
Multidimensional Poverty: How much substitution is allowed?

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Based on a paper with E. Maasoumi

Measuring poverty in 3 steps

1. Choose a well-being indicator

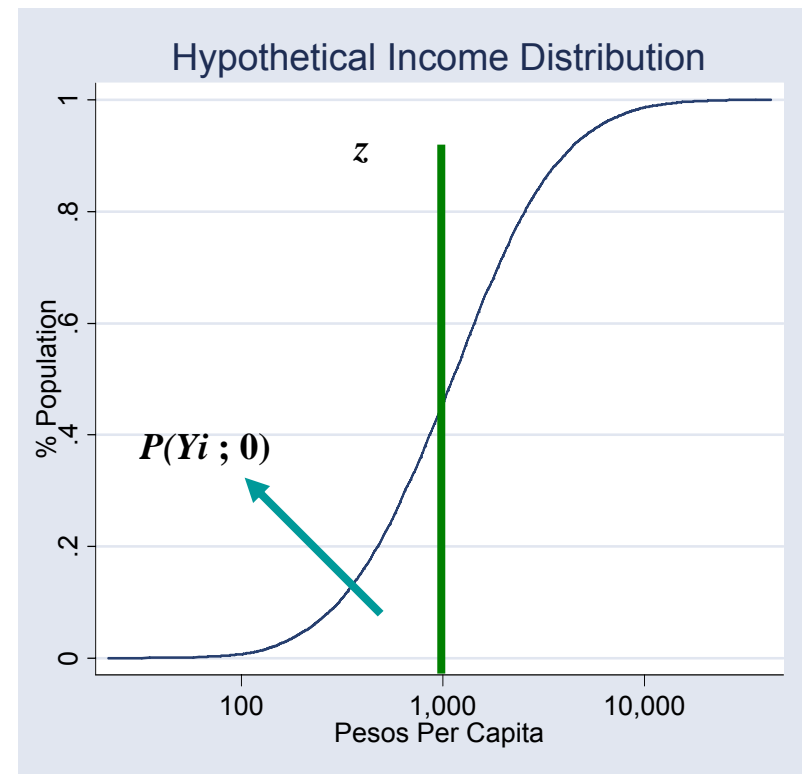
Y_i with distribution $F(Y_i)$

2. Choose poverty line

z poor; non-poor

3. Aggregate individuals

$P(Y_i; 0)$



Monetary approach to poverty

1. Well-being indicator: consumption or income

$$Y_i = \sum_{j=1}^J p_j x_{ij}$$

individual $i = 1, 2, \dots, n$
good $j = 1, 2, \dots, J$
prices p_1, p_2, \dots, p_J

2. Poverty line: value of “basic goods”

$$z = \sum_{j=1}^{J_0} p_j z_{j_0}$$

where z_{j_0} is the minimum quantity of good $j_0 \in J_0$ and $J_0 \subsetneq J$

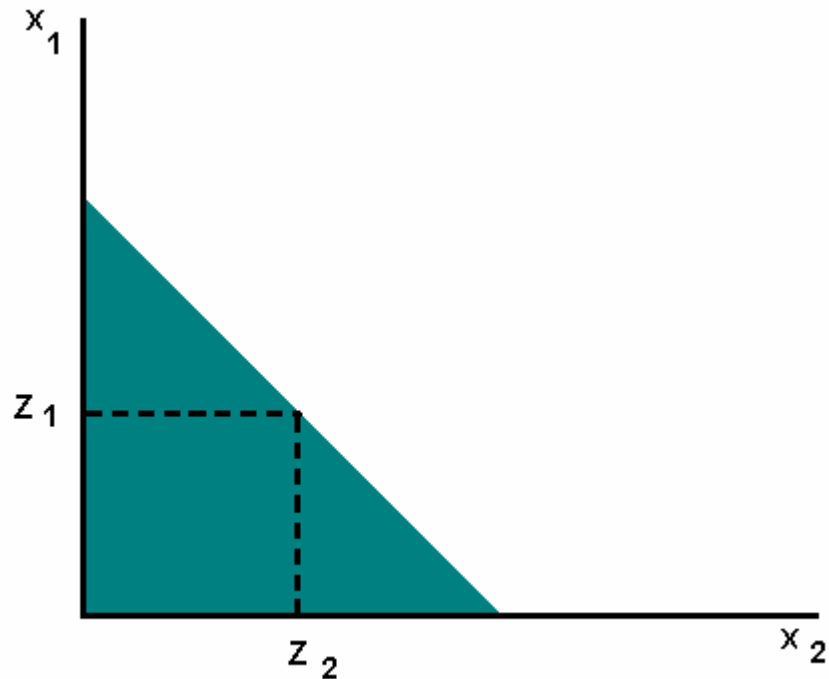
3. Poverty measure:

