Robustness analysis with the Alkire-Foster measures

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1. The poverty lines of each variable (i.e. the "first" cut-off): z_d.
2. The weights on each variable/dimension: w_d.
3. The threshold that the weighted sum of deprivations need to surpass in order to identify someone as (multidimensionally) poor (i.e. the "second" cut-off): k.

Statistical tools for the evaluation of robustness of rankings for the AF measures

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2. To derive conditions under which an ordering is robust for all lines, weights and multidimensional cut-offs.

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- Stochastic dominance conditions for H and M^0 that ensure robustness for all multidimensional thresholds, weights and lines.
- Some basic robustness tests (applied to weights).

The counting vector: key ingredient for the dominance conditions of H and M0

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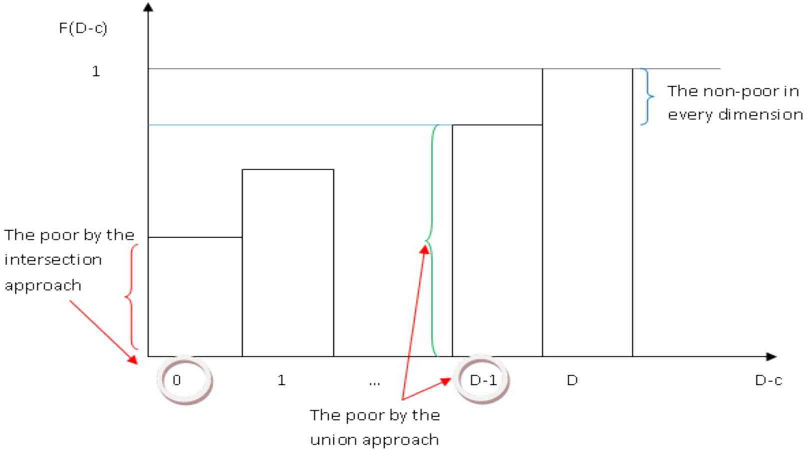
We then consider a distribution of deprivations, $D - c$, in the population, with values ranging from 0 (poor in every variable) to D (non-poor in every variable). A typical cumulative distribution is:

A typical cumulative distribution of D-c



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$$F^A(D-c) \leq F^B(D-c) \forall (D-c) \in [0, D] \leftrightarrow H^A \leq H^B \forall (D-c) \in [0, D]$$

$$H^A \leq H^B \forall (D-c) \in [0, D] \rightarrow M^A \leq M^B \forall (D-c) \in [0, D]$$

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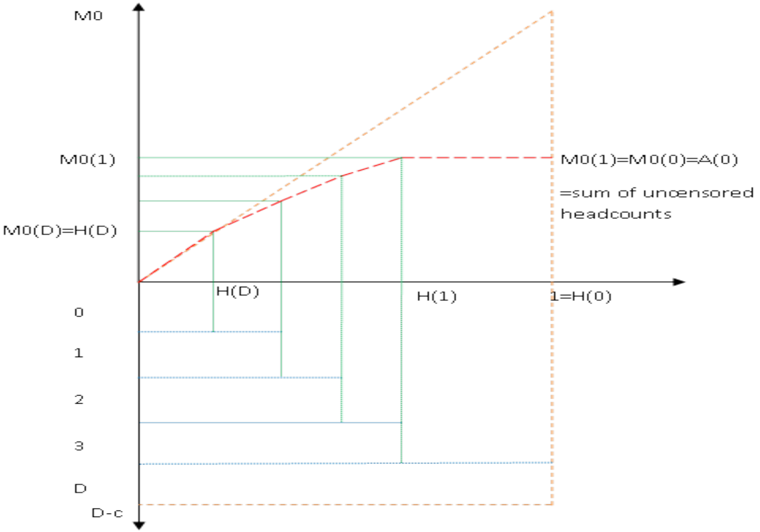

