, and .709, respectively. This represents an underrepresentation of the largest loading and a greater overrepresentation of the lowest loading relative to equal loading conditions. Second, there was a clear homogenization of the component loadings. The population factor loadings had a large range in value (.4 from highest to lowest) and a rather large 4 : 1 ratio between the highest and lowest loadings in explained variance (.8² : .4²). In contrast, the component pattern loadings had a rather narrow range of .062 from highest to lowest, and the approximate ratio of explained variance was less than 1.2 : 1 (.771² : .709²).

Third, and more important in the present context, the nondefining factor loadings, which were zero in the population, are represented as nonzero values in the unequal loading data. Of course, it would be possible to ensure one loading of precisely zero on each component; for example, if Component 1 were passed directly along the vector for Variable 2, the loading of Variable 2 on Component 2 would be reduced precisely to zero (as against its current .004 loading). However, Variables 1 and 3 would still have nonzero loadings on the second component. Moreover, in contrast to the population parameters that generated the data, there is no rotation in the two-dimensional component space that results in three loadings of precisely zero on each dimension. Thus, we must conclude that, in population common factor data with unequal loadings on oblique factors, the population parameters defined by component analysis will be inaccurate estimates of common factor defining loadings, common factor nondefining loadings, and intercorrelations among common factors; in short, component analysis

---

2 The ratio of variance explained by the highest and lowest loadings in the oblique component solution is only an approximation because (a) the components are correlated, and (b) the variables load at a nonzero level on both components. The 1.18 : 1 ratio of variance explained may, however, be interpreted accurately as a ratio of the direct effects of the given component on the highest and lowest loading indicators, respectively, holding constant the other component.
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will lead to differential representations of all parameters associated with the oblique common factor model.

Other Factors Affecting the Differential Representation of Parameters By Common Factor Analysis and Component Analysis

In the preceding sections, differences in the pattern loadings and intercorrelations among dimensions defined by component analysis and common factor analysis were investigated in a number of systematically varied conditions. Yet, two characteristics of the comparisons made in the preceding conditions were held constant: (a) each factor in a given model had an equal number of indicators, and (b) comparisons were made between data sets, rather than within data sets. In this section, the effects of relaxing these constraints are considered.

Effects of Differential Numbers of Indicators per Factor

In the left section of Table 6, a population factor model is presented. The population model has three factors and 13 observed variables. Factor 1 has six indicators, Factor 2 has four indicators, and Factor 3 has three indicators. On each factor, the indicators have loadings that vary between .8 and .4, and the nonzero loadings average .6 on all factors. The nondefining loadings are precisely zero on all factors, and the three factors correlate .5. The population correlation matrix was a 13 × 13 matrix. Two matrices were subjected to common factor and component analyses: (a) the matrix of correlations among the first nine variables, in which each factor had an equal number of indicators that had equal distribution of population loadings, and (b) the full 13 × 13 matrix of correlations among all variables, in which factors had different numbers of indicators, although the mean population loading and range of population loadings were comparable across factors.

The common factor analysis of the 9 × 9 matrix resulted in a perfect reproduction of the population values. Each factor had three nonzero loadings, one each at .8, .6, and .4; the nondefining loadings were precisely zero; and the factor intercorrelations were a uniform .5. The common factor analysis of the 13 × 13 matrix also resulted in a perfect reproduction of the population values given in Table 6, with defining loadings represented at their respective population values, nondefining loadings represented at precisely zero, and factor intercorrelations represented at .50. Thus, using common factor analysis, the results were identical for analysis of the 9 × 9 matrix as for the 13 × 13 matrix; in both analyses, the population parameters were perfectly reproduced.
Table 6
Context Effects on Population Parameters Defined by Common Factor Analysis and Component Analysis: Effects of Number of Indicators Per Factor

<table>
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Intercorrelations of dimensions

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The results of the component analysis of the 9 × 9 matrix are presented in the middle section of Table 6. Inspection of Table 6 reveals that the pattern loadings and component intercorrelations defined are identical for each of the three components. On each component, the indicator with the highest population loading had a component loading that was lower than the
population value, whereas the indicators with the two smaller nonzero population loadings (i.e., .6 and .4) had component loadings that were rather larger than the population values (i.e., .766 and .742). Although two of the six nondefining loadings on each factor were represented at their population values of zero, the remaining four nondefining loadings departed substantially from zero. Finally, there was a comparable, moderately large reduction in all component intercorrelations, which reflected an almost 60 percent reduction in the shared variance between pairs of dimensions (.327^2 = .107) relative to the population value (.5^2 = .25).