$$P(Y; z) = FGT_\alpha = \frac{1}{n} \sum_{i=1}^n \left(\max \left(\frac{z - y_i}{z}; 0 \right) \right)^\alpha$$

where each poor contributes to total poverty

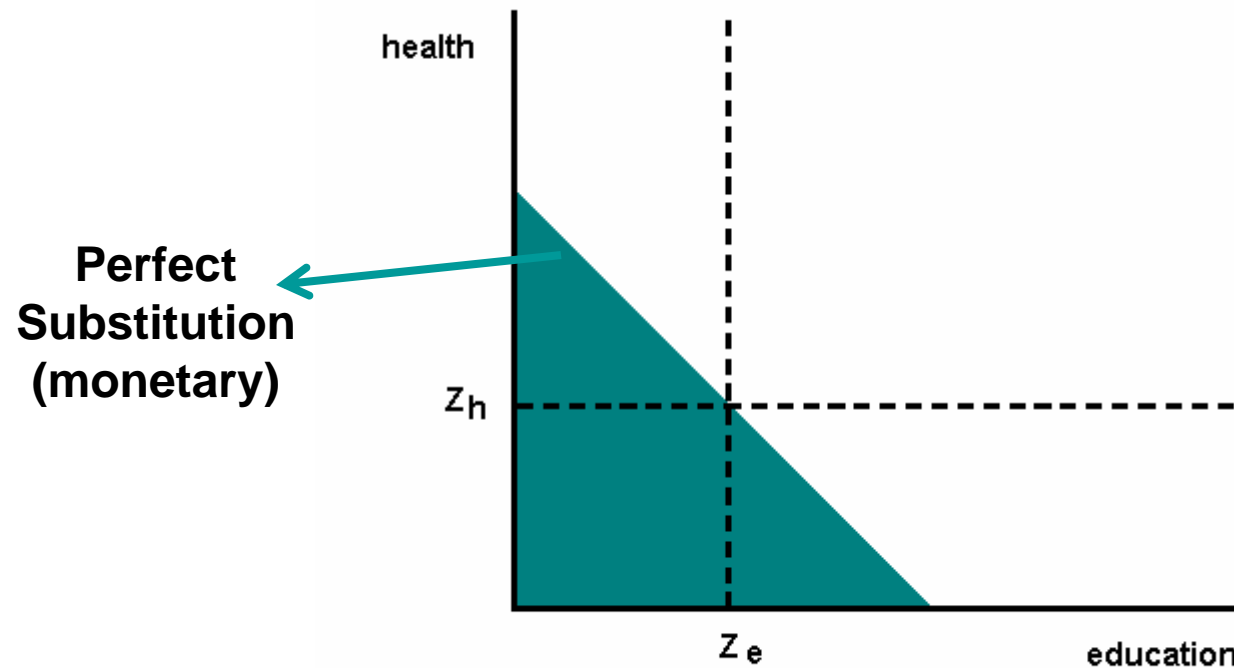
according to his/her relative shortfall from threshold $\left(\frac{z - y_i}{z} \right)^\alpha$