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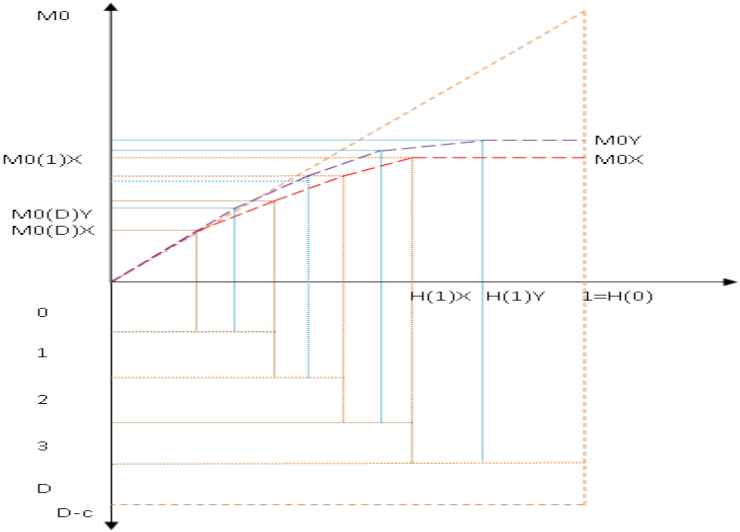

Proof, as explained by Alkire and Foster

Notice that $M0$ can be expressed in terms of H the following way:

$$M0(k) = \frac{1}{D}[H(D)D + \sum_{j=k}^{D-1} j[H(j) - H(j+1)]]$$

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Therefore: $H^A(k) \leq H^B(k) \forall k \in [1, D] \rightarrow M^A(k) \leq M^B(k) \forall k \in [1, D]$

More dominance results: incorporating weights and poverty lines

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- When there are only two variables.
- For any number of variables under extreme identification approaches (union and intersection).

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$$\overline{F^A}(x, y) \geq \overline{F^B}(x, y) \forall (x, y) \in [x_{min}, x_{max}] \times [y_{min}, y_{max}]$$

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When there are several variables and the poor are identified according to the *union* approach:

$$M_A^{\alpha}(X; min(w_d); Z) \leq M_B^{\alpha}(X; min(w_d); Z) \forall \alpha \in \mathbb{R}_0^+$$

$$\forall w_d \in \mathbb{R}^+ \mid \sum_{d=1}^{D} w_d = 1, \forall Z \leftrightarrow F_d^A(x_d) \leq F_d^B(x_d) \forall x_1, ..., x_D$$

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When there are several variables and the poor are identified according to the *intersection* approach:

$$M_A^{\alpha}(X; D; Z) \leq M_B^{\alpha}(X; D; Z) \forall \alpha \in \mathbb{R}_0^+$$

$$\forall w_d \in \mathbb{R}^+ \mid \sum_{d=1}^{D} w_d = 1, \forall Z \leftrightarrow$$

$$F^A(x_1, ..., x_D) \leq F^B(x_1, ..., x_D) \forall x_1, ..., x_D$$

Illustration of the conditions for $H(X; w, k, Z)$

$$A = \begin{array}{ccc} & y_1 & y_2 \\ x_1 & 0.3 & 0.1 \\ x_2 & 0.1 & 0.5 \end{array} \qquad B = \begin{array}{ccc} & y_1 & y_2 \\ x_1 & 0.2 & 0.2 \\ x_2 & 0.2 & 0.4 \end{array} \qquad C = \begin{array}{ccc} & y_1 & y_2 \\ x_1 & 0.2 & 0.15 \\ x_2 & 0.15 & 0.5 \end{array}$$

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Table 1: Multidimensional headcount ratios

Values of c_n	w_x vs w_y	z_x, z_y	H^A	H^B	H^C
w_x	$w_x > w_y$	x_1, y_1	0.4	0.4	0.35
w_x	$w_x \leq w_y$	x_1, y_1	0.5	0.6	0.5
w_x	$w_x > w_y$	x_1, y_2	0.4	0.4	0.35
w_x	$w_x \leq w_y$	x_1, y_2	1	1	1
w_x		x_2, y_1	1	1	1
w_y	$w_y > w_x$	x_1, y_1	0.4	0.4	0.35
w_y	$w_y \leq w_x$	x_1, y_1	0.5	0.6	0.5
w_y		x_1, y_2	1	1	1
w_y	$w_y > w_x$	x_2, y_1	0.4	0.4	0.35
w_y	$w_y \leq w_x$	x_2, y_1	1	1	1
$w_x + w_y$		x_1, y_1	0.3	0.2	0.2
$w_x + w_y$		x_1, y_2	0.4	0.4	0.35
$w_x + w_y$		x_2, y_1	0.4	0.4	0.35