The results of the component analysis of the 13 × 13 matrix are shown in the right section of Table 6. One important point to note is this: the trends in over- and underrepresentation of pattern loadings and component intercorrelations exhibited by the nine variables included in the component analysis of the 9 × 9 matrix were differentially moderated by the inclusion of an unequal number of additional indicators for the same population factors. That is, the defining loadings of the first three variables on the first component were rather closer to their population common factor values than was the case in the analysis of the 9 × 9 matrix, and this was due to the inclusion of three additional indicators for the first component. The defining loadings for Variables 4, 5, and 6 on the second component were somewhat more accurate than in the 9 × 9 analysis, due to inclusion of a single additional indicator; and the defining loadings of Variables 7, 8, and 9 on the third component were approximately as inaccurate as those from the 9 × 9 analysis, as no additional indicators were added for the third component.

The second important aspect of this demonstration is the form of the improved accuracy in defining and nondefining component loadings. Here, the improved accuracy in the defining loadings of indicators on one component is accompanied by improved accuracy of the nondefining loadings for those same indicators on the remaining components. That is, in terms of improved accuracy of defining loadings, Component 1 was most affected; more indicators for Component 1 were added in the 13 × 13 analysis, and the defining loadings for its indicators were the most accurate. But, increased accuracy of the defining loadings on Component 1 did not lead to increased accuracy of the nondefining loadings on that component. Instead, the nondefining loadings for indicators of Component 1 on Components 2 and 3 exhibited increased accuracy. So, the greater the number of indicators for a factor, the more accurate are the component loadings for those indicators across all components. In comparison, the indicators for Component 3 had the least accurate defining loadings, and their nondefining loadings on Components 1 and 2 were also the least
accurate of the nondefining loadings. Interestingly, by some criteria, Component 3 exhibited the best simple structure (i.e., the nondefining loadings on Component 3 tended to be closer to their population values of zero than was the case for Components 1 and 2).

It is unsurprising that accuracy in component intercorrelations was systematically related to accuracy in pattern loadings. For example, Components 1 and 2 had the largest numbers of indicators and most accurate nonzero pattern loadings and also exhibited the most accurate intercorrelation of any pair of components in the 13-variable component analysis.

*Effects of Moving a Variable from One Battery to Another*

The final condition examined in this article involves the effects on the factorial description of a variable when that variable is moved from one battery of measures to another. Thurstone (1947) stated that a “fundamental criterion of a valid method” for representing common factors is that the factorial description of a test, or the loadings of a test on a set of common factors, “must remain invariant when [the test] is moved from one battery to another which involves the same common factors” (p. 361).

To investigate this problem, a population factor model was specified, as shown in the left section of Table 7 (next page). The population factor pattern matrix had two common factors, each with five nonzero loadings. The five nonzero loadings on each factor consisted of two .8 loadings, a single .6 loading, and two .4 loadings. The population factors were correlated .5. The population correlation matrix was a $10 \times 10$ matrix. Two sets of analyses were performed on this matrix. First, analyses were performed on the correlations among Variables 1, 2, 3, 6, 7, and 8; these were termed analyses of a high communality battery of measures, as the population values for the defining loadings on each factor were .8, .8, and .6. Second, analyses were performed on the correlations among Variables 3, 4, 5, 8, 9, and 10; these were termed analyses of a low communality battery, because the population values for the defining loadings on each factor were .6, .4, and .4. The most crucial outcome of these analyses is the comparison across high and low communality battery analyses of the factorial description of Variables 3 and 8. By the specification of the population model, precisely the same common factors are present in the high communality and low communality batteries. Thus, the factorial descriptions of Variables 3 and 8 should be identical whether these variables are included in batteries with other indicators having higher or lower levels of communality.
Common factor analyses, using iterated communalities and orthoblique rotation, were performed on both the high communality battery and the low communality battery. The factor analysis of the high communality battery resulted in perfect reproduction of the population values. On both factors, the defining loadings consisted of two .8 loadings and a single .6 loading; the nondefining loadings were exactly zero; and the factors correlated .5. The common factor analysis of the low communality battery also perfectly reproduced the population values: defining loadings on each factor of .6, .4, and .4; nondefining loadings of exactly zero; and a factor intercorrelation of .5. More importantly, the factorial descriptions of Variables 3 and 8 were invariant across the two analyses. Variable 3 loaded .6 on Factor 1 and .0 on Factor 2, and Variable 8 showed the reversed pattern of loadings, regardless of whether these two variables were analyzed within batteries.
having higher or lower levels of communality. Thus, the common factor analyses satisfied Thurstone’s (1947) criterion for a valid method of isolating common factors.