Objections

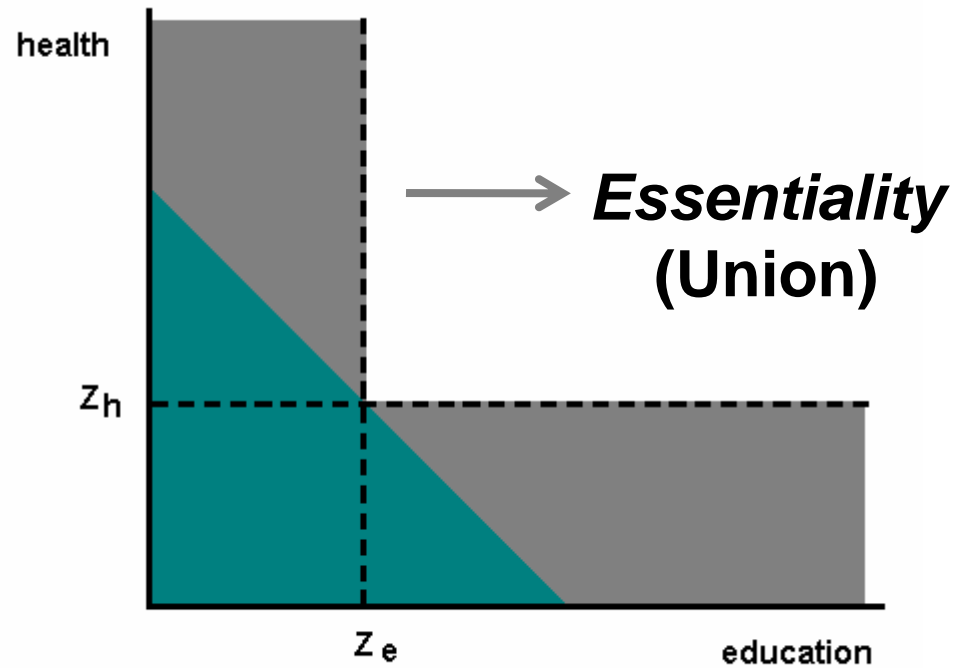


1. There are no markets (and hence prices) for all relevant ‘goods’ in J_0
2. Even if markets exist, prices in the market are not necessarily the best weights (Tsui, 2001)
3. **“Essentiality”**. Ass. perfect substitutability between attributes
Unless ... each attribute is seen as “essential” so that distance to z_j still matters