Illustration of the conditions for $H(X; w, k, Z)$: the cumulative and survival distributions

$$F^A = \begin{array}{ccc} & y_1 & y_2 \\ x_1 & 0.3 & 0.4 \\ x_2 & 0.4 & 1 \end{array} ; \quad F^B = \begin{array}{ccc} & y_1 & y_2 \\ x_1 & 0.2 & 0.4 \\ x_2 & 0.4 & 1 \end{array} ; \quad F^C = \begin{array}{ccc} & y_1 & y_2 \\ x_1 & 0.2 & 0.35 \\ x_2 & 0.35 & 1 \end{array}$$

$$\overline{F}^A = \begin{array}{ccc} & y_1 & y_2 \\ x_1 & 1 & 0.6 \\ x_2 & 0.6 & 0.5 \end{array} ; \quad \overline{F}^B = \begin{array}{ccc} & y_1 & y_2 \\ x_1 & 1 & 0.6 \\ x_2 & 0.6 & 0.4 \end{array} ; \quad \overline{F}^C = \begin{array}{ccc} & y_1 & y_2 \\ x_1 & 1 & 0.65 \\ x_2 & 0.65 & 0.5 \end{array}$$

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- What can we say about the degree of robustness of a whole ranking of countries, people, etc. when dominance conditions are not fulfilled for all possible pairs?
- We need other methods to quantify and assess the robustness of the orderings to changes in the parameters.

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In this presentation we will review some of the methods used in the report by Alkire et al. (2010):

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- Maybe the changes are in the weights, w'; poverty lines, z'; different thresholds, k'; even the number of dimensions might change to D'.

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- If $R_n = R'_n$ for all $n = 1, \ldots, N$ then the ordering is completely robust with respect to the alternative specification.
- In general, we want to measure how close two, or more, rankings, stand from a situation of perfect correlation, or concordance, and which classified elements may be repsonsible for any deviation from complete robustness.

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- With these concepts, γ is defined as:

$$\gamma = \frac{C - D}{C + D}$$

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- When the number of concordant pairs is equal to that of discordant pairs: $\gamma = 0$.

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  - When $R_n = R'_{N-n+1}$ for all n: $\rho = -1$.

Illustration: Robustness of weights of the MPI

Table 5: Correlations among MPIs adjusted by Survey

Pair of Rankings Compared	Correlation Coefficient	All Countries	Bottom 75 Countries	DHS Only	MICS Only	DHS & MICS	Non WHS	WHS Only
MPI with 50% weight Education and MPI with 50% weight on Health	Spearman	0.957	0.970	0.980	0.949	0.981	0.979	0.616
	Gamma	0.836	0.854	0.885	0.842	0.883	0.880	0.472
MPI with 50% weight Education and MPI with 50% weight on Living Stan.	Spearman	0.970	0.951	0.946	0.971	0.970	0.972	0.859
	Gamma	0.854	0.805	0.819	0.872	0.856	0.860	0.715
MPI with 50% weight Health and MPI with 50% weight on Living Stan.	Spearman	0.968	0.968	0.965	0.975	0.980	0.978	0.763
	Gamma	0.856	0.854	0.849	0.882	0.881	0.876	0.639
Total Number of Observations		**104**	**75**	**48**	**35**	**83**	**85**	**19**

Concordance indices for multiple rankings

Unlike correlation coefficients, these indices provide an evaluation of the degree of concordance of several rankings at the same time.

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- The index by Joe:

$$JO = \frac{\frac{1}{N}\sum_{n=1}^{N}\prod_{j=1}^{J} R_n^j - (\frac{N+1}{2})^2}{\frac{1}{N}\sum_{n=1}^{N} n^J - (\frac{N+1}{2})^2}$$

Illustration: Robustness of weights of MPI

Table 6: Indicators of rank concordance for all Survey Sources

Index	All Countries	Bottom 75 Countries	DHS Only	MICS Only	DHS & MICS	Non WHS	WHS Only
KF	0.981	0.981	0.981	0.980	0.987	0.987	0.863
K	0.975	0.974	0.975	0.973	0.983	0.983	0.817
J	0.983	0.974	0.968	0.984	0.982	0.983	0.862

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Notice that:

- When $p = 1$ and $J = 2$ rankings get reversed completely $(R_n = R'_{N-n+1})$.
- The maximum effect generated by just one country occurs when it moves from one extreme of the ranks to the other. If that is the only change then: $p = \frac{2}{N}$.