The component analyses of the high and low communality batteries offer a contrast to the common factor analyses of these batteries. The component analysis of the high communality battery is presented in the middle section of Table 7. All defining loadings were larger than the population values. Moreover, there was a clear homogenization of the component loadings, with a range of only .033 from highest to lowest defining loading as opposed to a .2 range in their population values. The lowest defining loading, .808, was more highly overrepresented than under conditions with equal population loadings of .6 per factor, which led to loadings of .757, as shown in Table 2.

The results of the component analysis of the low communality battery are presented in the right section of Table 7. Here, the homogenization of the component loadings resulted again in rather similar and overrepresented defining loadings. But, analysis of the low communality battery led to a lower level of overrepresentation in the defining loadings for the two variables with .6 population values, with loadings of .702, than under conditions with identical .6 loadings on each factor (i.e., .757, as shown in Table 2).

Many moderated patterns of representing defining and nondefining component pattern loadings and in the component intercorrelations presented in Table 7 could be discussed. However, the central issue addressed in the present section is the invariance of the factorial description of Variables 3 and 8 across the analyses of the high and low communality batteries. The direct effect of Factor 1 on Variable 3 in the population is .6 (representing 36 percent of the variance in Variable 3 explained by the factor). The .6 direct effect was reproduced precisely in the common factor analyses of both the high and low communality batteries, exhibiting precise invariance of the factorial description of Variable 3. Identical effects with regard to Factor 2 were observed for Variable 8. In contrast, the direct effect of Component 1 on Variable 3 represented in the component analysis of the high communality battery was .808, reflecting 65 percent of the variance explained by the direct effect; the parallel direct effect from the component analysis of the low communality battery was .702, representing 49 percent of the variance explained. Identical effects for Component 2 and Variable 8 were observed. The overrepresentation of the component loadings, and the direct effects they entail, was not surprising. However, more damaging was the lack of invariance of the weightings of Variables 3 and 8 across the component analyses of the high and low communality batteries. The lack of invariance
of component weights represented from error-free population data demonstrates that component analysis fails to meet Thurstone's fundamental criterion for a valid method of representing common factors.

**Discussion**

The primary aim of the present article was to investigate further several issues reviewed by Velicer and Jackson (1990a). Across conditions, differences between population parameters defined by common factor analysis and component analysis were demonstrated. These findings have implications for a number of data analytic and theoretical issues related to the choice of an analytic model (Velicer & Jackson, 1990a), issues that will be considered in turn.

First, it is important to consider the relative applicability of the common factor and component models for representing observed measures used in psychological research. Commonly, researchers state that measured variables having reliabilities in the range from .60 to .85 are adequate for research purposes. Although this may be an appropriate contention, it reflects a general acceptance of proportions of error variance in measures varying from .15 to .40. In addition to error variance, the unique variance of a particular measured variable represented by the common factor model includes specific variance, which is reliable variance of the variable that is not shared with the latent variables in the analysis. As a result, it is not uncommon to find that a common factor analysis explains from 30 to 50 percent of the total variance of a set of variables, and at times even less than 30 percent of the variance is captured by a common factor model. The common factor model was developed to represent data with these varying degrees of unique variance often encountered with empirical data. As unique variance is not represented explicitly in the component model, the component model should be used to estimate parameters in factor analytic models only if there is good evidence that model parameters represented by component analysis approximate those from common factor analysis.