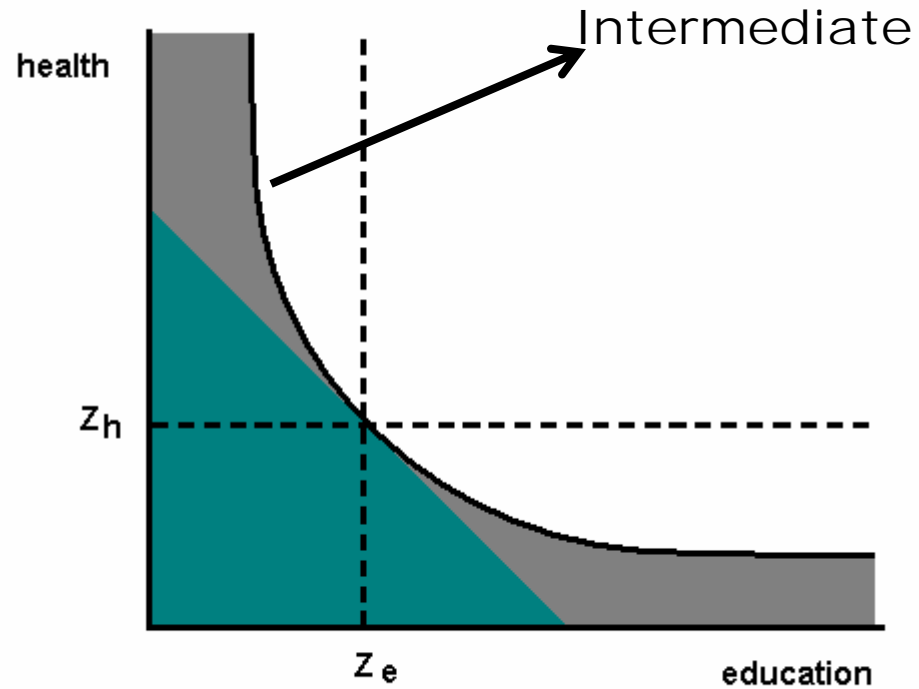
Poverty Approaches



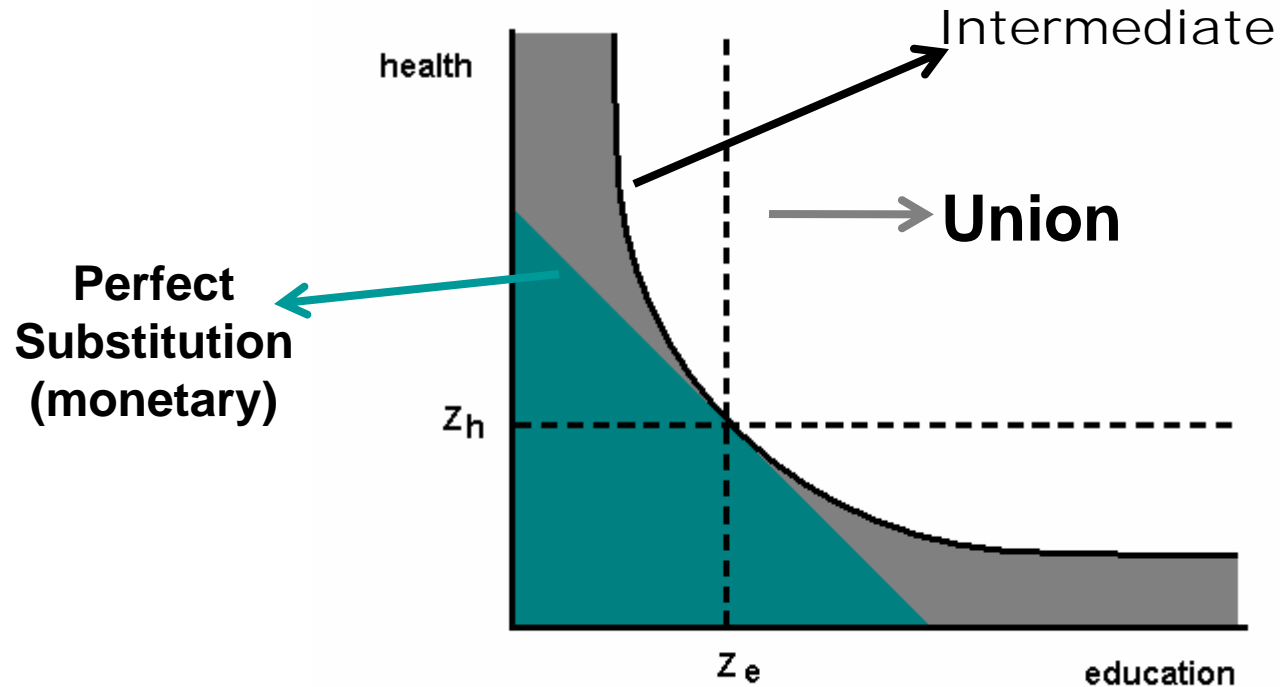
Poverty Approaches



Poverty Approaches



Framework for poverty measure



B. Non-monetary approach

1. **Well-being indicator:**
(from IT)
with $s_{ij} = f(x_{ij})$

$$Y_i = \begin{cases} \left(\sum_{j=1}^J w_j s_{ij}^\beta \right)^{1/\beta} \\ \prod_{j=1}^J s_{ij}^{w_j} & \text{if } \beta = 0 \end{cases}$$

2. **Poverty line:**

- I. Aggregate Poverty Line $\mathbf{z}(\mathbf{s})$... *shortfall of well-being* (Tsui 2002)
- II. Composite Poverty Line (\mathbf{z}_j) ... *'well-being' of shortfalls* (BC 2003)

3. **Poverty measure: FGT**

B. Non-monetary approach

$s_{ij} = f(x_{ij})$ will depend on two factors:

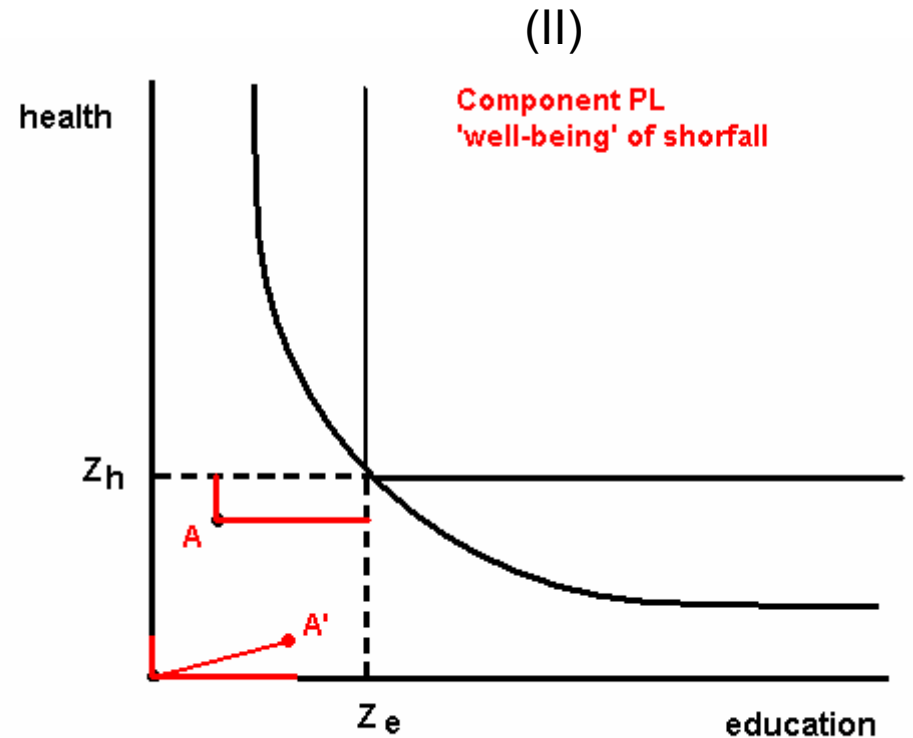
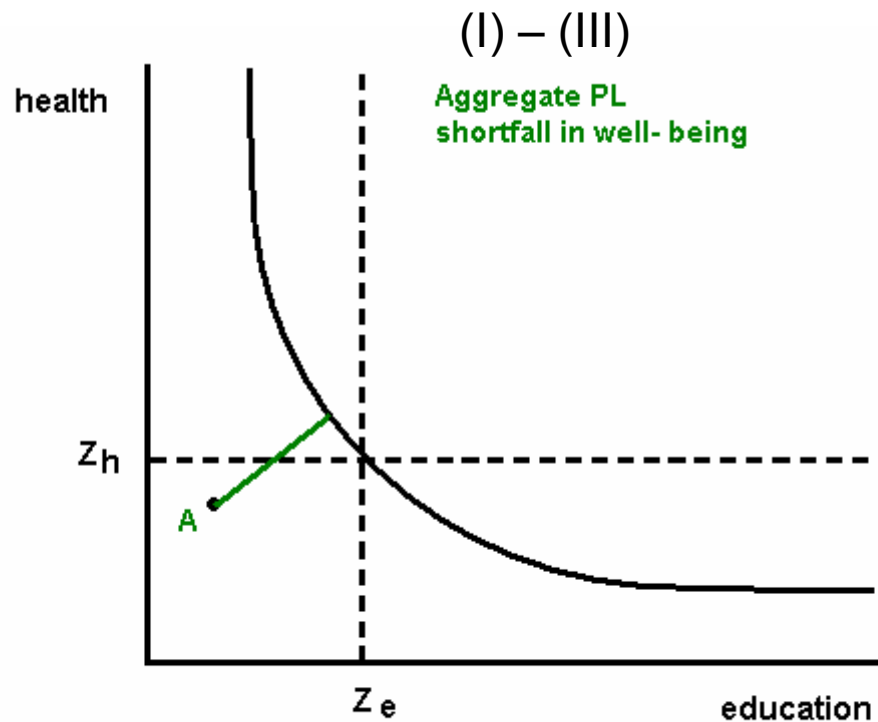
1. Poverty line (aggregate or composite)

2. Poverty focus

- **Strong:** if a person is poor in attribute j but rich in q , receiving more of q does not affect the level of poverty
=> Union a person is poor if he falls short of at least one attribute (no substitution between attributes above and below PL)
- **Weak :** allows for substitution between attributes above and below poverty line.

	Aggregate Poverty line	Composite Poverty Line
Strong	$s_{ij} = \min \{x_{ij}, z_j\}$ (I)	$s_{ij} = (z_j - x_{ij}) / z_j$ (II)
Weak	$s_{ij} = x_{ij}$ (III)	