Before turning to the results presented in the article and their implications, three aspects of the data and analyses employed are important to note. First, some researchers may consider population pattern matrices having congeneric patterns of loadings to be unlikely candidates for population matrices representing empirical data in any given domain (cf. MacCallum & Tucker, 1991). But, such population matrices were employed in this study to maintain comparability with several recent comparisons of common factor and component analysis (e.g., Snook & Gorsuch, 1989; Velicer et al., 1982) that used similar population matrices and to enable simpler comparisons of
results produced by common factor analysis and component analysis. Second, the use of a moderate correlation of .50 between factors in situations with oblique factors is not a limitation. In many studies of the ability domain (e.g., Cattell, 1963, 1971; Horn, 1988; Thurstone, 1938; Thurstone & Thurstone, 1941), correlations between factors of .50 and above frequently occur. Third, the analysis of population data is less common than are analyses of simulated sample data and may seem to require justification. But, analyses of population data were performed to bolster certain points of great theoretical interest, and analyses of population data have been used in other recent articles (e.g., MacCallum & Tucker, 1991). Moreover, estimation of parameters in common factor and component models is a rather complex undertaking, and analytic expressions for differences in parameters for many interesting cases are not available. But, if a model parameter is a particular, specified value in the population, yet is represented as a different value in an analysis of error-free population data, this finding, by itself, is a sufficient demonstration of the inaccurate representation of the parameter, an inaccuracy resulting from the method of analysis. With these points in mind, we now turn to the results reported in the present article and their implications.

In their first two data analytic issues, Velicer and Jackson (1990a) claimed that principal component analysis and common factor analysis led to similar pattern loadings, with differences appearing only when too many components or factors were extracted. However, Snook and Gorsuch (1989) disputed these contentions, demonstrating differences between common factor and component estimates of nonzero pattern loadings in situations in which overextraction of dimensions did not occur. The Snook and Gorsuch findings were closely replicated in the present study, using analyses of population matrices; the results were also consistent with Equation 6, an analytic expression for the pattern loadings under specifiable conditions of communality estimation. The resolution of these competing claims regarding differential representation of parameters centers on the indices of difference used. In Snook and Gorsuch and in the present study, differences in pattern weights were investigated separately for nonzero and zero population loadings. In both studies, differences were documented in nonzero loadings represented by component analysis, whereas population loadings of zero were usually accurately represented. In contrast, Velicer and his associates (e.g., 1977; Velicer et al., 1982; Velicer & Fava, 1987) used a single summary measure that averaged across nonzero and zero loadings. This inadvertently minimized the differences in common factor component pattern loadings, by averaging the affected nonzero loadings with a much
larger number of unaffected zero loadings, a point also noted by Snook and Gorsuch.

A more definitive demonstration of the source of differences between common factor and component pattern loadings was obtained in the present study through the extension of the Snook and Gorsuch (1989) study design. Snook and Gorsuch concluded that differences in pattern loadings were a function of the number of observed variables in the analysis and that results from component analysis and common factor analysis would not diverge importantly once approximately 40 observed variables were included in an analysis; Velicer and Jackson (1990b) agreed in general with this contention. However, the extension of the Snook and Gorsuch study unconfounded the effects of the number of indicators per factor and the number of observed variables in the analysis. The results clearly showed that differences in pattern loadings between common factor analysis and component analysis are a function of the number of indicators per factor, rather than the number of observed variables in the analysis. Thus, the same, substantial differences in nonzero pattern loadings will occur in analyses of 6 observed variables or 96 observed variables if each of the sets of variables consists of factors having only three indicators apiece. Moreover, for the rather restricted conditions considered in this section — an equal number of indicators per factor, indicators having equal population loadings, and orthogonal factors — the differences in common factor and component loadings result directly from the number of indicators per dimension and the deviation of true from estimated communalities. Hence, in these rather restricted conditions, the deviation of common factor and component parameters can be derived analytically, with no need to resort to simulation studies (Bentler & Kano, 1990; Lohmöller, 1989; McArdle, 1990; Widaman, 1990).

In the present study, extending the investigation of differences in model parameters represented by common factor analysis and component analysis to more general cases of population data allowed a more complete characterization of these differences. The most general case considered — which is virtually certain to reflect more accurately the situation confronting researchers when analyzing empirical data — involved population models having varied nonzero loadings on oblique factors. In such populations, several trends were noted. First, there were moderated trends in representing nonzero component pattern loadings; nonzero component loadings were typically rather larger than the population common factor loadings, but there were occasional instances in which they were smaller. Second, there was a clear homogenization of nonzero component loadings, with much smaller differences among the defining loadings represented by component analysis than existed in the population parameters that generated the data.
Third, zero loadings represented by component analysis were also affected, leading to positive or negative deviations about their population factor values of zero. Fourth, there was a consistent lowering of the estimates of component intercorrelations, which was directly related to the elevation of component pattern loadings. Fifth, common factor analyses of data, using iterated communality estimates, led to precise reproduction of nonzero and zero pattern loadings and interfactor correlations. These findings of differential estimates of all types of model parameters are added evidence that component analysis and common factor analysis can lead to rather different numerical representations of data even when the correct number of dimensions is retained for analysis. Taken together, these results from the present study appear to refute the first two conclusions stated by Velicer and Jackson (1990a).

The third issue of Velicer and Jackson (1990b) to which the current results speak concerns the identification of component analysis as a manifest variable technique and common factor analysis as a latent variable technique, a contrast favoring the latter model. Presumably, the parameters of a manifest variable technique are limited to the space of the variables included in the analysis, whereas the parameters of a latent variable technique have generalizability beyond the space of the observed variables. Velicer and Jackson claimed that the purported advantage favoring common factor analysis had not yet been demonstrated empirically. However, at least two sets of results from the present study seem to provide such empirical support. First, the results provided in Table 6 provided a comparison of analyses with equal and unequal numbers of indicators per factor. Common factor analyses of these data led to precise reproduction of all parameters in analyses of both the restricted, 9-variable battery as well as in the extended, 13-variable battery. Thus, parameters defined by common factor analysis generalized perfectly beyond the original 9-variable space to the 13-variable space. Such was not the case with parameters defined by component analysis. Certain component parameters were defined by the analysis of the 9-variable matrix, but different parameters were revealed in the analysis of the 13-variable matrix. Comparing the analysis of the 13-variable space with the 9-variable space, the component with the largest number of additional indicators had defining loadings that were more accurate estimates of the population factor loadings, yet its nondefining loadings were still poor estimates of their population common factor values. In contrast, the component with no additional indicators had defining loadings that were still poor estimates, but nondefining loadings that were rather better estimates of the population factor loadings of zero. In typical research situations in which exploratory common factor or component analyses are
used, it is impossible to predict how many variables will load on each factor that emerges. Hence, it is likely that differing numbers of indicators per factor will arise. In such situations, certain population factor parameters will be fairly accurately represented and others will not if component analysis is used. But, accurate estimates of population factor pattern loadings and factor intercorrelations will accompany the use of common factor analysis.

Second, the results of analyses of high communality and low communality batteries representing the same two common factors also exhibited differential generalizability of parameter estimates (cf. Table 7). For the two observed variables included in both batteries, the pattern loadings obtained from common factor analyses of both batteries were identical. Such was not the case for estimates based on component analysis. Due to the homogenization of nonzero loadings with component analysis, defining pattern loadings for the two variables included in both batteries were represented at relatively higher values in the analysis of the high communality battery than in the analysis of the low communality battery. These findings demonstrate the differential generalizability of parameters defined using the two analytic models, differential results favoring the common factor parameters. The findings with regard to the high and low communality batteries are especially important, providing differential support for common factor analysis with regard to the crucial criterion stated by Thurstone (1947, p. 361) for a "valid method" of isolating common factors.

The results from the present study also have implications for typical analytic practices. In the three major statistical program packages — SPSS, BMDP, and SAS — the default options for estimating common factor or component models are virtually identical: principal axes for estimating model parameters, unities as communality estimates, retention of factors or components with eigenvalues greater than or equal to unity, and then either varimax rotation (SPSS and BMDP) or no rotation (SAS; MacCallum, 1983; SAS Institute, 1985). Thus, the default method of analysis across packages is component analysis, retaining components according to the eigenvalue one rule; these procedures, coupled with varimax rotation, comprise the most commonly used set of procedures in analyses of data in published research. Now, Zwick and Velicer (1982, 1986) showed that the eigenvalue one rule typically suggests a number of components equal to approximately one-third the number of observed variables. This would tend to lead to conditions in which the average component has few defining loadings. But, these conditions — components that each have few defining loadings — are precisely the conditions in the present study in which the differences between the parameters defined by common factor analysis and
component analysis were the greatest. Therefore, results from the present study provide empirical demonstrations against the use of the default options operating in the most commonly used computer program packages.