B. Non-monetary approach

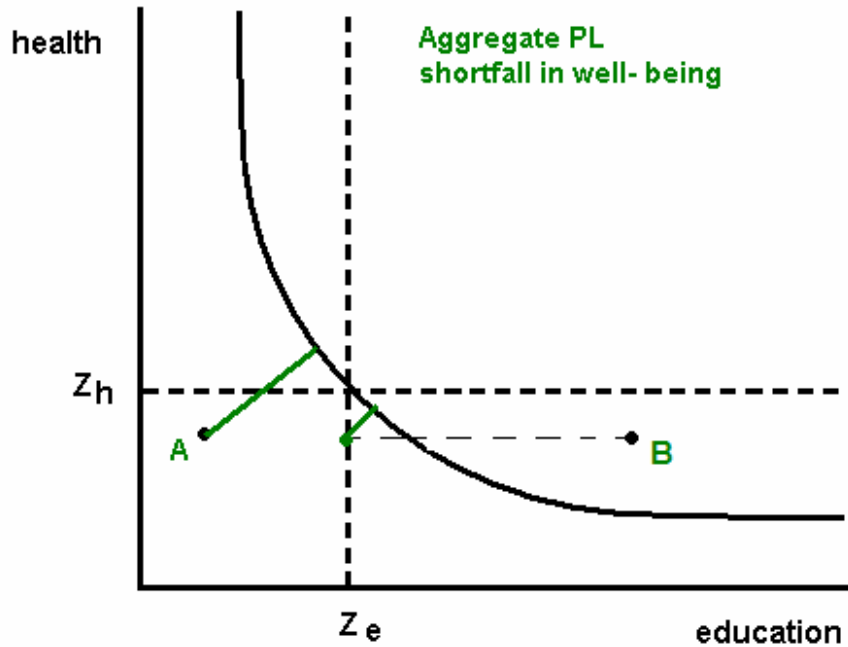


$$p_i = \max\left(\frac{S_z - S_i}{S_z}; 0\right) = \max\left(1 - \frac{\left(\sum_{j=1}^m w_j x_{ij}^\beta\right)^{1/\beta}}{\left(\sum_{j=1}^m w_j z_j^\beta\right)^{1/\beta}}; 0\right)$$

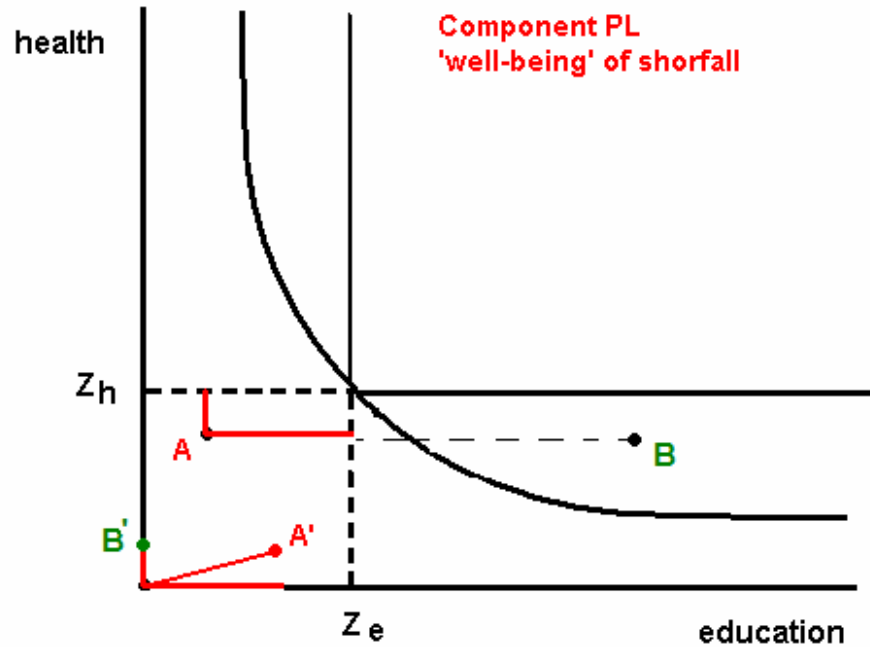
$$p_i = \left(\sum_{j=1}^m w_j \max\left(1 - \frac{x_{ij}}{z_j}; 0\right)^\beta\right)^{1/\beta}$$

B. Non-monetary approach

(I) – (III)



(II)



$$p_i = \max\left(\frac{S_z - S_i}{S_z}; 0\right) = \max\left(1 - \frac{\left(\sum_{j=1}^m w_j x_{ij}^\beta\right)^{1/\beta}}{\left(\sum_{j=1}^m w_j z_j^\beta\right)^{1/\beta}}; 0\right)$$

$$p_i = \left(\sum_{j=1}^m w_j \max\left(1 - \frac{x_{ij}}{z_j}; 0\right)^\beta\right)^{1/\beta}$$

Properties

- **Continuity**: to avoid jumps in the function – technical requirement
- **Symmetry** w.r.t. individuals: the identity of the individuals is not relevant for the evaluation of poverty (characteristics outside X do not matter)
- **Replication Invariance**: the size of the population does not affect the measurement. Necessary for cross-population comparisons
- **Monotonicity**: poverty index does not increase when the condition of the poor improve
- **Subgroup consistency**: if poverty decreases for a subgroup, total poverty cannot increase / **Subgroup Decomposability**
- **Scale Invariance**: poverty index is not sensitive to the unit of measurement

- **Poverty Criteria Invariance**: no dramatic change in the evaluation of poverty for changes in the poverty line z not affecting the number of the poor
- **Poverty Non-increasing Minimal Transfer**: Variation of Pigou-Dalton principle, among poor
- **Poverty-Nondecreasing Rearrangement**: more correlation, higher poverty, lower welfare.

- Poverty focus (strong or weak)