In summary, the results presented in the present article support three conclusions that are at variance with those presented by Velicer and Jackson (1990a). Specifically, the present results suggest that there are systematic differences, which may be considerable, between parameters defined by component and common factor analysis, that these differences are present even when the correct number of dimensions are retained, and that the parameters defined by common factor analysis appear to be more generalizable than those defined by component analysis. Of course, only certain variations on models and parameters could be considered within the scope of the present study; investigating outcomes across other variations in models and parameters would be a most useful extension of the present study. Both Velicer and Jackson (1990a) and Lohmöller (1989) deemphasized the differences between component and common factor estimates; interestingly, both used examples in which the differences between component and common factor loadings should be relatively small (e.g., population factor loadings of .8, with 6 and 3 indicators, respectively). But, this leaves a very wide array of situations that are likely to be met in practice in which the differences in parameters defined by the two analytic models are sufficiently large to affect significantly the interpretation of results. As an example, only well-identified dimensions, with 3 or more indicators, were examined in the present article; but, component analyses in the published literature often present one or more components each having only one or two high-loading indicators. Consideration of Equations 6, 13, and 14 leads to the conclusion that differences between component and common factor parameter estimates in such cases may be profound.

The remaining four issues discussed by Velicer and Jackson (1990a)— contrasting component and common factor analysis on acceptability of parameter estimates, relative efficiency of parameter estimation, determinacy of scores, and the confirmatory versus exploratory nature of the analyses—are unaffected by the present results. Of these four issues, the indeterminacy of factor scores is probably the most important, has received the most intense interest in the research literature (e.g., Guttman, 1955; McDonald & Mulaik, 1979; Schönemann & Steiger, 1976, 1978; Schönemann & Wang, 1972; Steiger & Schönemann, 1978), and is a most serious concern. But, all four issues should be subjected to additional scrutiny to determine whether the conclusions offered by Velicer and Jackson (1990a) on these issues also require modification. In addition to the preceding issues, future research should determine whether the differences in parameters defined by
common factor analysis and principal component analysis would generalize to other forms of component analysis, such as image component analysis. Other forms of component analysis may yield parameter estimates that approximate those from common factor analysis more closely than is the case for principal component analysis; comparing and contrasting parameters defined by different methods of component analysis would be a useful extension of the present work.

The term bias has been used in recent articles (e.g., Snook & Gorsuch, 1989; Widaman, 1990) to characterize differences between parameters defined by component and common factor analysis; component parameters have been termed biased, common factor parameters unbiased. This is perhaps an unfair use of terms (cf. Velicer & Jackson, 1990b); to remedy this, the term bias has been used sparingly in the present article. Rather, we may say that component analysis and common factor analysis define different parameters in the population, as shown in the present article, and therefore estimate different parameters in samples from that population, as shown by Snook and Gorsuch (1989). Furthermore, the parameters defined by component analysis and common factor analysis differ systematically and, in many situations, nontrivially.

The differences in parameters defined by the two models arise because the two models serve different aims. If the goal of an analysis is data reduction, a researcher is interested only in reducing the number of dimensions in the analysis by obtaining a set of weighted sums of observed variables, warts (i.e., error variance) and all. With this goal, no attempt should be made to interpret the observed variables as manifestations of unitary underlying entities (as there are none), and component analysis is the method of choice. On the other hand, the researcher may wish to characterize patterns of covariation among observed variables in terms of the influence of one or more unitary, latent constructs. To attain this goal, the effects of both error and specific variance on the common factor pattern loadings and factor intercorrelations should be minimized; this is achieved through communality estimation. The common factor parameters representing the patterns of covariation among observed variables then reflect the influence of the latent variables. With the choice put in these terms and considering the types of data typically obtained in psychological and social scientific investigations (e.g., with considerable amounts of error variance), it seems that the prudent researcher should rarely, if ever, opt for a component analysis of empirical data if his/her goal were to interpret the patterns of observed covariation among variables as arising from latent variables or factors.
References


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